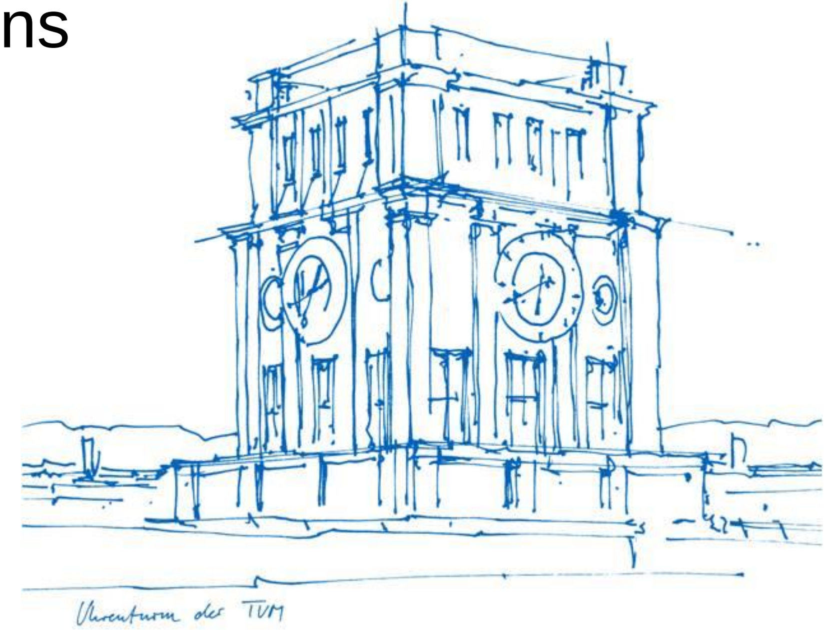
The Resource-Constrained Project Scheduling Problem with Flexible Resource Profiles: Models, Methods, and Applications



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Technical University of Munich

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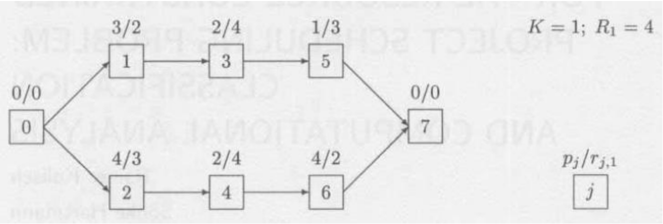
Outline

1. Introduction of the FRCPSP
2. Solution Approaches
   - Comparison of MIPs
   - Hybrid Metaheuristic
3. Instances
4. Applications
   - Lead Optimization in Pharmaceutical Research
   - Engineering-to-Order Production Planning
5. Research Opportunities

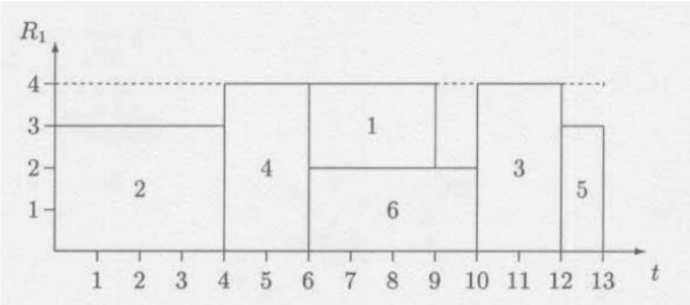
Resource-Constrained Project Scheduling Problem (RCPSP)

Decisions:

Scheduling (start time of activities)



Source: Kolisch and Hartmann (1999)



Flexible RCPSP (FRCPSP)

Decisions:

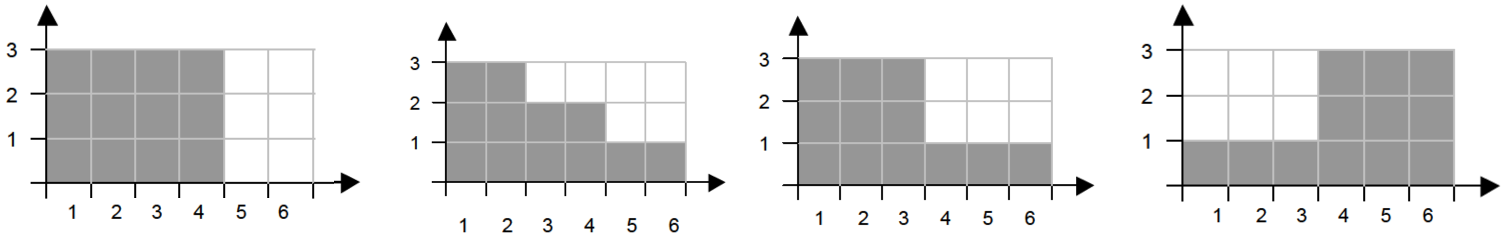
- Scheduling: Start time of activities (RCPSP)

- Resource Allocation and activity demand profile for each activity j:

  Resources have to be allocated to the periods such that $[\underline{u}_j, \overline{u}_j]$

  1) the work content w_j is covered within the time window $[ES_j, LF_j]$
  2) there are no idling periods between the start and the finish period of the activity (no preemption)
  3) if resources are allocated in a period the amount has to be within interval
  4) resource allocation has to be constant for at least l periods except the final periods (minimum block length)

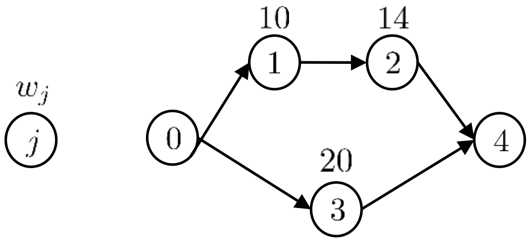
Resource Allocation: Example

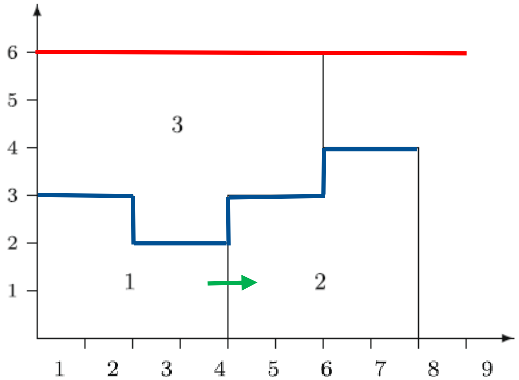


$w = 12$
$l = 2$
$\underline{u} = 1,\ \overline{u} = 3$
$ES = 1, LF = 6$

Decisions of the FRCPSP

Resource allocation and scheduling subject to precedence and resource constraints.





Source: Fündeling and Trautmann (2010)

$l = 2$
$\underline{u} = 2,\ \overline{u} = 4$

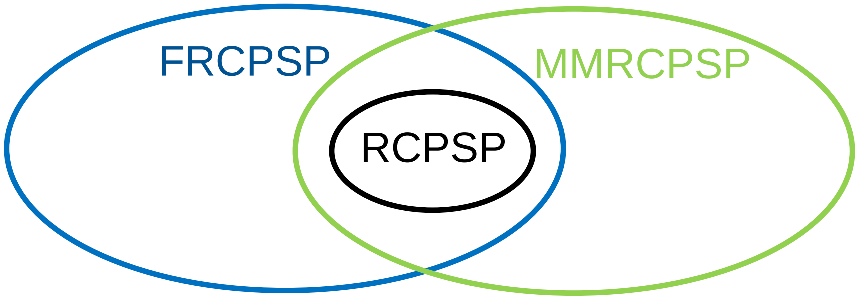
Trautmann et al. term the FRCPSP as “RCPSP with work content constraints“.

Variants of the FRCPSP

Resource Allocation		Time: Continuous	Time: Discrete
	Discrete		Operational problem (periods days) • Kolisch et al. (2003) • Fündeling and Trautmann (2010) • Baumann et al. (2015)
	Continuous	• Naber (2017)	Tactical problem (periods weeks) • Tritschler et al. (2017) • Naber and Kolisch (2014)

FRCPSP in the Project Scheduling Landscape

- The FRCPSP is a general case of the RCPSP and thus many scheduling problems such as the JSP, the FSP, the simple ALB, SP, as well as single and parallel machine scheduling problems.
- Under certain assumptions the FRCPSP covers Multi-mode RCPSP problems and vice versa.
- A feasible solution can be obtained in polynomial time if the time windows are sufficiently large.
- Solving the FRCPSP to optimality is NP-hard.



Solution Approaches for the FRCPSP

MIPs

- Kolisch et al. (2003)
- Baumann and Trautmann (2013)
- Naber and Kolisch (2014)

Heuristics

- Kolisch et al. (2003): Priority Rule
- Fündeling and Trautmann (2010): Priority Rule
- Ranjbar and Kianfar (2010): Genetic Algorithm
- Tritschler et al. (2017): Hybrid Metaheuristic
- Zimmermann (2017): Matheuristic

MIP-Formulations: Variables

Binary variables:

$z_{j,t} = 1$, if j is processed in t (On/Off)

$x_{j,t} = 1$, if j starts at the beginning of t (Pulse)

$x^a_{j,t} = 1$, if j starts at the beginning of t or before (Step)

$y_{j,t} = 1$, if j is completed at the end of t (Pulse)

$y^a_{j,t} = 1$, if j is completed at the end of $t-1$ or before (Step)

$\delta_{j,r,t} = 1$, if resources allocated to j change from $t-1$ to t

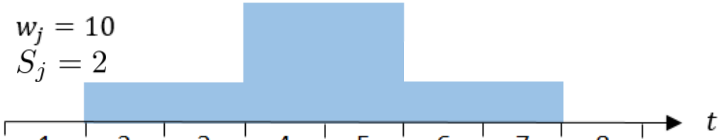
Continuous variables:

$q_{j,r,t} \geq 0$ Resources allocated to j in t

$v^c_{j,t} \in [0,1]$ Cumulated intensity of j in period t

$$v^c_{j,t} = v^c_{j,t-1} + \frac{q_{j,r,t}}{w_{j,r}}$$

$S_j \geq 0$ Starting period of j



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$q_{j,t}$	0	1	1	3	3	1	1	0
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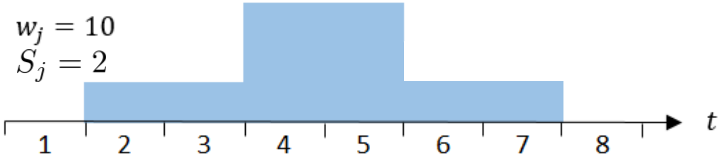

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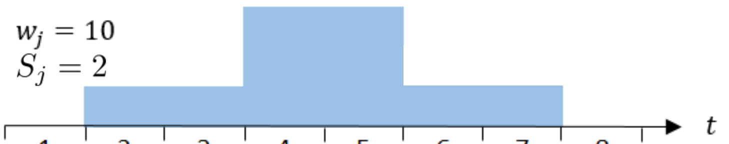

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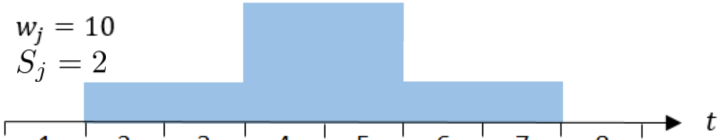

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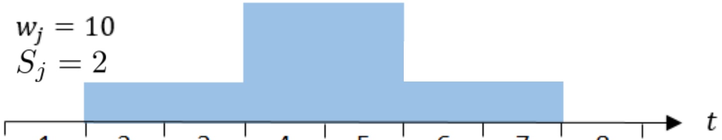

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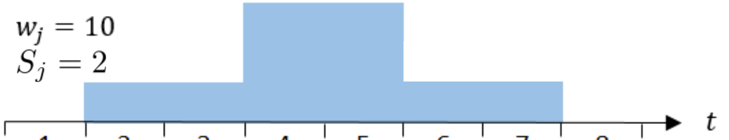

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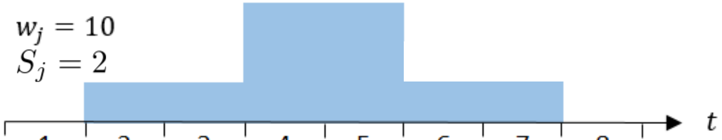

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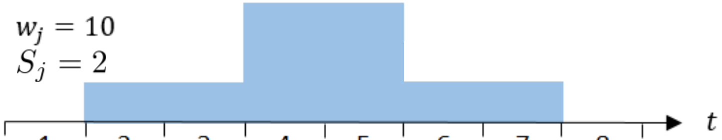

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MIP-Formulations

Model FP-DT	Based on	Binary variables						Continuous variables			
		x_{jt}	x^a_{jt}	y_{jt}	y^a_{jt}	z_{jt}	δ_{rjt}	v^c_{jt}	s_j	q_{rjt}	C_{max}
1	Formulation 2 of Klein (2000)					•	•			•	•
2	Ranjbar and Kianfar (2010)	•		•			•			•	•
3	Bianco and Caramia (2013) and Kis (2005)		•		•		•	•		•	•
4	Sabzehparvar and Seyed-Hosseini (2008)					•	•		•	•	•

Source: Naber and Kolisch (2014)

<u>FP-DT1:</u>
Based on Klein (2000), same binary variables in Baumann and Trautmann (2013).
On/Off variable $z_{j,t}$ (1, if j is processed in t).

<u>FP-DT2</u>
Based on Ranjbar and Kianfar (2010).
Pulse variables $x_{j,t}$ (1, if j starts at the beginning of t) and $y_{j,t}$ (1, if j is completed by the end of period t).

<u>FP-DT3</u>
Based on Bianco and Caramia (2013) and Kis (2005).
Step variables $x^a_{j,t}$ (1, if j starts at the beginning of t or before) and
$y^a_{j,t}$ (1, if j is completed by the end of period $t - 1$ or before) and continuous variable $v^c_{j,t}$ (cumulated intensity)

<u>FP-DT4</u>
Based on Sabzehparvar and Seyed-Hosseini (2008)
On/Off variable $z_{j,t}$ (1, if j is processed in t) and continuous start variable s_j

Comparison of MIP-Formulations: Model Sizes

Bechnmark instances of Fündeling and Trautmann (2010)

Model FP-DT	1	2	3	4
2–10 Activities with 1 resource (46.3%), 3 resources (0.5%), and 4 resources (53.2%) in total 551 instances				
Binary variables	**403**	622	710	575
Continuous variables	**216**	424	588	435
Rows	**1417**	2837	3411	2913
Nonzero A's (%)	0.475%	0.786%	**0.204%**	0.394%
11–20 Activities with 1 resource (35%) and 4 resources (65%) in total 569 instances				
Binary variables	**1770**	2039	2217	1771
Continuous variables	**1294**	1294	1796	1314
Rows	**8161**	8813	10,392	8912
Nonzero A's (%)	0.141%	0.452%	**0.069%**	0.162%
21–53 Activities with 1 resource (27.3%) and 4 resources (72.7%) in total 553 instances				
Binary variables	**6165**	7358	7683	6166
Continuous variables	**4543**	4543	6180	4582
Rows	**28,878**	31,410	35,790	31,070
Nonzero A's (%)	0.058%	0.251%	**0.021%**	0.065%

Source: Naber and Kolisch (2014)

MIP-Formulations: Performance

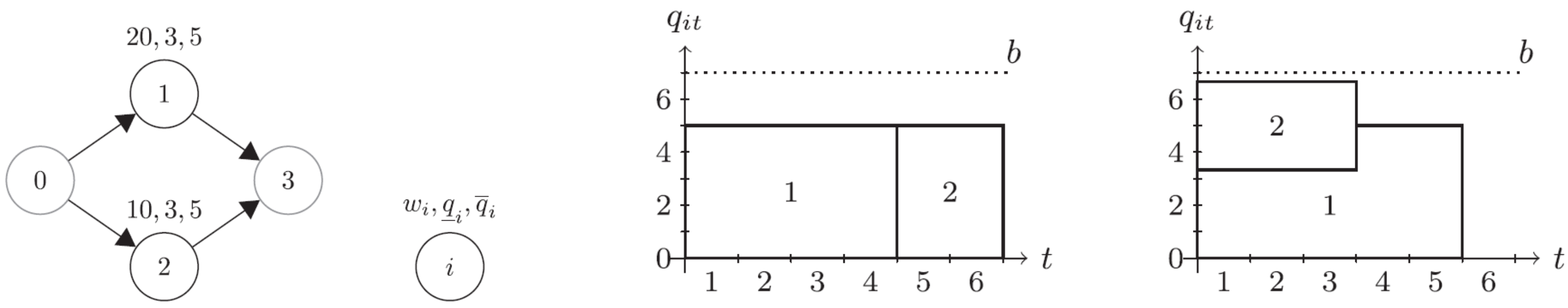
Model FP-DT	1 (%)	2 (%)	3 (%)	4 (%)
2–10 Activities with 1 resource (46.3%), 3 resources (0.5%), and 4 resources (53.7%) in total of 551 instances				
Instances attaining optimality	98.73	97.82	**100.00**	97.64
Instances with good solutions (≤5%)	99.46	99.64	**100.00**	99.64
Instances with feasible solutions	100.00	100.00	**100.00**	100.00
Avg. gap of best solution – best known	0.06	0.04	**0.00**	0.09
Avg. gap of LP – best known (9.15%)	2.92	2.91	**1.30**	3.67
Instances with LP = best known	65.15	65.34	**79.31**	59.89
11–20 Activities with 1 resource (35%) and 4 resources (65%) in total of 569 instances				
Instances attaining optimality	86.12	80.84	**92.79**	81.72
Instances with good solutions (≤5%)	89.46	84.71	**99.82**	85.59
Instances with feasible solutions	99.47	93.32	**100.00**	94.02
Avg. gap of best solution – best known	1.35	2.26	**0.02**	2.00
Avg. gap of LP – best known (8.83%)	3.39	3.33	**2.38**	3.93
Instances with LP = best known	57.29	58.00	**64.85**	51.14
21–53 Activities with 1 resource (27.3%) and 4 resources (72.7%) in total of 553 instances				
Instances attaining optimality	65.34	55.96	**71.25**	59.03
Instances with good solutions (≤5%)	70.52	62.39	**86.62**	64.56
Instances with feasible solutions	74.73	66.06	**86.98**	68.77
Avg. gap of best solution – best known[1]	1.40	2.47	**0.08**	2.71
Avg. gap ofLP – best known[1] (5.68%)	4.12	4.11	**3.56**	4.82
Instances with LP = best known[1]	60.87	60.25	**67.49**	44.72
All instances in total of 1673 instances				
Instances attaining optimality	83.40	78.21	**88.05**	79.46
Instances with good solutions (≤5%)	86.49	82.25	**95.52**	83.26
Instances with feasible solutions	91.47	86.51	**95.70**	87.65
Avg. gap of best solution – best known[2]	0.92	1.56	**0.03**	1.56
Avg. gap ofLP – best known[2] (7.99%)	3.45	3.42	**2.36**	4.11
Instances with LP = best known[2]	61.07	61.20	**70.62**	52.21

Source: Naber and Kolisch (2014)

Hybrid Metaheuristic of Tritschler et al. (2017)

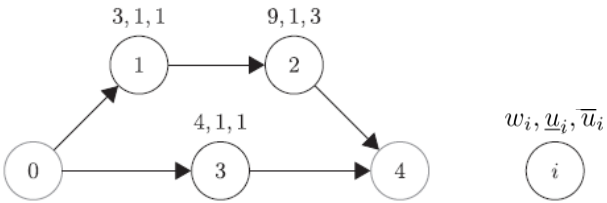
- FRCPSP-adapted parallel schedule generation scheme of Kolisch (1996)
- FRCPSP-adapted GA of Hartmann (1998)
- VNS-based transfer of resources

Example: Greedy Resource Assignment May Not Be Optimal



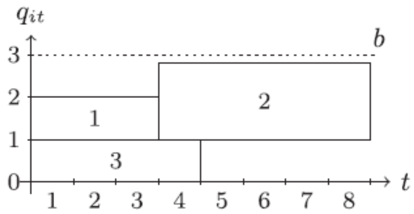
Source: Tritschler et al. (2017)

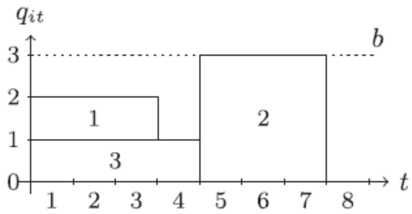
Example: Early Start May Not Be Optimal



Source: Tritschler et al. (2017)

$l = 3,\ b = 3$





Solution Encoding of the GA

- Activity list $\lambda = (j_1, j_2, \ldots, j_n)$
- Resource allocation limit list $\rho = (\rho_1, \rho_2, \ldots, \rho_n)$ with $\rho_i \in \{0, \ldots, \overline{\rho}_i\}$

$$\overline{\rho}_i = \min(\overline{d}_i - \underline{d}_i, LF_i - ES_i)$$

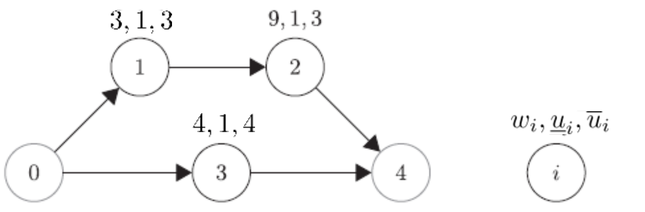
$$q_{i,r,t} = \min\left(\overline{b}_{r,t}, \frac{w_{i,r}}{\underline{d}_i + \rho_i}\right)$$

$$\overline{b}_{r,t} = \text{Left-over capacity in period } t$$

$$\underline{d}_i = \text{Min duration of activity } i$$

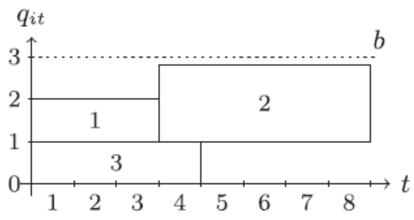
- Start delay list $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ with $\sigma_i \in \{0, 1, \ldots, LS_i - ES_i\}$

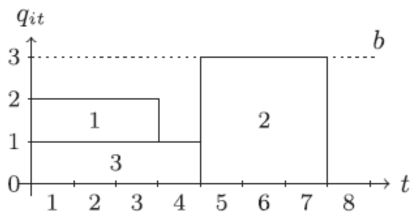
Mapping of (λ,ρ,σ) Into a Solution: Adapted SGS



Source: Tritschler et al. (2017)

$l = 3,\ b = 3$





$\lambda = (1, 3, 2)$

$\rho = (2, 0, 3)$

$\sigma = (0, 1, 0)$

Period t = 1:

Select activity 1, $S_1 = ES_1' + \sigma_1 = 1 + 0 = 1,\ q_{1,1} = \min(\bar{b}_1, \frac{w_1}{\underline{d}_1 + \rho_1}) = \min(3, \frac{3}{1+2}) = 1$

Select activity 3, $S_3 = ES_3' + \sigma_3 = 1 + 0 = 1,\ q_{3,1} = \min(\bar{b}_1, \frac{w_3}{\underline{d}_3 + \rho_3}) = \min(2, \frac{4}{1+3}) = 1$

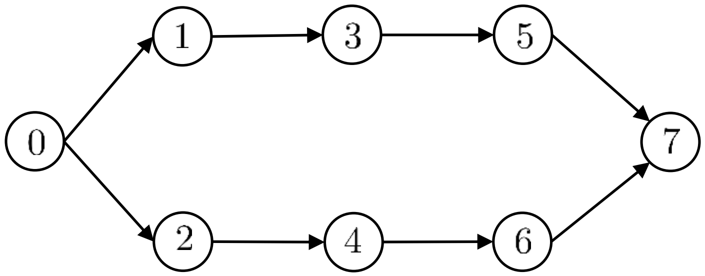
….

Period t = 4:

…

Select activity 2, $S_2 = ES_2' + \sigma_2 = 4 + 1 = 5,\ q_{2,5} = \min(\bar{b}_5, \frac{w_2}{\underline{d}_2 + \rho_2}) = \min(3, \frac{9}{3+0}) = 3$

Adapted Two-Point Crossover



Parents

$$\lambda_1^P = (1, 3, 5, 2, 4, 6) \quad \lambda_2^P = (2, 4, 6, 1, 3, 5)$$
$$\rho_1^P = (1, 2, 4, 3, 1, 2) \quad \rho_2^P = (0, 0, 1, 2, 3, 3)$$
$$\sigma_1^P = (4, 5, 6, 1, 2, 3) \quad \sigma_2^P = (0, 0, 0, 2, 2, 2)$$

Children

$$\lambda_1^C = (1, 3, 2, 4, 5, 6) \quad \lambda_2^C = (2, 4, 1, 3, 5, 6)$$
$$\rho_1^C = (1, 2, 1, 2, 1, 2) \quad \rho_1^C = (0, 0, 4, 3, 1, 2)$$
$$\sigma_1^C = (4, 5, 0, 2, 2, 3) \quad \sigma_2^C = (0, 0, 6, 1, 2, 2)$$

Resource Transfers

Resource Transfers:

- Determine the set of critical activities (on the critical path) $\mathcal{V}^c$
- Determine pair of activities $(i, j), \; i \in \mathcal{V}^c, \, j \in \mathcal{V} \setminus \mathcal{V}^c$
- For each pair (i, j) determine the max resource flow from j to i
- Update list ρ to ρ' to consider the resource flow
- Obtain the new schedule with the adapted SGS and lists λ, ρ' and σ

Variable Neighborhood Search: Consider the neighborhood of $k = 1, \dots, k^{max}$ resource transfers.

Computational Results

Based on instance sets of
Fündeling and Trautmann (2010)

HM = Hybrid Method (GA + VNS)
SGA = Self adapting SGS based on Hartmann (2002)
PRS = Parallel SGS with random sampling
SRS = Serial SGS with random sampling

	HM		SGA		PRS		SRS	
Set / Ω	Δ_{mip}	Δ_{lb}	Δ_{mip}	Δ_{lb}	Δ_{mip}	Δ_{lb}	Δ_{mip}	Δ_{lb}
$A_{\leq 55}$	**0.03**	**5.60**	**2.12**	**7.82**	**2.64**	**8.41**	**3.89**	**9.83**
1000	0.64	6.27	2.52	8.30	3.26	9.09	4.66	10.68
5000	0.01	5.58	2.13	7.83	2.67	8.45	3.95	9.89
15,000	−0.24	5.30	1.95	7.63	2.37	8.10	3.57	9.47
25,000	−0.27	5.25	1.88	7.53	2.26	7.98	3.40	9.28
B_{10}	**0.24**	**5.40**	**1.31**	**6.57**	**1.55**	**6.81**	**1.59**	**6.88**
1000	0.44	5.62	1.34	6.61	1.56	6.83	1.62	6.91
5000	0.22	5.38	1.30	6.56	1.55	6.82	1.58	6.87
15,000	0.15	5.31	1.30	6.56	1.55	6.81	1.58	6.87
25,000	0.14	5.28	1.30	6.56	1.55	6.81	1.58	6.87
B_{20}	**0.28**	**4.24**	**0.75**	**4.77**	**1.28**	**5.34**	**1.31**	**5.40**
1000	0.60	4.60	0.86	4.90	1.63	5.73	1.70	5.84
5000	0.27	4.23	0.75	4.78	1.31	5.37	1.30	5.39
15,000	0.15	4.10	0.70	4.71	1.12	5.16	1.15	5.23
25,000	0.10	4.04	0.69	4.70	1.07	5.10	1.09	5.16
B_{40}	**−1.88**	**4.08**	**−1.73**	**4.29**	**−0.05**	**6.21**	**0.32**	**6.74**
1000	−1.66	4.35	−1.53	4.55	0.49	6.84	0.93	7.44
5000	−1.88	4.09	−1.72	4.31	−0.01	6.26	0.42	6.85
15,000	−1.98	3.96	−1.82	4.18	−0.29	5.94	0.04	6.41
25,000	−2.01	3.93	−1.85	4.13	−0.40	5.81	−0.09	6.26
B_{100}		**3.94**		**4.05**		**7.15**		**8.09**
1000		4.07		4.21		7.68		8.68
5000		3.94		4.08		7.21		8.14
15,000		3.89		3.97		6.92		7.84
25,000		3.87		3.93		6.80		7.71
B_{200}		**3.41**		**3.55**		**7.12**		**8.20**
1000		3.46		3.65		7.53		8.67
5000		3.40		3.57		7.16		8.27
15,000		3.39		3.51		6.95		7.98
25,000		3.39		3.48		6.83		7.88
Overall	**−0.33**	**4.46**	**0.63**	**5.20**	**1.37**	**6.86**	**1.81**	**7.55**

Impact of the VNS

Inst% = percentage of instances where the VNS improves the solution
ΔC_{max} = Absolute improvement of the makespan
ΔC_{max}% = Improvement of the makespan in percent

Set	Inst %	ΔC_{max}	ΔC_{max}%
$A_{\leq 55}$	5.59	1.06	2.27
B_{10}	1.56	1.06	2.44
B_{20}	7.57	1.23	1.39
B_{40}	20.63	1.34	0.85
B_{100}	54.24	2.12	0.57
B_{200}	61.84	3.84	0.55
Overall	**22.91**	**2.58**	**1.34**

Instances

Fündeling and Trautmann (2010)

- Test set A derived from PSPLIB-instances (553 instances)
- Test set B generated with ProGen/max (2.400 instances)

Brachmann et al. (2023)

- Test set with 360 instances derived from real-world ETO production planning problems

Instances of Brachmann et al. (2023)

- Time horizon 1.5 years
- 144 customer orders

Parameter	Values	Levels
No. of projects (No. of activities)	38 (622), 76 (1,219), 114 (1,839)	3
Resource availability	1.7, 1.6, 1.5, 1.4, 1.3, 1.2	5
Due date tightness	1.6, 1.5, 1.4, 1.3, 1.2	4
Resource flexibility	[0.10, 0.70], [0.15, 0.65], [0.20, 0.60], [0.25, 0.55]	4

=> $3 \cdot 5 \cdot 4 \cdot 4 = 360$ instances

- Resource availability: $Scaling\ Factor \cdot Average\ Resource\ Capacity\ in\ ERP\ Data$
- Due date tightness: $Project\ Arrival + Scaling\ Factor \cdot Critical\ Path\ Duration$
- Resource flexibility: $Scaling\ Factor \cdot Activity\ Work\ Content$

Brachmann et al. (2023) Instances: Characterization

We calculate the following instance measures for the generated instances:

Network topology:
- Coefficient of Network Complexity (CNC)
- Order Strength (OS)
- Serial-Parallel-Indicator (SPI)
- Activity Distribution (AD)

Resource scarcity:
- Resource Strength (RS)
- Normalized Average Resource Loading Factor (NARLF)
- Utilization Factor (UF)
- Variance of Utilization Factor (σ_{UF}^2)
- Resource Sharing Ratio (RSR)

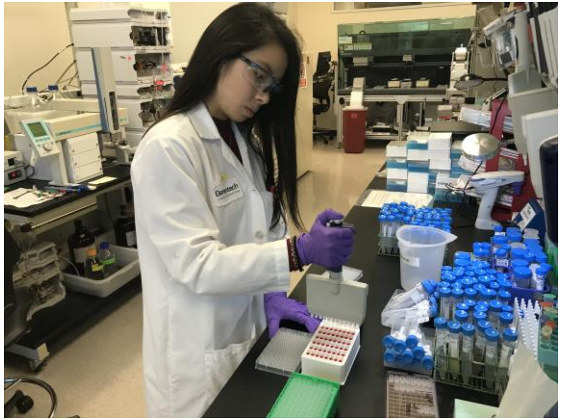
Projects	Activities	CNC	OS	SPI	AD	NARLF	RSR
38	622	1.52	0.0079	0.0064	0.48	-16.93	0.066
76	1,219	1.52	0.0040	0.0033	0.48	9.79	0.079
114	1,839	1.52	0.0027	0.0022	0.48	1.07	0.083

Selected Applications

- Kolisch et al. (2003): Scheduling lead optimization in pharmaceutical research
- Braune and Dörner (2017): Scheduling testing of input material for chemical production.
- Kis and Drótos (2017): Scheduling in digital factories.
- Baur and Rieck (2020): Scheduling of disaster response projects.
- Brachmann and Kolisch (2021): Scheduling engineer-to-order (ETO) production planning with working time accounts.
- Noureddine et al. (2023): Scheduling ship overhauls.
- Brachmann et al. (2023): Real-world instance set for ETO production planning.

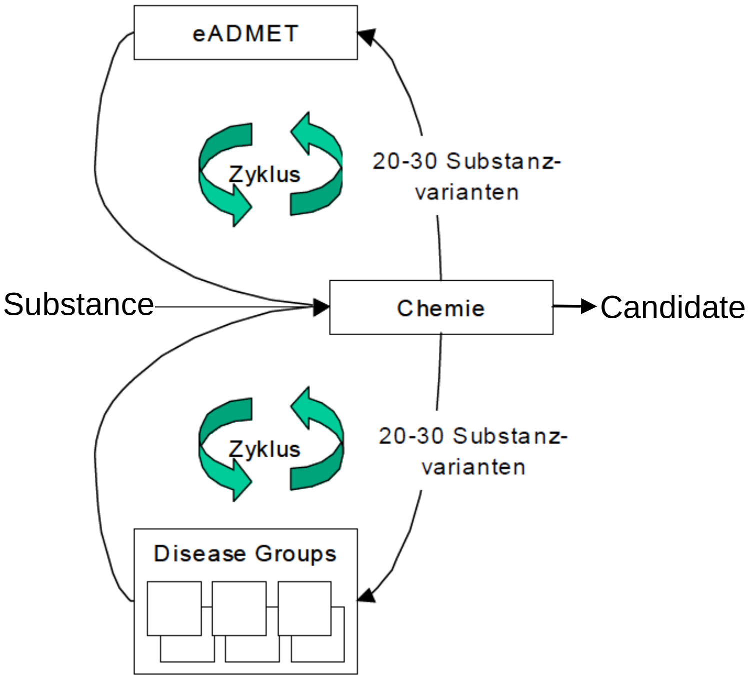
Lead Optimization in Pharmaceutical Research

- Kolisch, Meyer, Mohr, Schwindt, Urmann (2003): Ablaufplanung für die Leistrukturoptimierung in der Pharmaforschung, Zeitschrift für Betriebswirtschaft 78(8) 825-848.



Source: https://www.genengnews.com/insights/improved-assays-and-predictive-tools-for-drug-lead-optimization/

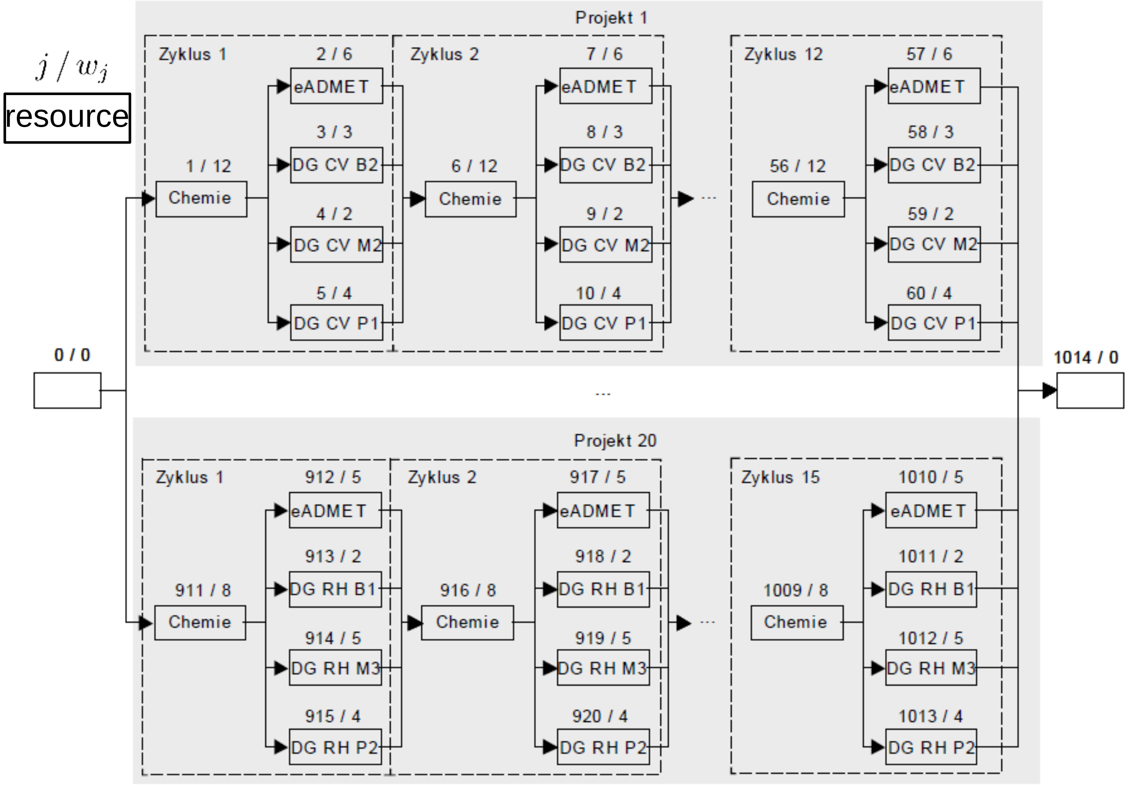
Research Cycle for Lead Optimization



Source: Kolisch et al. (2003)

- Iterative improvement from substance to candidate
- 15 – 20 cycles
- „Chemistry“ → alternation of substance
- „eADMET“ (early Administration, Distribution, Metabolism, Kinetics, Toxicology) → test of tolerability by the human body
- „DG“ (disease groups) → test of effectiveness of the substance

Modeling as FRCMPSP



Source: Kolisch et al. (2003)

- ~ 20 parallel research projects
- ~ 60 activities for each project
- Series-parallel network structure
- Work content w_j in laboratory unit weeks
- One activity requires one specific resource (specific type of laboratory unit)
- One laboratory unit consists of 1 chemical engineer, 2 chemical lab assistants and one laboratory including research equipment
- 27 DG resources with capacity [4,7]
- 1 eADMET resource with capacity 14
- 1 chemistry resource with capacity 50
- Periods are weeks
- Objective: Min makespan

Engineering-to-Order Production Planning

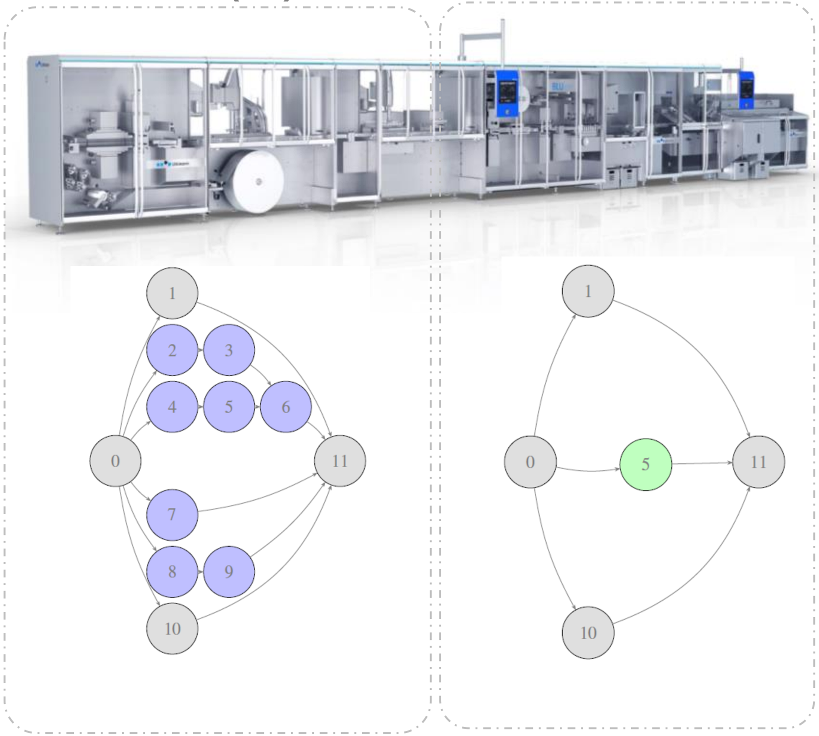
Brachmann, R. and Kolisch, R. (2021): The impact of flexibility on engineer-to-order production planning, International, Journal of Production Economics, Vol. 239, 108183.

Weber, H., Brachmann, R. and Kolisch R. (2023): Generation and characterization of real-world instances for the flexible resource-constrained multi-project scheduling problem, Working Paper.

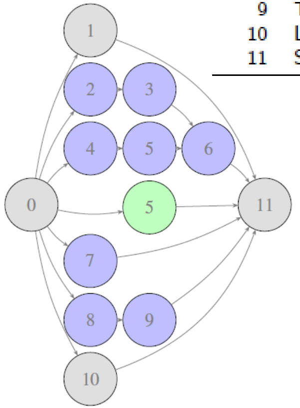
Source:https://www.uhlmann.de/news-media/newsroom/news-detail/sustainability-from-nice-to-haves-to-must-haves-with-innovative-solutions-from-uhlmann-pac-systeme-339/

Single Project

Module 1 (M1) Module 2 (M2)



M1 ∪ M2

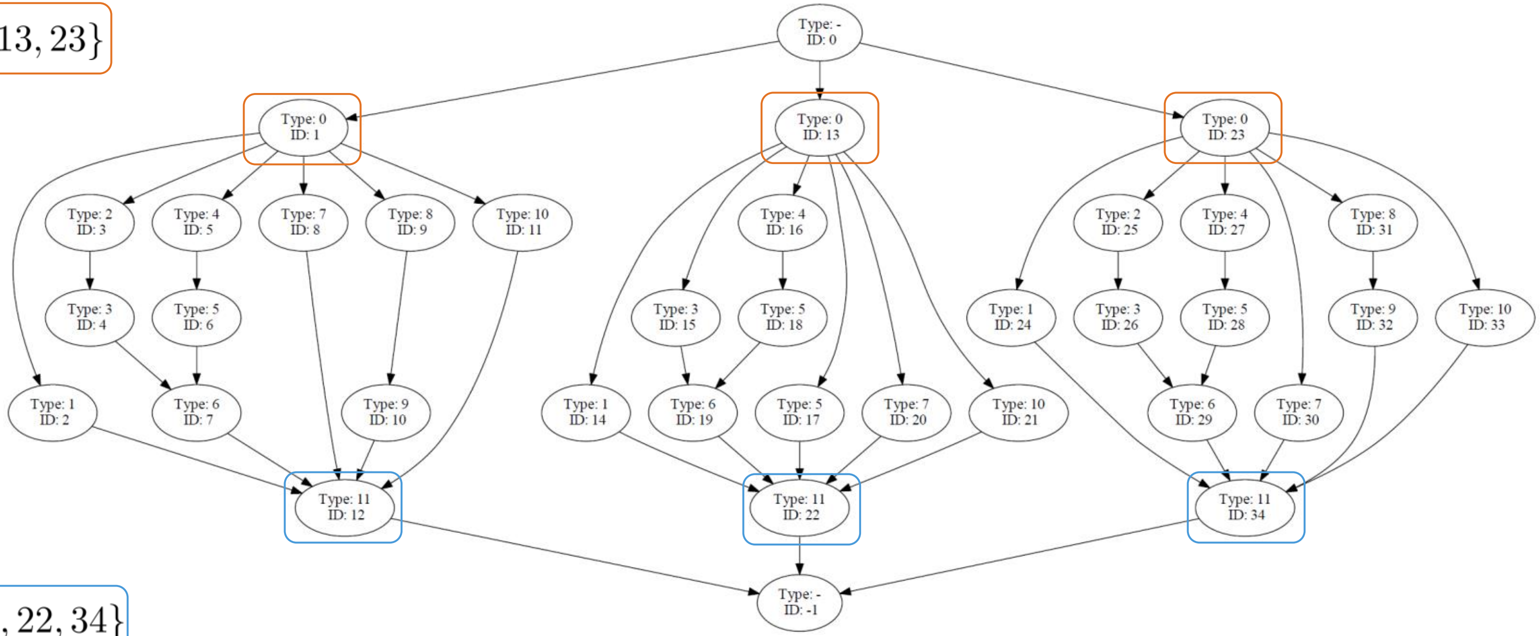


Activity		Resource	
Type	Description	Number	Description
0	Planning	1	Project office
1	Management	1	Project office
2	Mechanical design	2	Mechanical engineering
3	Mechanical pre-assembly	3	Mechanical assembly
4	Electrical engineering hardware	4	Electrical engineering
5	Electrical pre-assembly	5	Electrical assembly
6	Machine assembly	6	Machine assembly
7	Electrical software engineering	7	Software engineering
8	Tools engineering	8	Tools engineering
9	Tools assembly	9	Tools assembly
10	Layout design	10	Layout design
11	Synchronization	11	Synchronization

- Period length week
- Work content in worker days
- Planning horizon 1.5 years

Multi-Project Network

$$\mathcal{V}^S = \{1, 13, 23\}$$



$$\mathcal{V}^F = \{12, 22, 34\}$$

$$\text{Min} \sum_{j \in \mathcal{V}^F} w_j(\alpha \cdot E_j + \beta \cdot L_j)$$

Period length week

Solution Approach

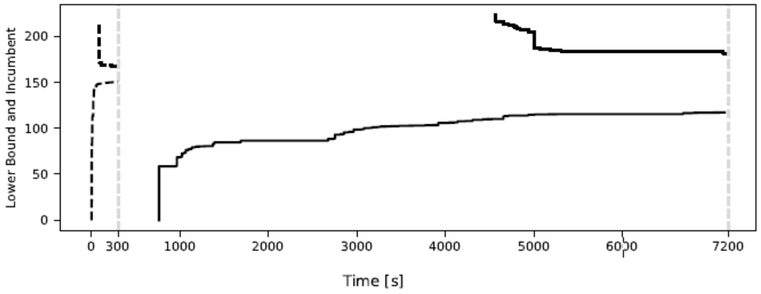
- MIP-Formulation based on FP-DT3.
- Use of the Lower Bound as detailed below.

Lower Bound

- MIP-Formulation based on FP-DT3.
- Observation: For each project, resource-constraints of activities have a block-angular structure.
- Determine processing time window for finish activities $\mathcal{V}^F$.
- Delete all activities from the MIP $\mathcal{V} \setminus \mathcal{V}^F$ and solve it to optimality.

Number of	FP-DT3	LB-FP-DT3
Continuous variables	21,938	1,256
Integer variables	53,143	2,368
Binary variables	52,991	2,216

LB-FP-DT3 FP-DT3



Computational Results

- Python-Gurobi implementation
- i7-8700K with 3.7GhHz, 32 GigaByte RAM
- 2 hours computation time

Size	Measure	Resource flexibility				
		All	Level 1 [0.1, 0.7]	Level 2 [0.15, 0.65]	Level 3 [0.2, 0.6]	Level 4 [0.25, 0.55]
624	Optimality (%)	92.50	100.00	100.00	83.33	86.67
	Solved (%)	100.00	100.00	100.00	100.00	100.00
	Objective value	4.35	1.57	2.57	4.73	8.53
	Best bound	4.22	1.57	2.57	4.53	8.23
	Gap (%)	1.47	0.00	0.00	2.29	3.57
	Improved optimality (%)	98.33	100.00	100.00	93.33	100.00
	Improved best bound	4.33	1.57	2.57	4.67	8.53
	Improved gap (%)	0.21	0.00	0.00	0.83	0.00
	CPU time FRCMPSP	716.39	21.36	230.04	1361.37	1252.78
	CPU time FRCMPSP-Rel	6.74	0.00	0.00	23.97	3.00
1221	Optimality (%)	68.33	86.67	76.67	60.00	50.00
	Solved (%)	100.00	100.00	100.00	100.00	100.00
	Objective value	27.93	20.17	24.00	29.90	37.63
	Best bound	22.52	16.67	19.77	23.90	29.77
	Gap (%)	6.61	3.25	4.34	8.24	10.58
	Improved optimality (%)	70.83	86.67	80.00	63.33	53.33
	Improved best bound	26.08	19.33	22.97	27.83	34.20
	Improved gap (%)	2.10	0.76	1.04	2.60	4.00
	CPU time FRCMPSP	2635.11	1469.43	2079.91	3213.30	3777.82
	CPU time FRCMPSP-Rel	86.42	40.02	48.63	111.45	145.56
1841	Optimality (%)	31.67	40.00	40.00	33.33	13.33
	Solved (%)	87.50	93.33	90.00	90.00	76.67
	Objective value	69.60	64.25	65.37	81.00	67.70
	Best bound	62.20	48.70	57.67	64.63	77.80
	Gap (%)	14.77	15.05	12.04	16.47	15.64
	Improved optimality (%)	35.83	46.67	40.00	36.67	20.00
	Improved best bound	73.90	63.57	68.77	80.13	83.13
	Improved gap (%)	7.17	7.46	5.54	6.70	9.26
	CPU time FRCMPSP	5492.80	4850.44	5281.24	5393.97	6445.54
	CPU time FRCMPSP-Rel	122.02	83.25	105.12	143.20	156.51

MIP

MIP+LB

Research Opportunities

- Improving MIP-formulations
- Improving metaheuristics
- New applications

Acknowledgement

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