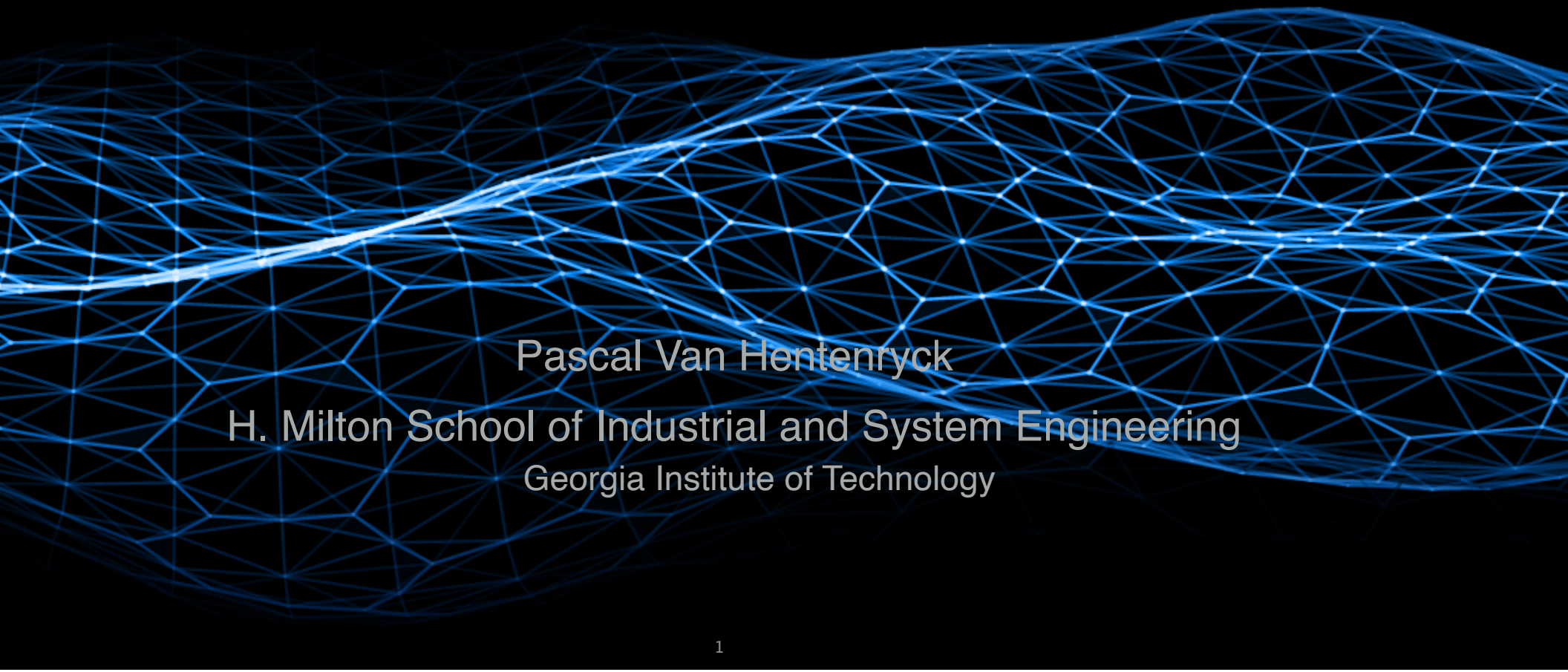
Constraint Programming for Scheduling



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Constraint Programming

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- IEEE Software 2010
  - "The application and importance of CP has grown remarkably in the past two decades"
- OR/MS August 2011
  - "A must-have tool for any O.R. Practitioner's toolkit"

Progress in Optimization

- "If you only knew optimization from 10 years ago, you probably don't have the techniques needed to solve real-world sport scheduling problems"
  - Mike Trick, Professor at CMU, 2008

- "The following do make a big difference (and are much more recent ideas)"
  - Complicated variables
  - Large neighborhood search
  - Constraint programming (ideally combined with integer programming).

Scheduling in the Last Decade

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Outline

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- Brief overview to constraint programming
- Scheduling with constraint programming
- Hybridizing constraint programming and MIP
- Learning-based constraint programming
- Learning Optimization Proxies

Constraint Programming

- A language for combinatorial optimization
  - Expressing substructures of applications
  - Expressing constraints at a high level of abstraction
  - Programming search
- A computational approach to optimization
  - Focus on feasibility
  - Focus on pruning infeasible solutions
    - Explicit representation of the search space


You Must Unlearn
What You Have Learned

Scheduling

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- One of the killer applications for constraint programming
  - heavily used in industry
  - many MIP models face significant challenges
- Modeling layer
  - high-level concepts for scheduling
    - interval variables, global constraints, …
- Dedicated solving capabilities
  - sophisticated algorithms
  - lower bounds
  - branching strategies

What is an Interval Variable?

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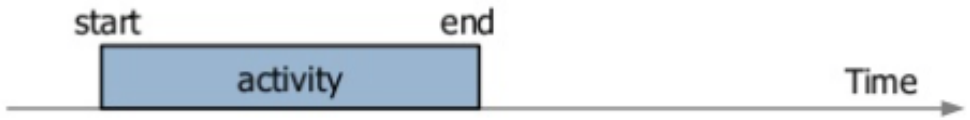
- An object that has
  - a start variable
  - an end variable
  - a duration variable (possibly a constant)
- Ideal for scheduling



Interval Variable

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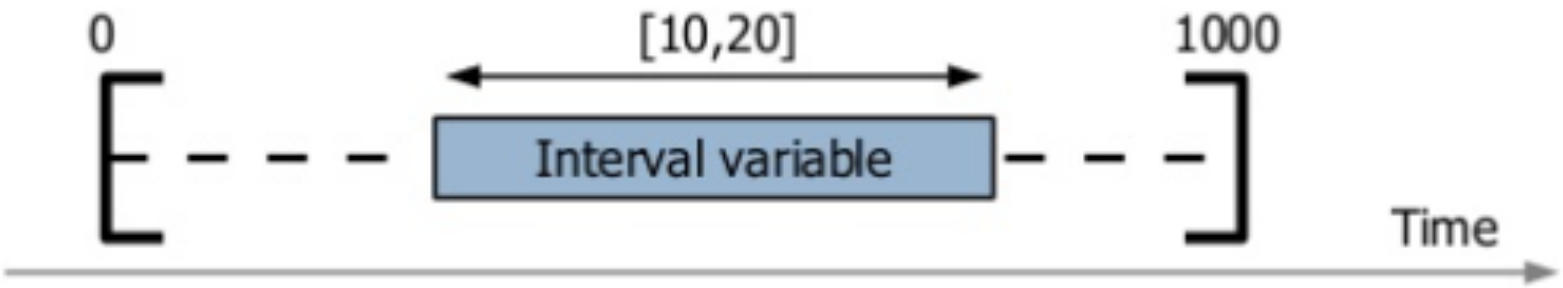
- Durations can be a variable as well

```
dvar interval x in 0..1000 size in 10..20;
```



Jobshop Scheduling

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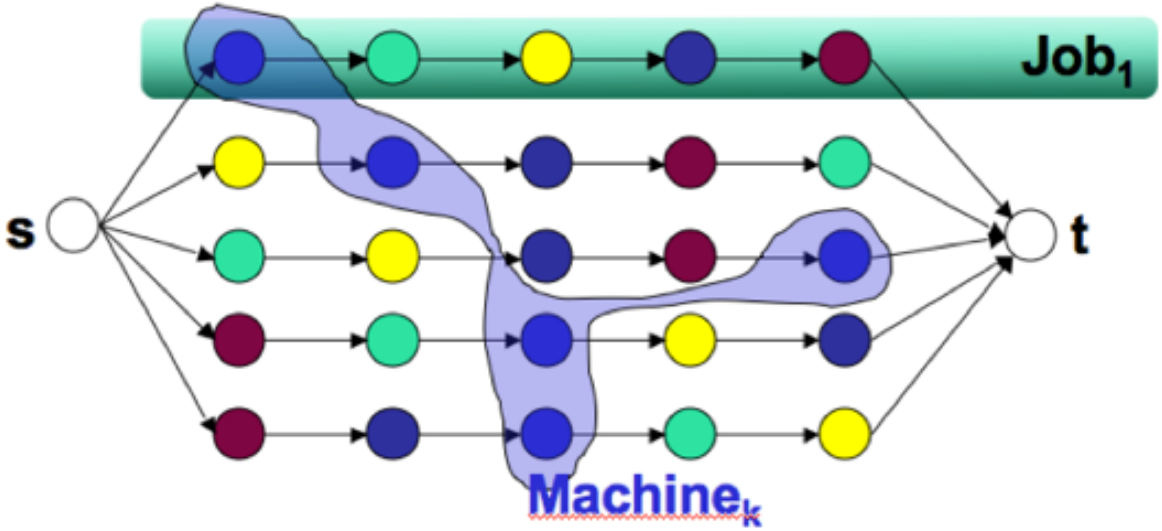
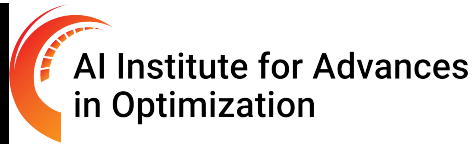


Jobshop Scheduling



- We are given a set of jobs
  - each job is a sequence of tasks
  - each
  - each
  - each
- We ar
  - each
- The g
  - the to

```
nbJobs = 6;
nbMachines = 6;

op = [
 [ <5,4>, <1,3>, <4,3>, <3,2>, <0,1>, <2,2> ],
 [ <1,3>, <0,8>, <5,7>, <2,2>, <4,9>, <3,3> ],
 [ <3,1>, <4,9>, <1,9>, <0,7>, <5,5>, <2,5> ],
 [ <3,8>, <4,2>, <1,1>, <5,7>, <2,8>, <0,9> ],
 [ <1,6>, <3,2>, <4,5>, <5,5>, <0,3>, <2,1> ],
 [ <4,10>, <2,4>, <0,4>, <3,3>, <1,2>, <5,3> ]
];
```

Jobshop Scheduling


```
int nbJobs = ...;
int nbTasks = ...;
range Jobs = 0..nbJobs-1;
range Tasks = 0..nbtasks-1;
tuple Operation {int machine; int pt; };
Operation op[Jobs,Tasks] = ...;

dvar interval act[j in Jobs,o in Tasks] size op[j,o].pt;
dvar sequence machine[m in Machines] in
     all(j in Jobs,o in Tasks: op[j,o].machine == m) act[j,o];

minimize max(j in Jobs) endOf(act[j,nbTasks-1]);
subject to {
  forall (m in Machines)
    noOverlap(machine[m]);
  forall (j in Jobs, o in 0..nbTasks-2)
    endBeforeStart(act[j,o],act[j,o+1]);
}
```

Jobshop Scheduling

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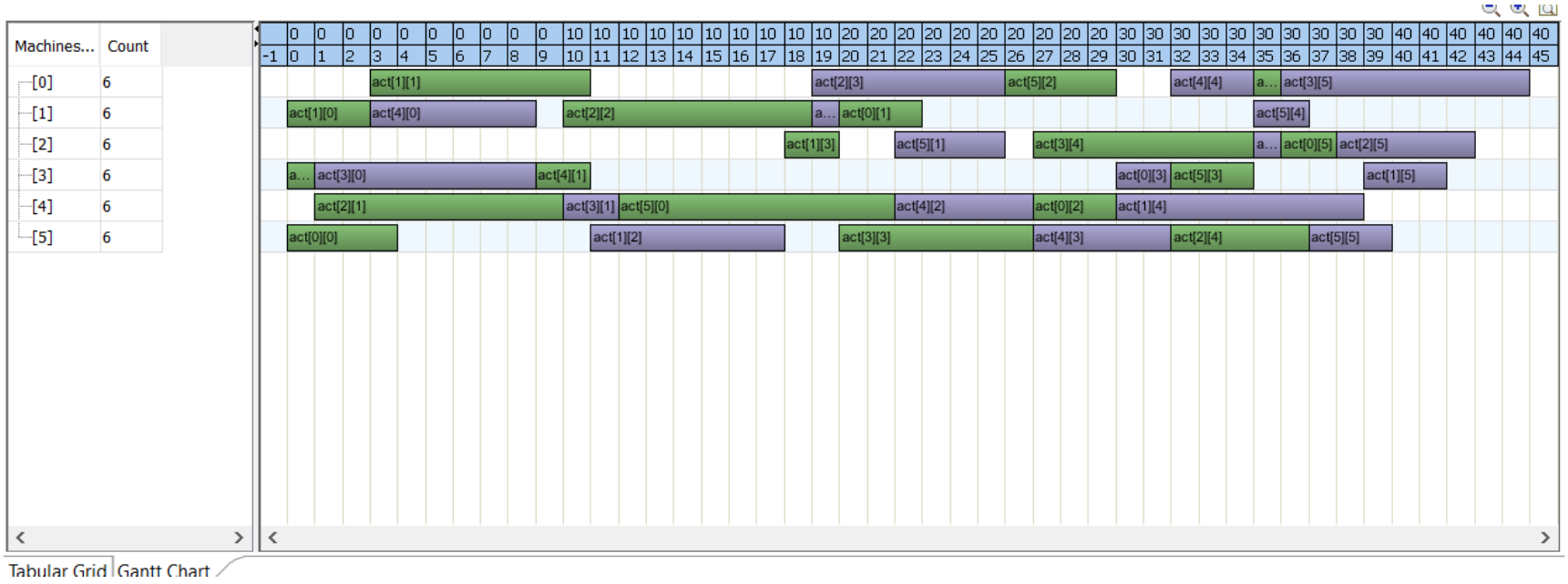
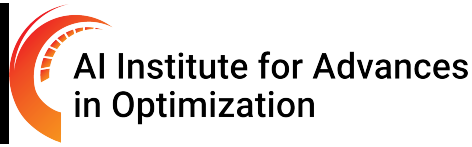


Computational Model of CP



- Branch and prune
  - Iterate two steps: pruning and branching
- Pruning: reduce the search space
  - Explicit representation of the search space
  - Each variable has its set of possible values
  - Use constraints to remove infeasible values
- Branching: decompose the problem into subproblems
  - Use feasibility information to determine how to branch

Constraint Programming For Scheduling

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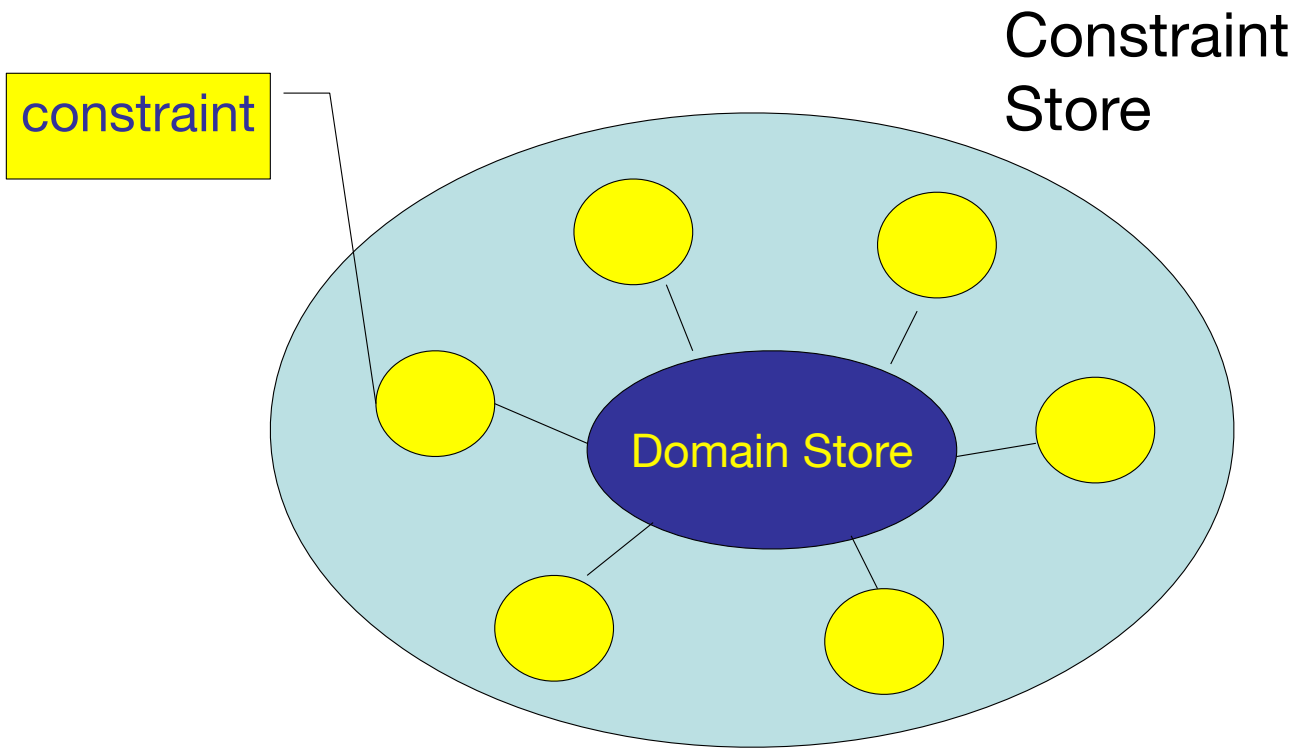

Constraint Solving

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- What does a constraint do?
  - Feasibility Checking
    - Can the constraint be satisfied given the variable domains?
  - Pruning
    - remove values from the domains if they do not appear in any solution of the constraint
    - Holy grail: domain consistency
- How we solve these algorithmic problems, if fast enough, is irrelevant
    - dedicated to each constraint

Constraint Propagation

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```
dvar int x[1..5] in 1..4;
constraints {
   allDifferent(all(i in 1..4) x[i]);
   x[2] <= x[5];
   sum(i in 1..4) x[i] <= 9;
}
```

	x[1] =1	alldiff	x[2]≤ x[5]	x[5] >2	x[2] ≤ x[5]	alldiff	sum≤9	alldiff
x[1]	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1*
x[2]	*1..4*	*2..4*	*2..4*	*2..4*	*2*	*2*	*2*	*2*
x[3]	*1..4*	*2..4*	*2..4*	*2..4*	*2..4*	*3..4*	*3*	✖
x[4]	*1..4*	*2..4*	*2..4*	*2..4*	*2..4*	*3..4*	*3*	✖
x[5]	*1..4*	*1..4*	*2..4*	*3..4*	*2*	*2*	*2*	*2*

Jobshop Scheduling

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```
int nbJobs = ...;
int nbTasks = ...;
range Jobs = 0..nbJobs-1;
range Tasks = 0..nbtasks-1;
tuple Operation {int machine; int pt; };
Operation op[Jobs,Tasks] = ...;

dvar interval act[j in Jobs,o in Tasks] size op[j,o].pt;
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}
```

Jobshop Scheduling

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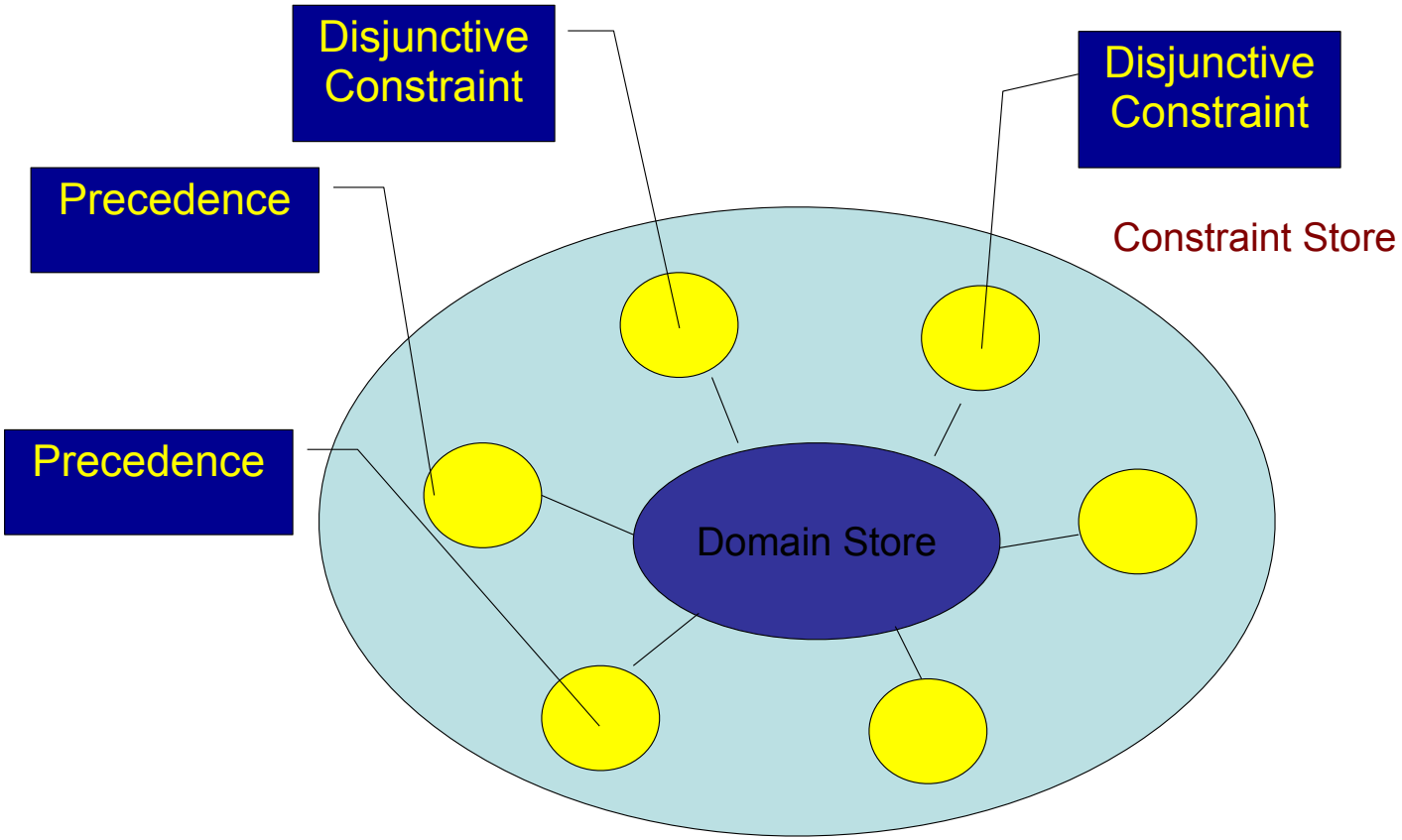

No-Overlap Constraints

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- Given a set of tasks
  - each task has a release date, due date, and duration
- Determine if there exist start dates for the tasks
  - so that no two tasks overlap in time
- One-Machine Problem

No-Overlap Constraints

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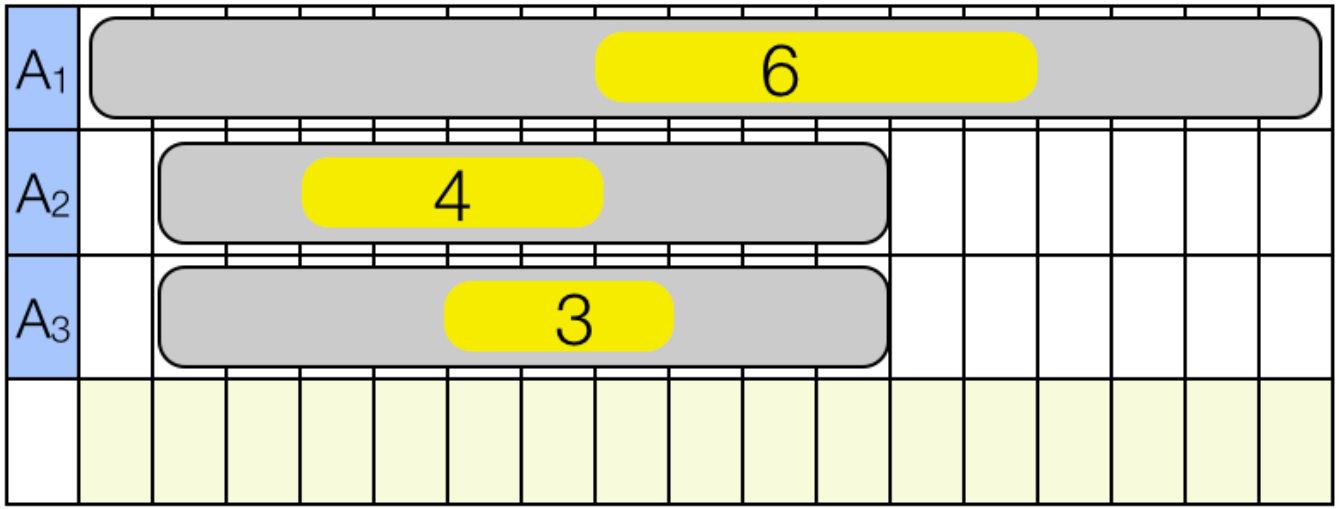
- One-Machine Feasibility



One-Machine Feasibility is NP-Hard

No-Overlap Constraints

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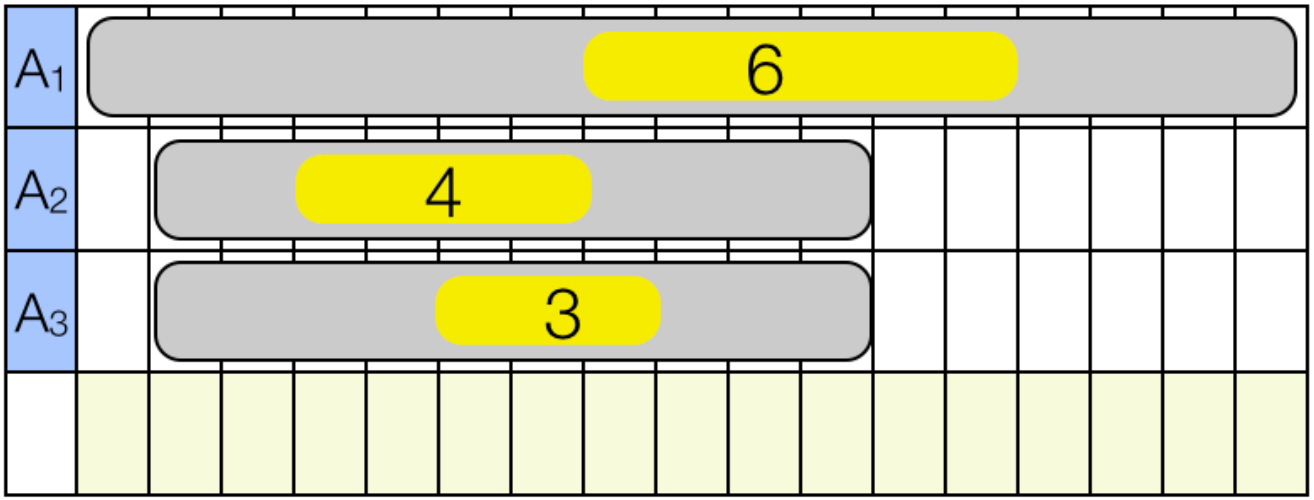
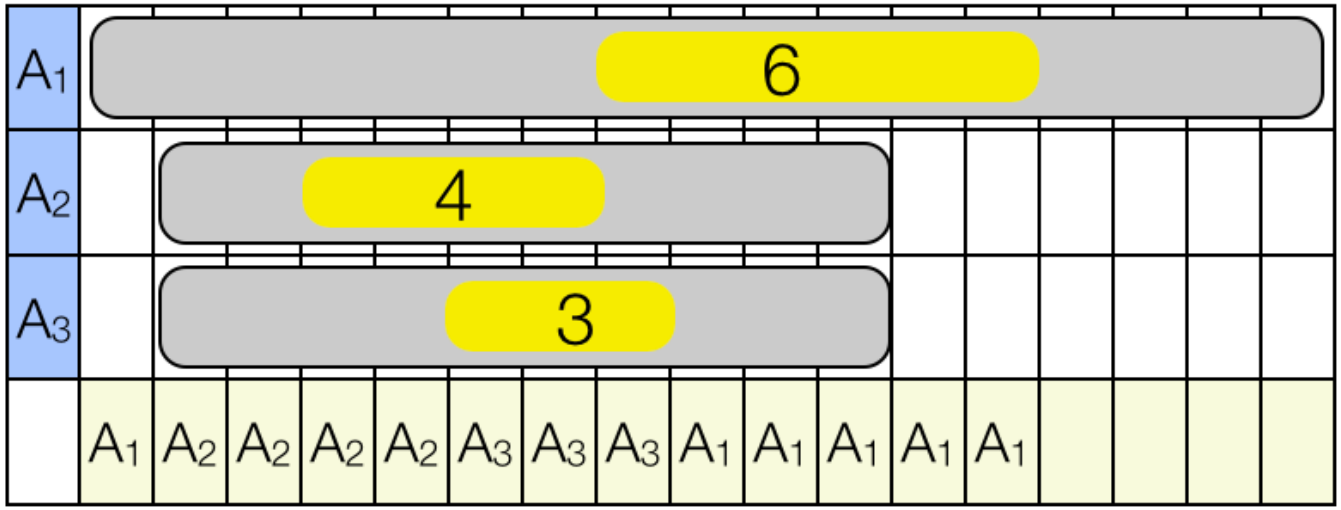
- Relax! One-Machine Preemptive Feasibility



- One-Machine Preemptive Feasibility can be computed in O(n log n) time.

Pruning: Edge Finder Rules

- Pruning: Must A_1 start after A_2 and A_3?



Pruning: Edge Finder Rules

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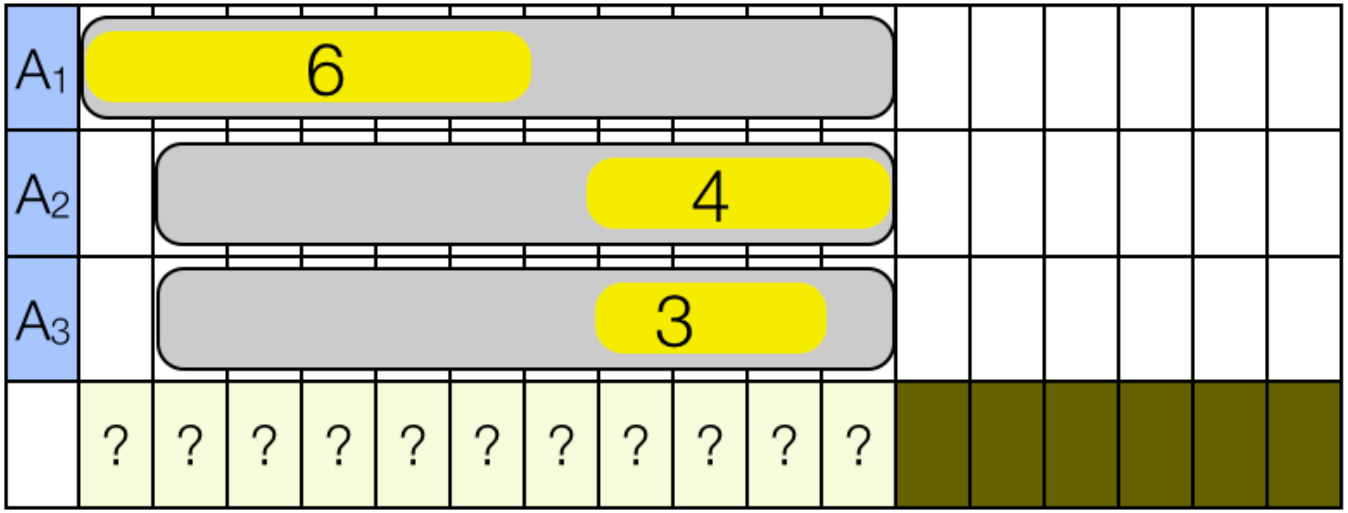

Pruning: Edge Finder Rules

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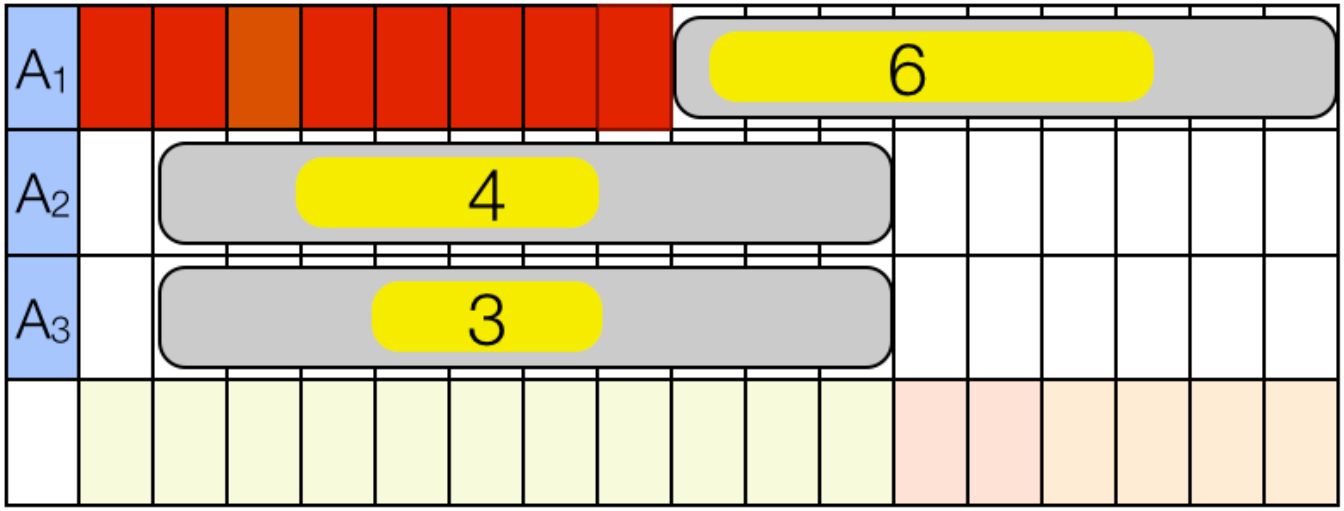
- Pruning: A_1 must start after A_2 and A_3



Cumulative Constraints

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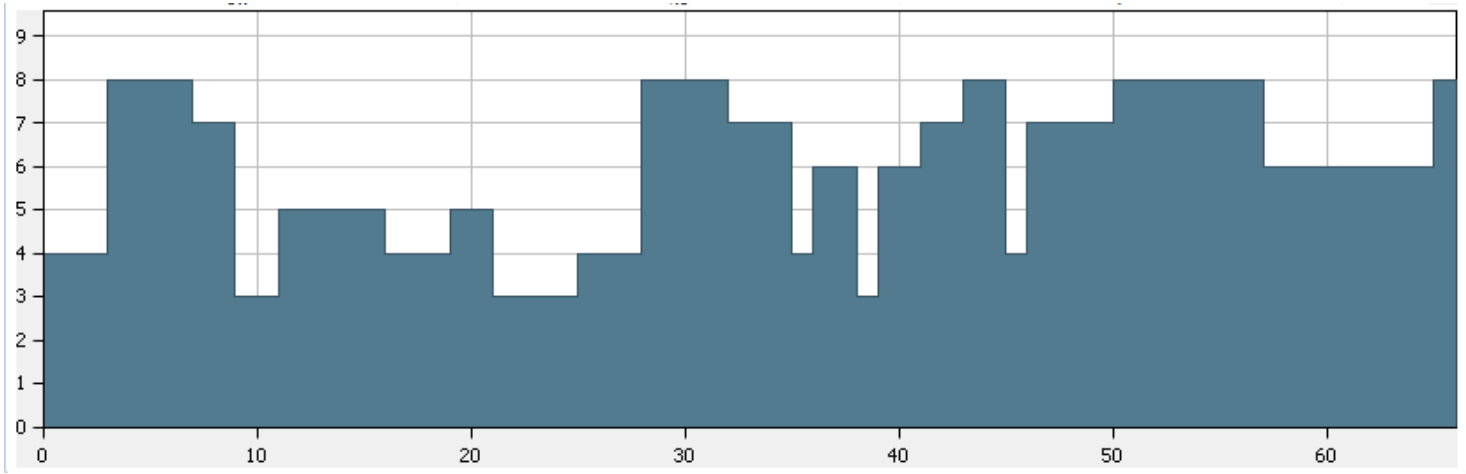

Cumulative Constraints

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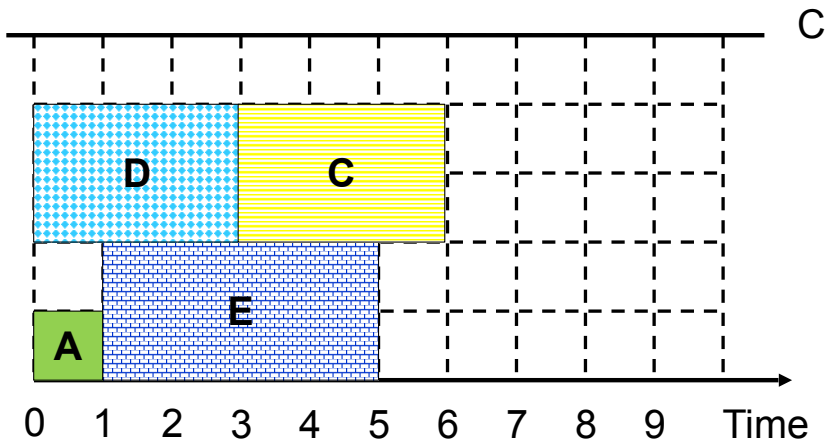
- Given a task set T and a resource with a capacity of C then an assignment S of the start times is a solution iff

$$\forall t \in [t, t_{\max}): \sum_{i \in T: S(i) \le t < S(i)+p_i} c_i \le C$$



Modern CP Solvers

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Key Modeling Features

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- Variables
  - interval variables
  - optional interval variables
  - sequences
- Constraints
  - no-overlap, cumulative, state constraints
  - sequence dependent transition times and costs
  - alternative constraints
- Other features
  - calendars
  - …

Key Features of the Solver

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- Lower bounds
  - temporal and linear relaxations
- Primal solutions
  - large neighborhood search
  - restarts
- Search no-goods
  - never explore the same subtree twice
- primal and dual threads
  - dedicated search methods for proving optimality

Outline

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Production, Manufacturing, Transportation and Logistics

Constraint programming models for integrated container terminal operations☆

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Container Terminals

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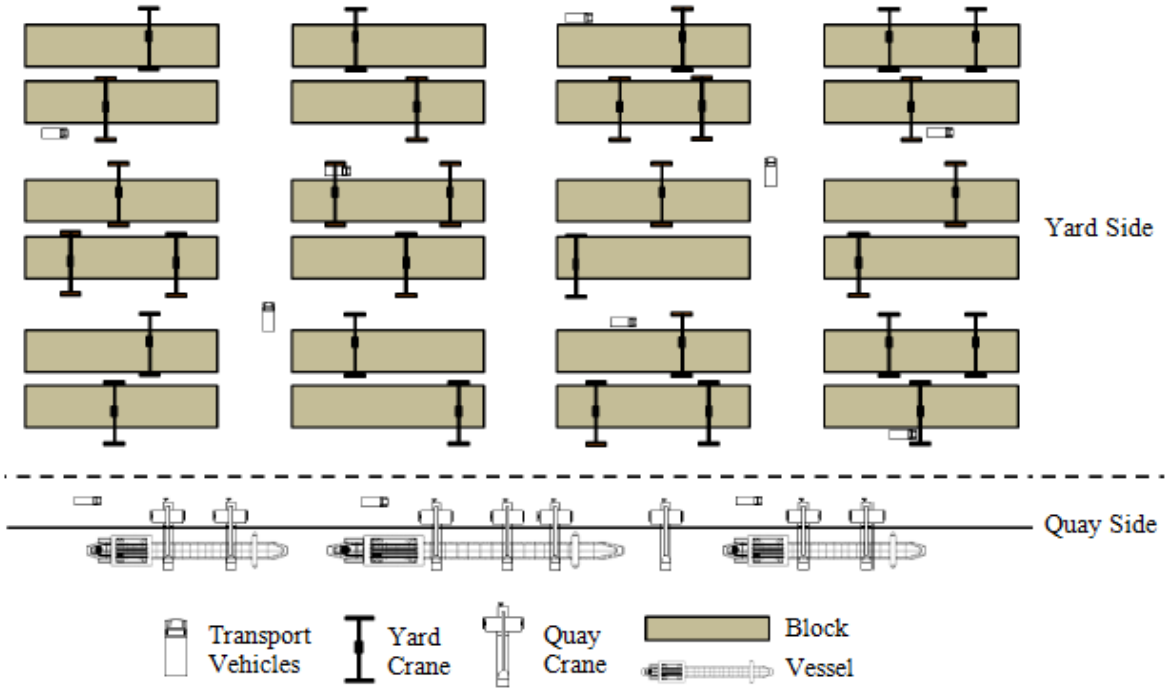
General Picture of the Container Terminal



Quay Cranes

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Yard Cranes

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Yard Cranes

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Yard Trucks

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Container Terminals

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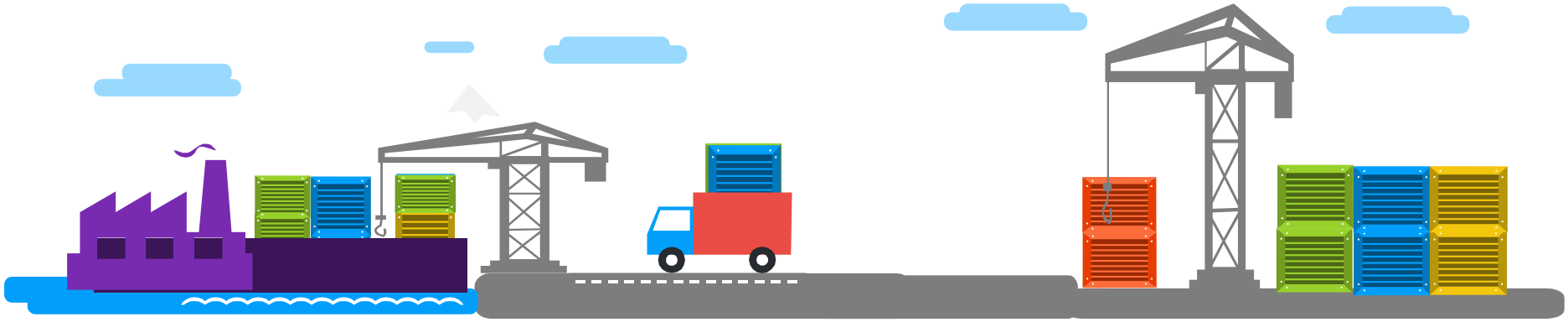
General Operations in Container Terminal



Berth Allocation Problem (BAP): When and in which part of the port the coming vessel should berth. Vessel Stowage Plan has known.

BAP

Quay Crane Assignment and Scheduling Problem (QCASP): When loading and discharging operations should start by which QC.

QCASP

Yard Truck Dispatching Problem (YTDP): When containers should be transported between quay and yard by which YT.

TDP

Yard Crane Assignment and Scheduling Problem: Storage location and stacking position of containers should be determined.

YCASP

Yard Location Assignment and Stacking Problem: Storage location and stacking position of containers should be determined.

Stacking

Container Terminal in Izmir

- Berth allocation factors:
  - Priorities between customers
  - Berthing privileges of certain vessels at specific ports
  - Vessel sizes
  - Depth of water

Problem Definition

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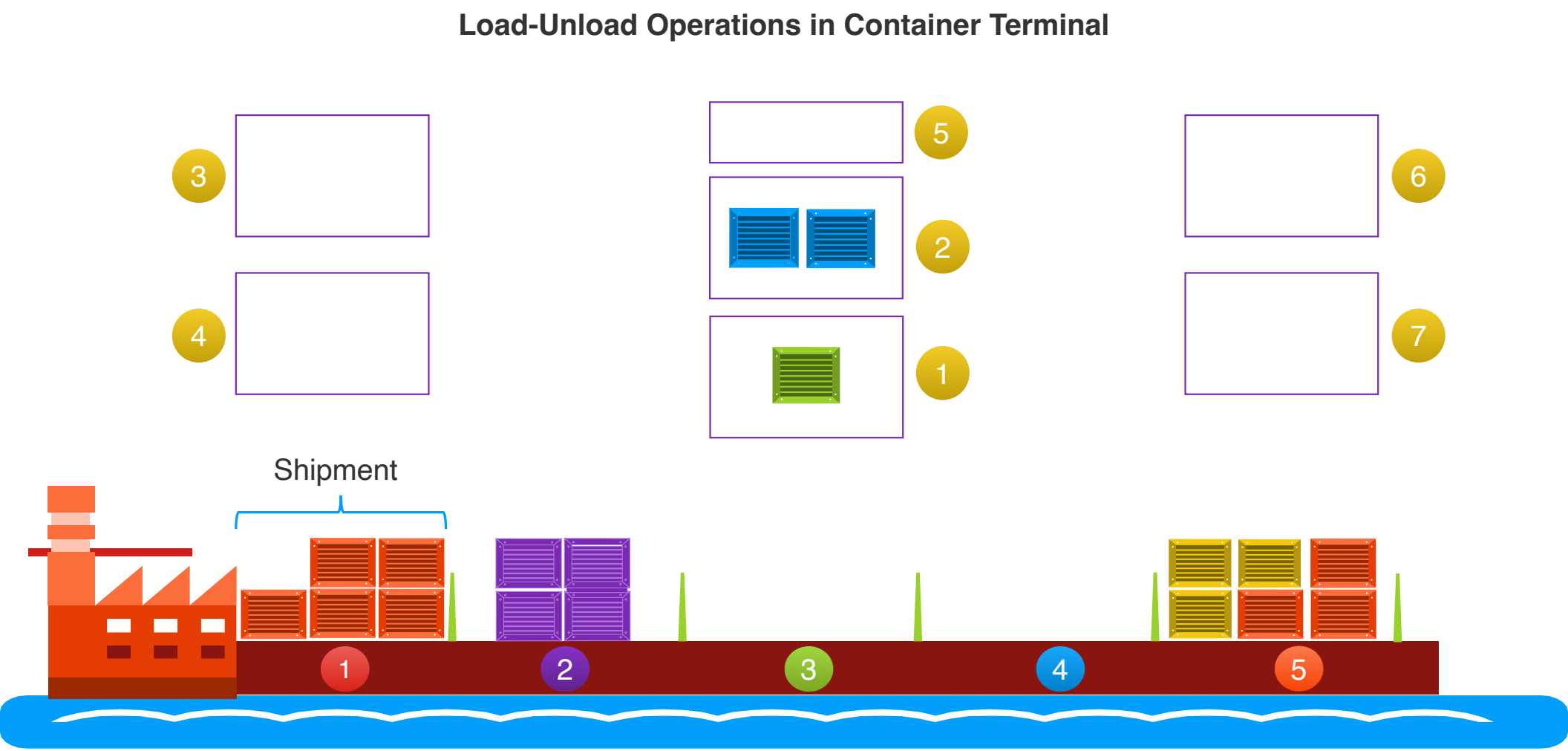

Problem Definition

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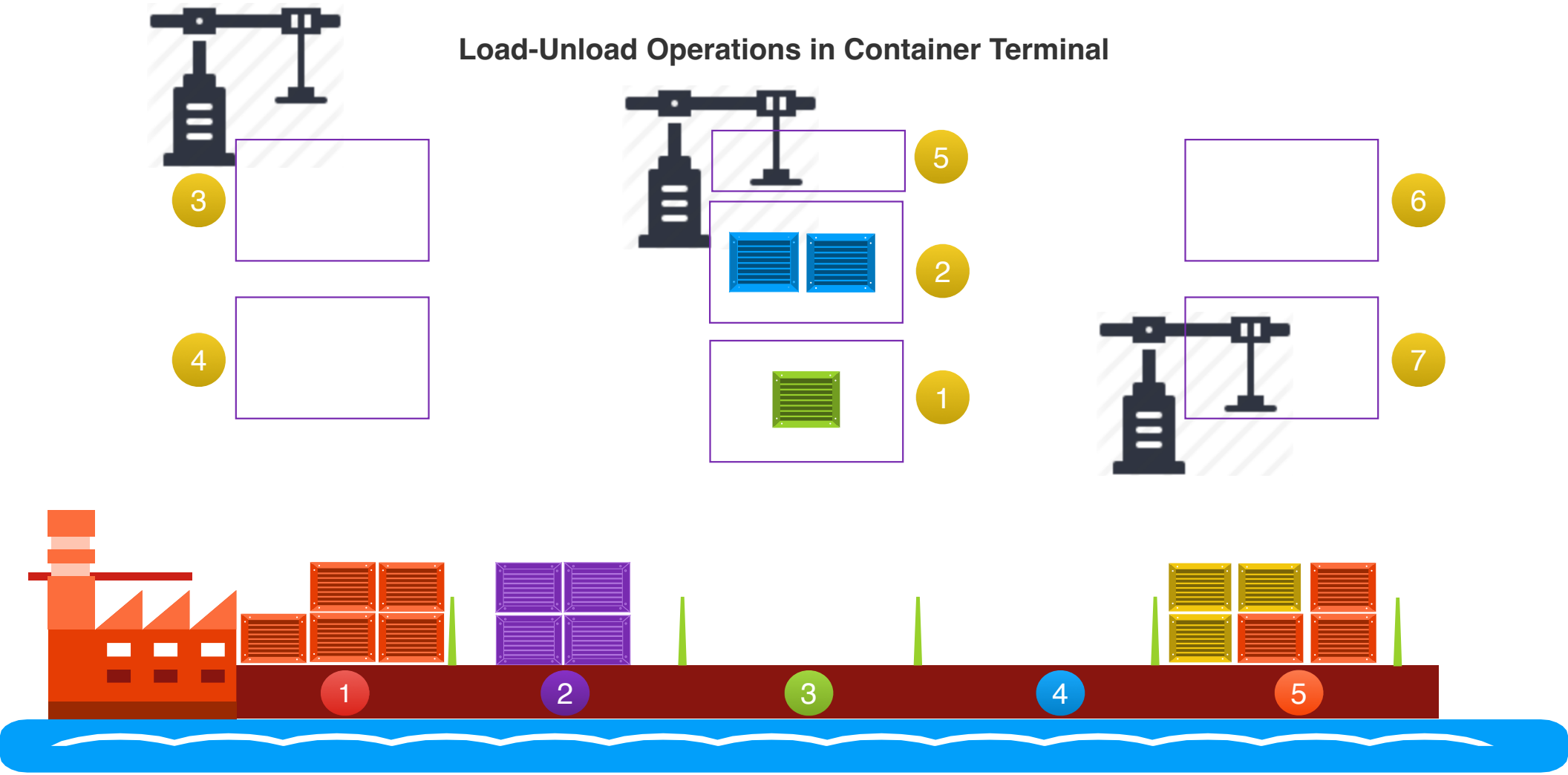

Problem Definition

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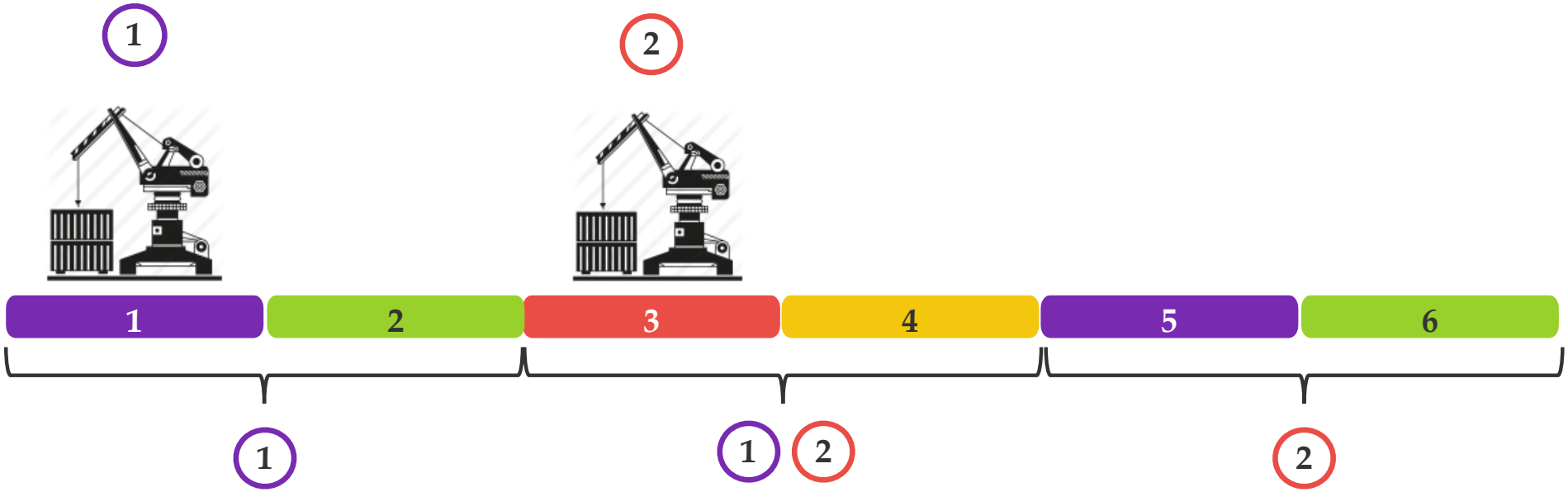
QC Eligibility



Overview

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qc_i : Interval variable for the QC handling of shipment i
yc_i : Interval variable for the YC handling of shipment i
$aqc_{i,j}$: Optional interval variable for shipment i on QC j with duration between Q_i and M
$ayc_{i,k}$: Optional interval variable for shipment i on YC at yard location k with duration between Y_i and M located at yard location k on YT j with duration between 0 and M
qcs_j : Sequence variable for QC j over $\{aqc_{i,j} \mid i \in C\}$
ycs_j : Sequence variable for YC j over $\{ayc_{i,k} \mid i \in C \wedge YC(k) = j\}$
$interfere_{i,v,j,w}$: Sequence variable over $\{aqc_{i,v}, aqc_{j,w}\}$

Listing 1. The simplified CP model: variables.

Objective

$$\min \sum_{s \in S} w_s \left(max\left(\max_{i \in C_u^s} \text{ENDOF}\left(yc_i\right), \max_{j \in C_l^s} \text{ENDOF}\left(qc_j\right)\right)\right) \quad (01)$$

Constraints

$$\text{ALTERNATIVE}\left(qc_i, \text{all}\left(q \text{ in } QC\left(i\right)\right) aqc_{i,q}\right) \quad \forall i \in C \quad (02)$$

$$\text{ALTERNATIVE}\left(yc_i, \text{all}\left(k \text{ in } L_u\right) ayc_{i,k}\right) \quad \forall i \in C_u \quad (03)$$

$$\sum_{i \in C_u} \left(CN_i * \text{PRESENCEOF}\left(ayc_{i,k}\right)\right) \leq Cap_k \quad \forall k \in L_u \quad (04)$$

$$\text{PRESENCEOF}\left(ayc_{i,l_i}\right) = 1 \quad \forall i \in C_l \quad (05)$$

$$\text{STARTBEFORESTART}\left(aqc_{i,n}, ayc_{i,k}, Qt_i + tt_k\right) \forall i \in C_u, k \in L_u, n \in qc_i \quad (06)$$

$$\text{STARTBEFORESTART}\left(ayc_{i,l_i}, aqc_{i,n}, Yt_i + tyt_i\right) \quad \forall i \in C_l, n \in qc_i \quad (07)$$

$$\text{STARTBEFORESTART}\left(aqc_{i,m}, aqc_{j,n}\right) \quad \forall i, j \in P, m \in qc_i, n \in qc_j \quad (08)$$

$$\text{NOOVERLAP}\left(qcs_m, eqc_{i,j}\right) \quad \forall m \in QC \quad (09)$$

$$\text{NOOVERLAP}\left(ycs_m, eyc_{i,j}\right) \quad \forall m \in YC \quad (10)$$

$$\text{NOOVERLAP}\left(interfere_{i,v,j,w}, \Delta_{i,j}^{v,w}\right) \quad \forall i, j \in C, v \in qc_i, w \in qc_j, \Delta_{i,j}^{v,w} > 0 \quad (11)$$

Listing 2. The simplified CP Model: Objective and Constraints.

Decision Variables

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Variables

qc_i : Interval variable for the QC handling of shipment i

yc_i : Interval variable for the YC handling of shipment i

$aqc_{i,j}$: Optional interval variable for shipment i on QC j with duration Q_i

$ayc_{i,k}$: Optional interval variable for shipment i on YC at yard location k with duration Y_i

No Overlap

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qcs_j : Sequence variable for QC j over $\{aqc_{i,j} \mid i \in C\}$

ycs_j : Sequence variable for YC j over $\{ayc_{i,k} \mid i \in C \wedge YC(k) = j\}$

$$\text{NOOVERLAP}\,(ycs_m, eyc_{i,j}) \quad \forall m \in YC$$

$$\text{NOOVERLAP}\,(qcs_m, eqc_{i,j}) \quad \forall m \in QC$$

Precedence Constraints

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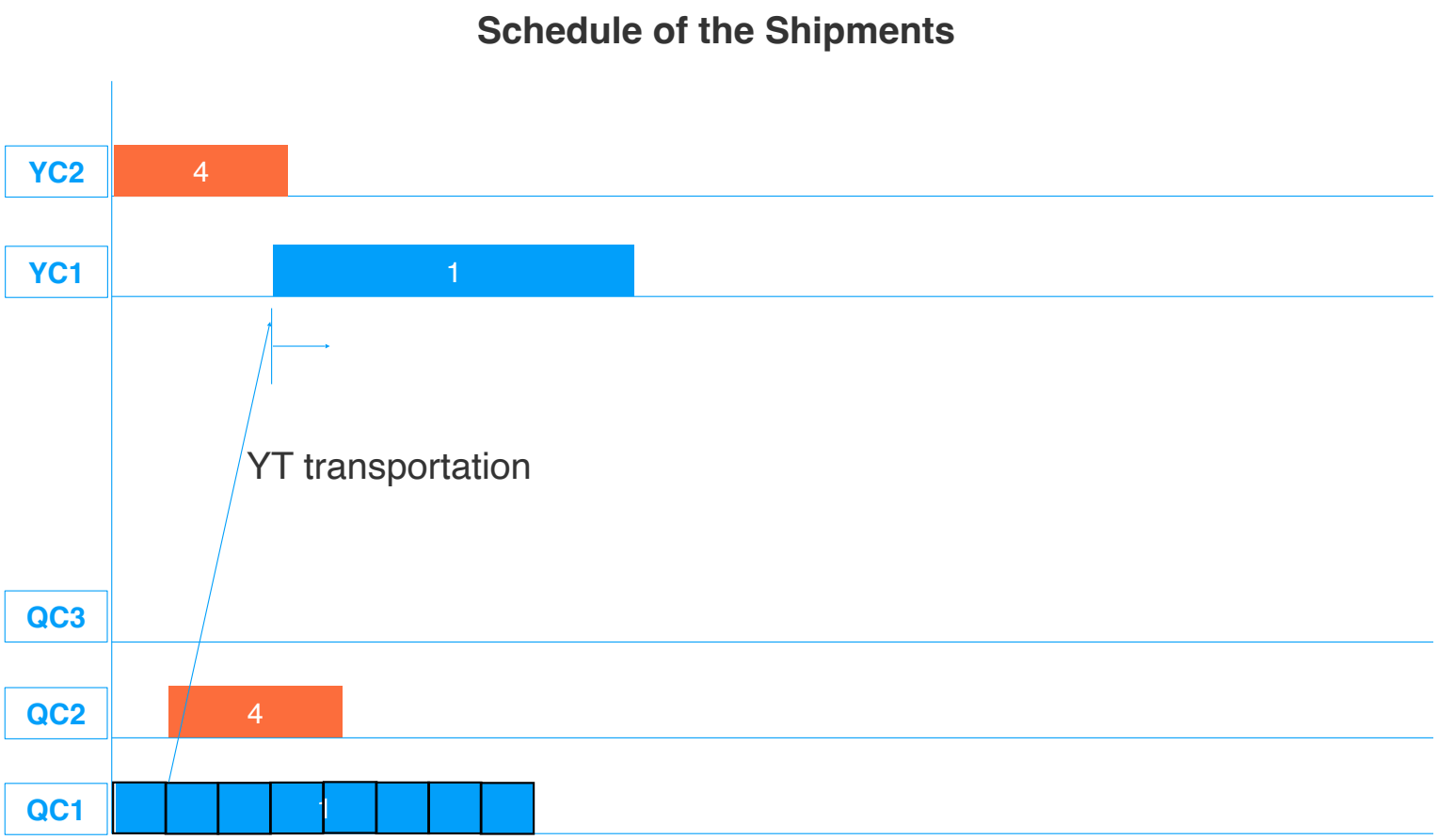

Interferences

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- No overlap + distance

$interfere_{i,v,j,w}$: Sequence variable over $\{aqc_{i,v}, aqc_{j,w}\}$

$$\text{NOOVERLAP}\left(interfere_{i,v,j,w}, \Delta_{i,j}^{v,w}\right) \quad \forall i,j \in C, v \in qc_i, w \in qc_j, \Delta_{i,j}^{v,w} > 0$$

The Model

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- Minimizing the weighted completion times of each vessel

$$\min \sum_{s \in S} w_s \left(max \left(\max_{i \in C_u^s} \text{ENDOF}\left(yc_i\right), \max_{j \in C_l^s} \text{ENDOF}\left(qc_j\right) \right) \right)$$

Scheduling Yard Trucks

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yt_i : Interval variable for the YT handling of container i
$ayt_{i,k,j}$: Optional interval variable for container i transported to or initially located at yard location k on YT j with duration between 0 and M
yts_j : Sequence variable for YT j over $\{ayt_{i,j} \mid i \in K\}$
$contQC_i$: Interval variable for container i on QC with duration $Qtime_i$
$contYC_i$: Interval variable for container i on YC with duration $Ytime_i$

Listing 3. The CP Model for the IPCTP: Additional variables.

$$\text{ALTERNATIVE}\left(yt_i, \text{all}\left(v \text{ in } YT\right) ayt_{i,l_j,v}\right) \quad \forall i \in H_j, j \in C_l \tag{12}$$
$$\text{ALTERNATIVE}\left(yt_i, \text{all}\left(v \text{ in } YT, k \text{ in } L_u\right) ayt_{i,k,v}\right) \quad \forall i \in K_u \tag{13}$$
$$\sum_{v \in YT} \text{PRESENCEOF}\left(ayt_{i,l_j,v}\right) = 1 \quad \forall i \in H_j, j \in C_l \tag{14}$$
$$\sum_{v \in YT} \text{PRESENCEOF}\left(ayt_{i,k,v}\right) = \text{PRESENCEOF}\left(ayc_{j,k}\right) \quad \forall i \in H_j, j \in C_u, k \in L_u \tag{15}$$
$$\sum_{v \in YT} \text{SIZEOF}\left(ayt_{i,l_j,v}\right) \geq tyt_j \quad \forall i \in H_j, j \in C_l \tag{16}$$
$$\sum_{v \in YT} \text{SIZEOF}\left(ayt_{i,k,v}\right) \geq tt_k * \text{PRESENCEOF}\left(ayc_{j,k}\right) \forall k \in L_u, i \in H_j, j \in C_u \tag{17}$$
$$\text{STARTOF}\left(contQC_i\right) = \sum_{q \in QC} \text{STARTOF}\left(aqc_{j,q}\right) \quad i = \min\left(H_j\right) \tag{18}$$
$$\text{STARTOF}\left(contQC_j\right) \geq \text{ENDOF}\left(contQC_i\right) \quad \forall k \in C, i, j \in H_k | j = i + 1 \tag{19}$$
$$\text{STARTOF}\left(contYC_i\right) = \sum_{k \in L} \text{STARTOF}\left(ayc_{j,k}\right) \quad i = \min\left(H_j\right) \tag{20}$$
$$\text{STARTOF}\left(contYC_j\right) \geq \text{ENDOF}\left(contYC_i\right) \quad \forall k \in C, i, j \in H_k | j = i + 1 \tag{21}$$
$$\max_{i \in H_j} \text{ENDOF}\left(contQC_i\right) = \sum_{q \in QC} \text{ENDOF}\left(aqc_{j,q}\right) \quad \forall j \in C \tag{22}$$
$$\max_{i \in H_j} \text{ENDOF}\left(contYC_i\right) = \sum_{k \in L} \text{ENDOF}\left(ayc_{j,k}\right) \quad \forall j \in C \tag{23}$$
$$\text{ENDBEFORESTART}\left(contQC_i, ayt_{i,k,v}\right) \quad \forall i \in K_u, k \in L_u, v \in YT \tag{24}$$
$$\text{ENDBEFORESTART}\left(ayt_{i,k,v}, contYC_i\right) \quad \forall i \in K_u, k \in L_u, v \in YT \tag{25}$$
$$\text{ENDBEFORESTART}\left(contYC_i, ayt_{i,l_j,v}\right) \quad \forall i \in H_j, j \in C_l, v \in YT \tag{26}$$
$$\text{ENDBEFORESTART}\left(ayt_{i,l_j v}, contQC_i\right) \quad \forall i \in H_j, j \in C_l, v \in YT \tag{27}$$
$$\text{NOOVERLAP}\left(yts_m, eyt_{i,j}\right) \quad \forall m \in YT \tag{28}$$

Listing 4. The CP Model for the IPCTP: Additional Constraints.

Import-Export Rate 20%

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			MIP			CP		
U-L Rate	# of Bays	# of Shipment	Avg. Obj.	Avg. CPU	Avg. GAP%	Avg. Obj.	Avg. CPU	RPD%
2	4	5	293	0.40	0.00	293	0.05	0.00
		10	542$^{0/5}$	3600.05	0.36	542	9.41	0.00
		15	740$^{0/5}$	3601.68	0.69	740^{1}	734.91	0.00
		20	822$^{0/5}$	3601.39	0.75	820^{1}	788.81	0.00
		25	1018$^{2/3}$	3600.81	0.77	998^{2}	1791.34	0.00
	6	5	179	0.16	0.00	179	0.14	0.00
		10	360$^{0/1}$	1717.16	0.05	360	6.20	0.00
		15	544$^{0/5}$	3600.50	0.58	541	36.47	0.00
		20	661$^{1/4}$	3600.75	0.67	647	274.15	0.00
		25	NA	NA	NA	752	479.15	0.00
	8	5	427	0.16	0.00	427	0.40	0.00
		10	507	98.30	0.00	507	10.26	0.00
		15	844$^{0/5}$	3600.26	0.48	841	150.59	0.00
		20	967$^{2/3}$	3600.13	0.57	1052	542.60	0.00
		25	1466$^{4/1}$	3614.05	0.64	1320	1255.55	1.26

Import-Export Rate 20%

AI Institute for Advances in Optimization

			MIP			CP		
U-L Rate	# of Bays	# of Shipment	Avg. Obj.	Avg. CPU	Avg. GAP%	Avg. Obj.	Avg. CPU	RPD%
3	4	5	207	**0.40**	0.00	207	**0.21**	0.00
		10	426$^{0/4}$	**3297.84**	0.31	426	**6.80**	0.00
		15	643$^{0/5}$	**3605.24**	0.65	641	**79.30**	0.00
		20	865$^{0/5}$	**3600.71**	0.75	860^{1}	**897.30**	0.00
		25	824$^{3/2}$	**3601.27**	0.73	842^{2}	**1895.17**	0.00
	6	5	248	**0.16**	0.00	248	**0.15**	0.00
		10	401$^{0/1}$	**1449.26**	0.08	401	**8.70**	0.00
		15	482$^{0/1}$	**3600.39**	0.50	482	**539.06**	0.00
		20	626$^{1/4}$	**3600.50**	0.65	614^{1}	**778.93**	0.00
		25	988$^{2/3}$	**3600.22**	0.74	856^{4}	**2938.55**	0.00
	8	5	418	**0.22**	0.00	418	**1.27**	0.00
		10	655	**449.44**	0.00	655	**26.02**	0.00
		15	895$^{0/5}$	**3600.45**	0.47	882	**160.79**	0.00
		20	1272$^{1/4}$	**3611.86**	0.63	1113^{1}	**1524.57**	0.38
		25	NA	**NA**	NA	1382^{3}	**2681.45**	2.87

Outline

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- Brief Overview to constraint programming
- Scheduling with constraint programming
- Hybridizing constraint programming and MIP
- Learning-based constraint programming
- Learning Optimization Proxies

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ARTICLE

A branch-and-price-and-check model for the vehicle routing problem with location congestion

Authors: Edward Lam, Pascal Van Hentenryck Authors Info & Claims

Constraints, Volume 21, Issue 3 • July 2016 • pp 394–412 • https://doi.org/10.1007/s10601-016-9241-2

CP and MIP

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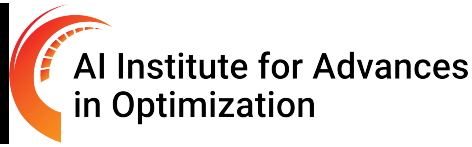
- Trends in combinatorial optimization
  - logical Benders decomposition, and branch & check
  - generalizes Benders decomposition with combinatorial subproblems
- Branch and price and check
  - generalizes the idea to branch and price
- Beautiful synergies between
  - mathematical and constraint programming

VRP with Location Constraints



- The problem
  - multiple vehicles
  - pickup and delivery constraints
  - time windows
  - capacity constraints
- Location constraints
  - number of parking slots at an airport
  - number of landings and takeoffs at an airport in a given intervals

Traditional VRP

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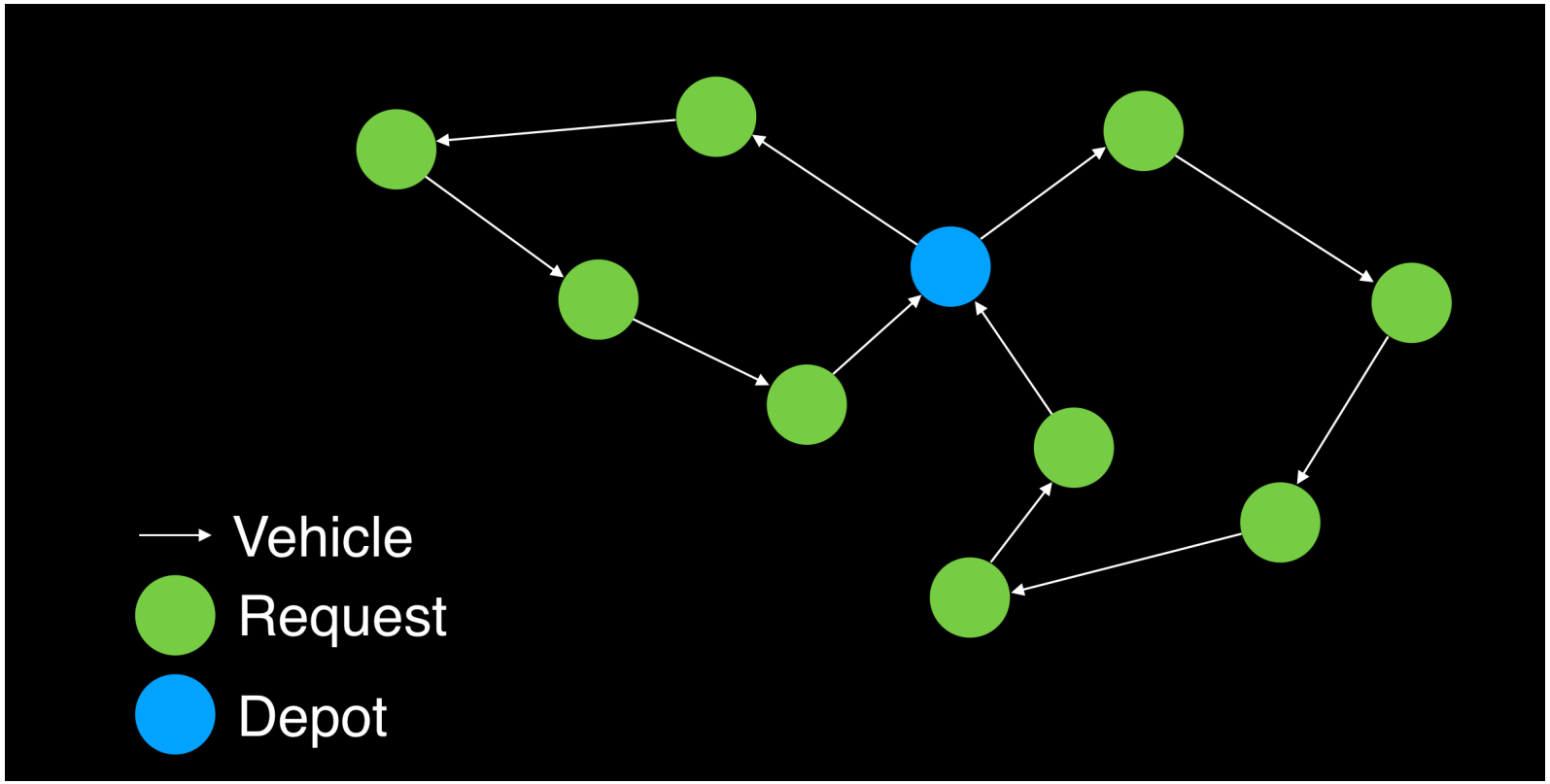

Locations

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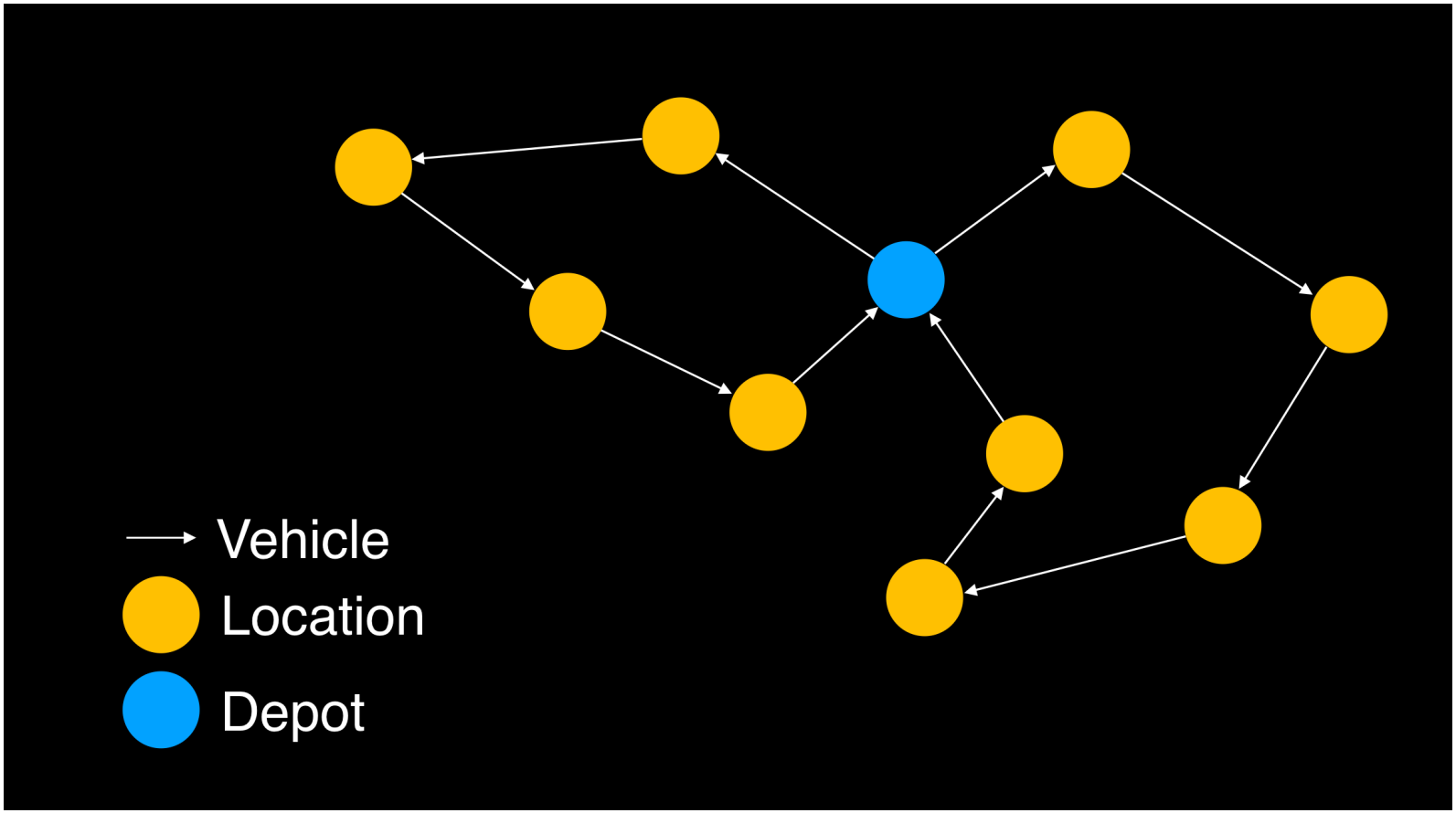

Locations and Requests

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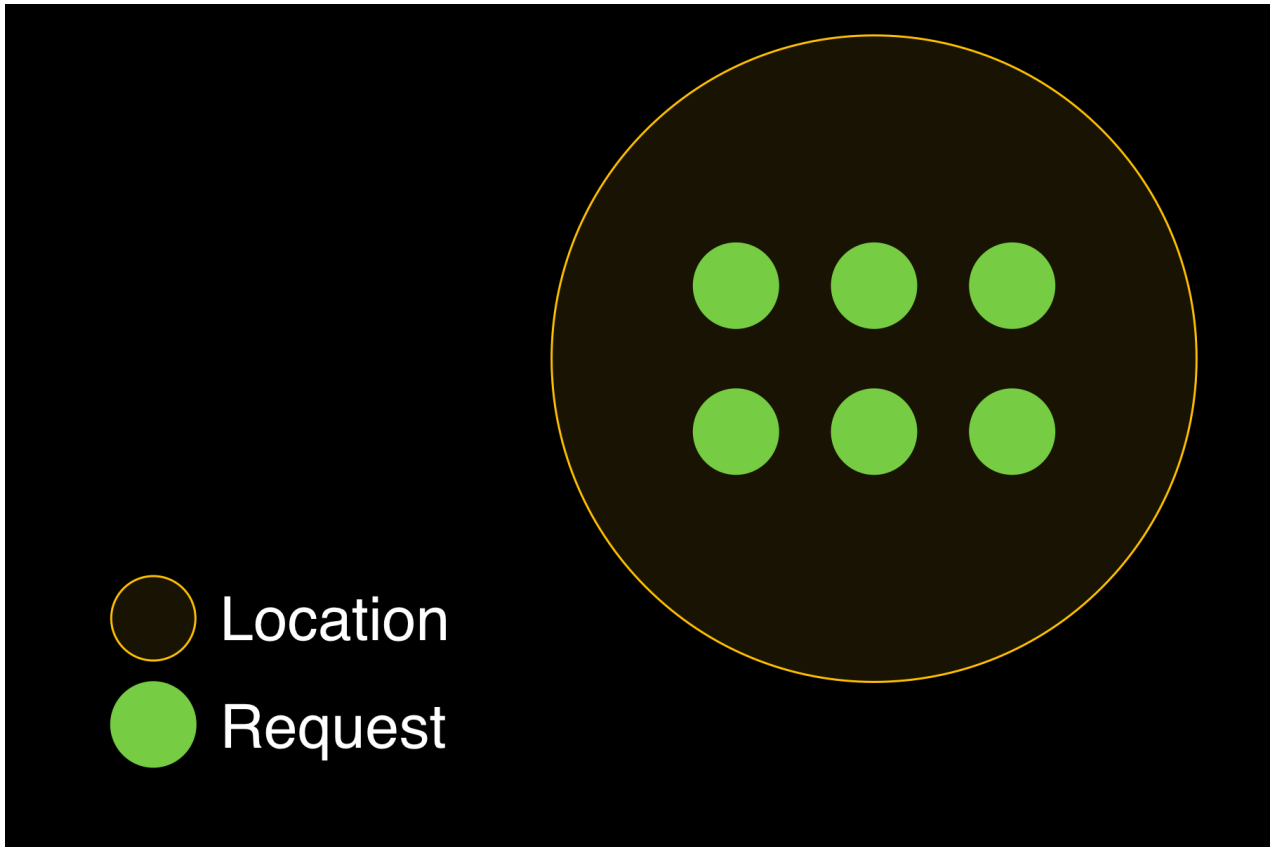

Presence Constraints

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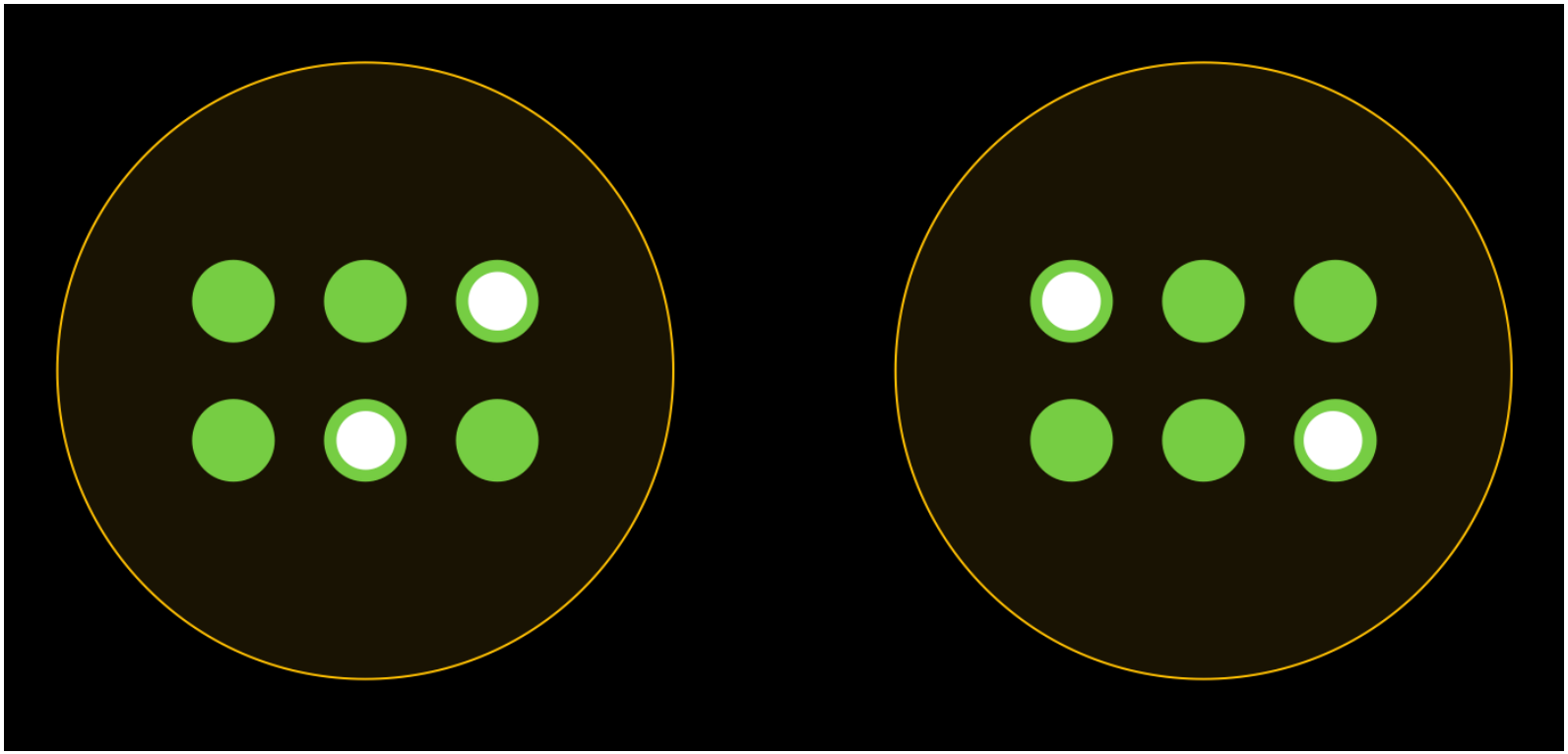

Motivating Applications

- Humanitarian and military logistics
  - number of possible landings in a base
  - number of parking spots available at a base
  - fuel available at an airport
- City logistics
  - parking at hubs
  - number of possible spots at transit centers

Branch and Price and Check

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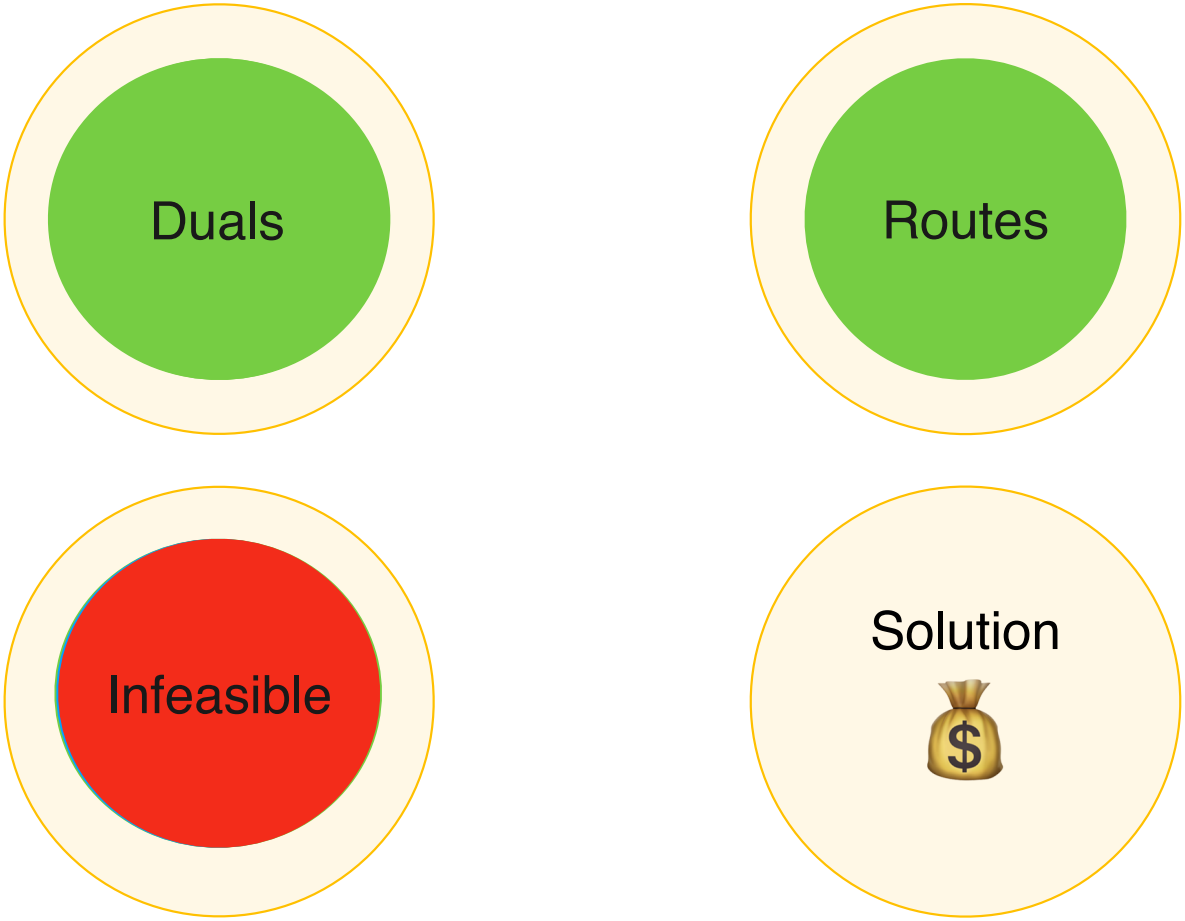

The Master Problem

r is a route

every pickup is covered by a single route

$$\min \sum_{r \in \Omega} c_r x_r$$

subject to

$$\sum_{r \in \Omega} a_{i,r} x_r = 1, \qquad \forall i \in \mathcal{P},$$

$$x_r \in [0,1], \qquad \forall r \in \Omega.$$

The Pricing Problem

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- Standard labeling algorithm
  - solving a resource-constrained shortest path with
    - time windows
    - pickup and delivery constraints
    - capacity constraints on the load of the vehicles

- Reduced costs

$$\overline{d}_{ij} = \begin{cases} d_{ij} - \pi_i + \sum_{B \in \mathcal{B}} 1_{Bij} \mu_B, & \forall i \in \mathcal{P}, j \in \mathcal{N}, \\ d_{ij} \qquad\quad + \sum_{B \in \mathcal{B}} 1_{Bij} \mu_B, & \forall i \in \mathcal{N} \setminus \mathcal{P}, j \in \mathcal{N}, \end{cases}$$

The Separation Problem

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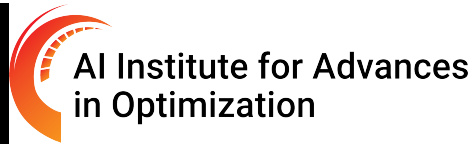
- The subproblem is heavily combinatorial
  - no simple Benders composition

$$
\begin{aligned}
&\text{arr}(i) \leq \text{serv}(i), && \forall i \in \mathcal{R},\\
&\text{serv}(i) + t(i) \leq \text{dep}(i), && \forall i \in \mathcal{R},\\
&\text{arr}(i) = \text{serv}(i) = \text{dep}(i), && \forall i \in \mathcal{S} \cup \mathcal{E},\\
&\text{dep}(i) + d(i, \text{succ}(i)) = \text{arr}(\text{succ}(i)), && \forall i \in \mathcal{R} \cup \mathcal{S},\\
&\textsc{Cumulative}(\{\text{serv}(i) : i \in \mathcal{R}_l\}, \{t(i) : i \in \mathcal{R}_l\}, \mathbf{1}, C_l), && \forall l \in \mathcal{L}.
\end{aligned}
$$

The Separation problem



- How do we exclude incompatible routes?
  - in other words, what are the combinatorial Benders cuts?

Combinatorial Benders Cuts

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$$\sum_{r \in \Omega} B_r x_r \leq |B| - 1, \qquad \forall B \in \mathcal{B},$$

Numbers of arcs in B for a route r

Arcs in the infeasible routes

Branch and Price and Check

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1. Master
   - solve the master problem
2. Separation
   - check if the “integer” routes satisfy the cumulative constraints; add the cut if infeasible & go to (1)
3. Feasibility
   - if all routes are integral, a solution has been found; go to (6)
4. Pricing
   - generate new routes; if any, go to (1)
5. Branching
   - consider a route and branch on the prefixes
6. Node selection
   - take an open node and go to (1)

VRP with Location Constraints

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- MIP
  - cumulative constraints implemented with logical constraints (no time-indexed formulation)
- CP
  - VRP + cumulative constraints
- Branch and Price and Check
  - branch and price
  - checking cumulative constraints with CP

Experimental Results

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			Instance Set 1								Instance Set 2								Instance Set 3							
			TS	MIP			CP	BPC			TS	MIP			CP	BPC			TS	MIP			CP	BPC		
$\|\mathcal{L}\|$	$\|\mathcal{P}\|$	C_l	UB	UB	LB	Gap	UB	UB	LB	Gap	NG	UB	UB	LB	Gap	UB	UB	LB	Gap	NG	UB	UB	LB	Gap	UB	UB	LB	Gap	NG
8	80	1	–	–	0	–	×	–	746	–	3,171	–	–	0	–	×	–	1,176	–	28,124	–	–	0	–	×	–	390	–	2,865
		2	–	–	0	–	×	–	746	–	3,169	–	–	0	–	×	–	1,176	–	27,082	–	–	0	–	–	–	390	–	17
		3	–	–	0	–	–	–	746	–	13,897	–	–	0	–	×	–	1,176	–	28,660	–	–	0	–	–	–	391	–	2,730
		4	–	–	0	–	–	845	755	10.7%	2,971	–	–	0	–	–	–	1,176	–	28,040	393	–	0	–	–	**393**	393	0.0%	111
		5	794	–	0	–	–	799	755	5.5%	1,903	–	–	0	–	–	1,370	1,182	13.7%	20,805	393	–	0	–	–	**393**	393	0.0%	0
		6	794	–	0	–	–	795	761	4.3%	149	1,195	–	0	–	–	1,195	1,182	1.1%	2,565	393	–	0	–	–	**393**	393	0.0%	0
		7	794	–	0	–	–	794	762	4.0%	0	1,195	–	0	–	–	1,194	1,184	0.8%	1,729	393	–	0	–	–	**393**	393	0.0%	0
		8	794	–	0	–	–	794	762	4.0%	0	1,195	–	0	–	–	1,195	1,184	0.9%	2,242	393	–	0	–	–	**393**	393	0.0%	0
		9	794	–	0	–	–	794	762	4.0%	0	1,195	–	0	–	–	1,195	1,186	0.8%	0	393	–	0	–	–	**393**	393	0.0%	0
		10	794	–	0	–	–	794	762	4.0%	0	1,195	–	0	–	–	1,195	1,186	0.8%	0	393	–	0	–	–	**393**	393	0.0%	0
		11	794	–	0	–	–	794	762	4.0%	0	1,195	–	0	–	–	1,195	1,186	0.8%	0	393	–	0	–	–	**393**	393	0.0%	0
		12	794	–	0	–	–	794	762	4.0%	0	1,195	–	0	–	–	1,195	1,186	0.8%	0	393	–	0	–	–	**393**	393	0.0%	0
		13	794	–	0	–	–	794	762	4.0%	0	1,195	–	0	–	–	1,195	1,186	0.8%	0	393	–	0	–	–	**393**	393	0.0%	0
		14	794	–	0	–	–	794	762	4.0%	0	1,195	–	0	–	–	1,195	1,186	0.8%	0	393	–	0	–	–	**393**	393	0.0%	0
		15	794	–	0	–	–	793	763	3.8%	0	1,195	–	0	–	–	1,195	1,186	0.8%	0	393	–	0	–	–	**393**	393	0.0%	0

			Instance Set 1									Instance Set 2									Instance Set 3								
			TS	MIP			CP	BPC				TS	MIP			CP	BPC				TS	MIP			CP	BPC			
$\|\mathcal{L}\|$	$\|\mathcal{P}\|$	C_l	UB	UB	LB	Gap	UB	UB	LB	Gap	NG	UB	UB	LB	Gap	UB	UB	LB	Gap	NG	UB	UB	LB	Gap	UB	UB	LB	Gap	NG
11	80	1	–	–	4	–	×	–	876	–	8,763	–	–	0	–	×	–	1,260	–	24,022	–	–	0	–	×	–	411	–	10,388
		2	–	–	4	–	×	–	872	–	12,877	–	–	0	–	×	–	1,260	–	25,102	–	–	0	–	–	–	407	–	12
		3	–	–	4	–	–	–	875	–	9,282	–	–	0	–	×	–	1,260	–	24,733	–	–	0	–	–	–	410	–	780
		4	–	–	4	–	–	–	881	–	914	–	–	0	–	–	–	1,260	–	25,974	413	–	0	–	–	413	411	0.5%	184
		5	883	–	4	–	–	**883**	883	0.0%	32	–	–	0	–	–	1,340	1,260	6.0%	11,902	413	–	0	–	–	413	412	0.2%	0
		6	883	–	4	–	–	**883**	883	0.0%	0	–	–	0	–	–	1,319	1,266	4.0%	5,011	413	–	0	–	–	413	412	0.2%	0
		7	883	–	4	–	–	**883**	883	0.0%	0	–	–	0	–	–	1,312	1,269	3.3%	2,083	413	–	0	–	–	413	412	0.2%	0
		8	883	–	4	–	–	**883**	883	0.0%	0	1,280	–	0	–	–	1,280	1,270	0.8%	2,165	413	–	0	–	–	413	412	0.2%	0
		9	883	–	4	–	–	**883**	883	0.0%	0	1,280	–	0	–	–	1,280	1,272	0.6%	0	413	–	0	–	–	413	412	0.2%	0
		10	883	–	4	–	–	**883**	883	0.0%	0	1,280	–	0	–	–	1,280	1,272	0.6%	0	413	–	0	–	–	413	412	0.2%	0
		11	883	–	4	–	–	**883**	883	0.0%	0	1,280	–	0	–	–	1,280	1,272	0.6%	0	413	–	0	–	–	413	412	0.2%	0
		12	883	–	4	–	–	**883**	883	0.0%	0	1,280	–	0	–	–	1,280	1,272	0.6%	0	413	–	0	–	–	413	412	0.2%	0
		13	883	–	4	–	–	**883**	883	0.0%	0	1,280	–	0	–	–	1,280	1,272	0.6%	0	413	–	0	–	–	413	412	0.2%	0
		14	883	–	4	–	–	**883**	883	0.0%	0	1,280	–	0	–	–	1,280	1,272	0.6%	0	413	–	0	–	–	413	411	0.5%	0
		15	883	–	4	–	–	**883**	883	0.0%	0	1,280	–	0	–	–	1,280	1,272	0.6%	0	413	–	0	–	–	413	412	0.2%	0

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- Learning Optimization Proxies

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Constraints (2011) 16:250–282
DOI 10.1007/s10601-010-9103-2

Explaining the `cumulative` propagator

Andreas Schutt · Thibaut Feydy · Peter J. Stuckey · Mark G. Wallace

Learning

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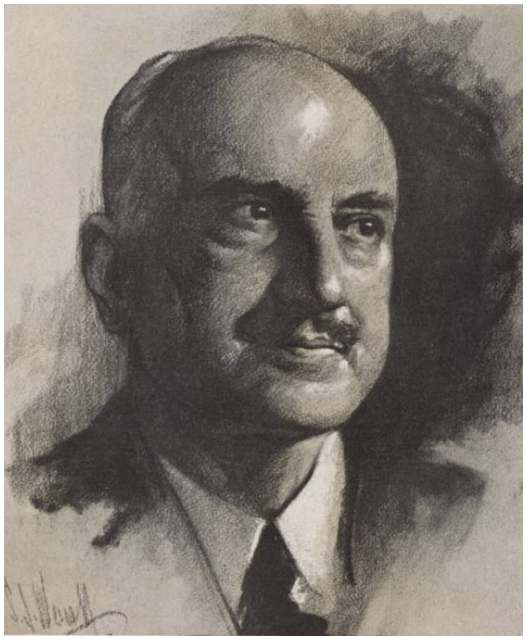
- Those who forget the past are doomed to repeat it



George Santayana

How Much Search is Repeated?

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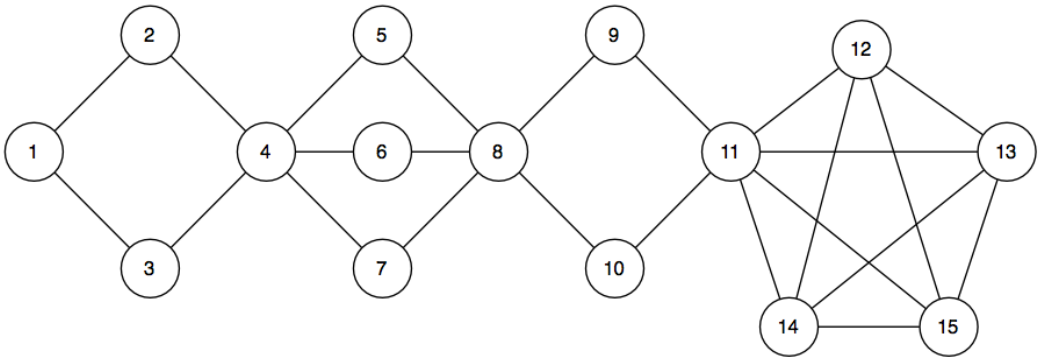
- Color the following graph with 4 colors



- Traditional search: 462672 failures
- Constraint Programming with Learning
  - 18 failures

How Much Search is Repeated?

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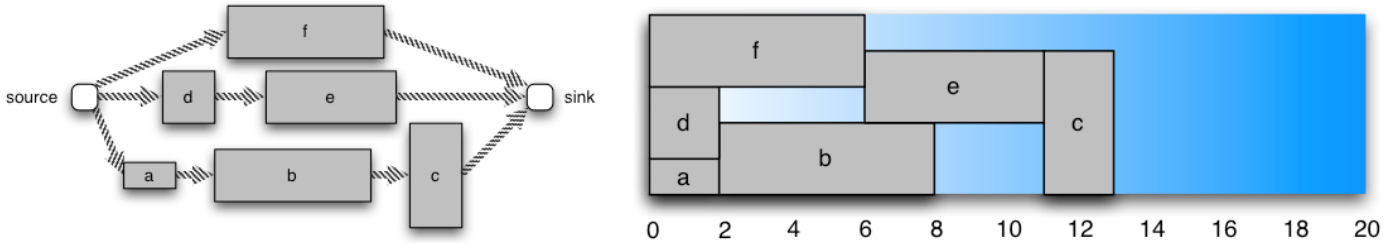
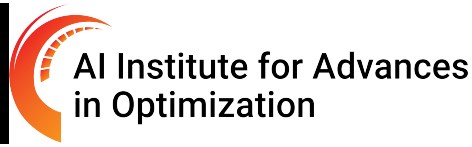
- Resource Constrained Project Scheduling
  - BL instance (20 tasks)



- Input order: 934,535 failures
  - With learning: 931 failures
- Smallest start time order: 296,567 failures
  - With learning: 551 failures
- Activity-based search: > 2,000,000 failures
  - With learning: 1144 failures

Conflict-Based Learning in CP



- Can we learn from failures?
  - A long history (from at least 1986)
  - but it has not really worked until recently
- Conflict-based learning
  - learn a new clause (1UIP) when encountering a failure

Inference Graph

- What does pruning do in CP?
  - $x = v$, $x \neq v$, $x \geq v$, $x \leq v$ (in its simplest form)
    - domain events
  - denote them as $[x = v]$, $[x \neq v]$, $[x \geq v]$, $[x \leq v]$
- The graph captures the CP Inferences
  - e.g. $[x \leq 2]$ and $x \geq y$ implies that $[y \leq 2]$
  - inference: $[x \leq 2] \rightarrow [y \leq 2]$

Inference Graph

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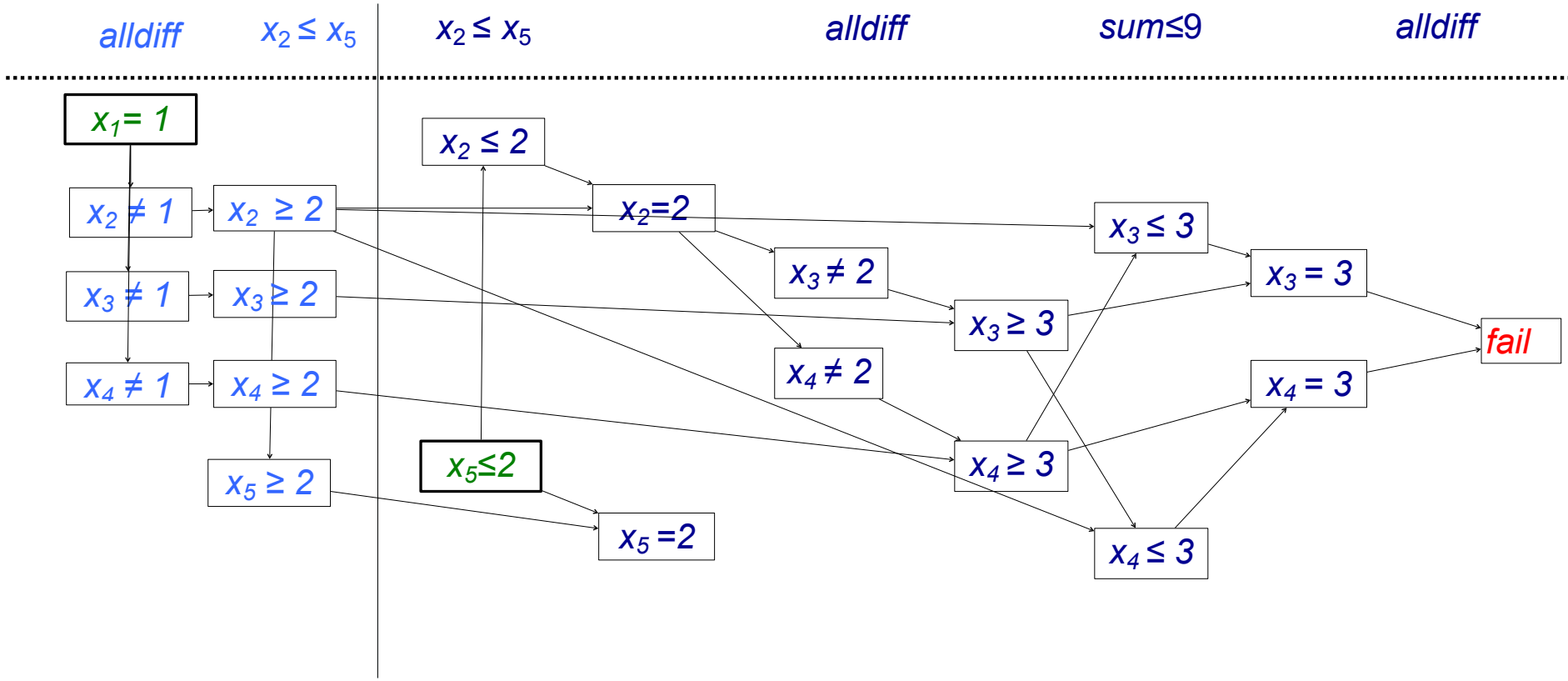

Nogood Learning

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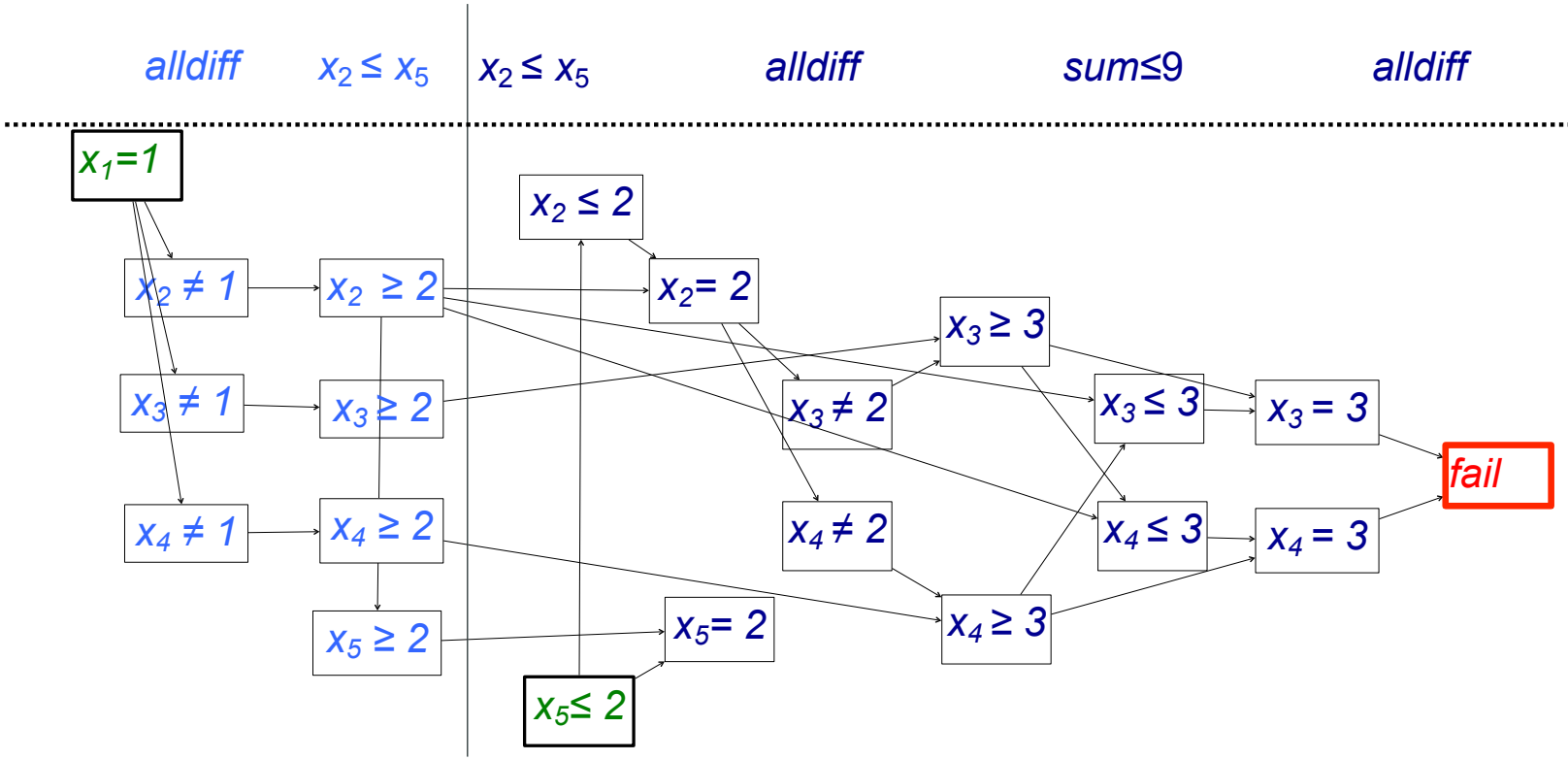

Nogood Learning

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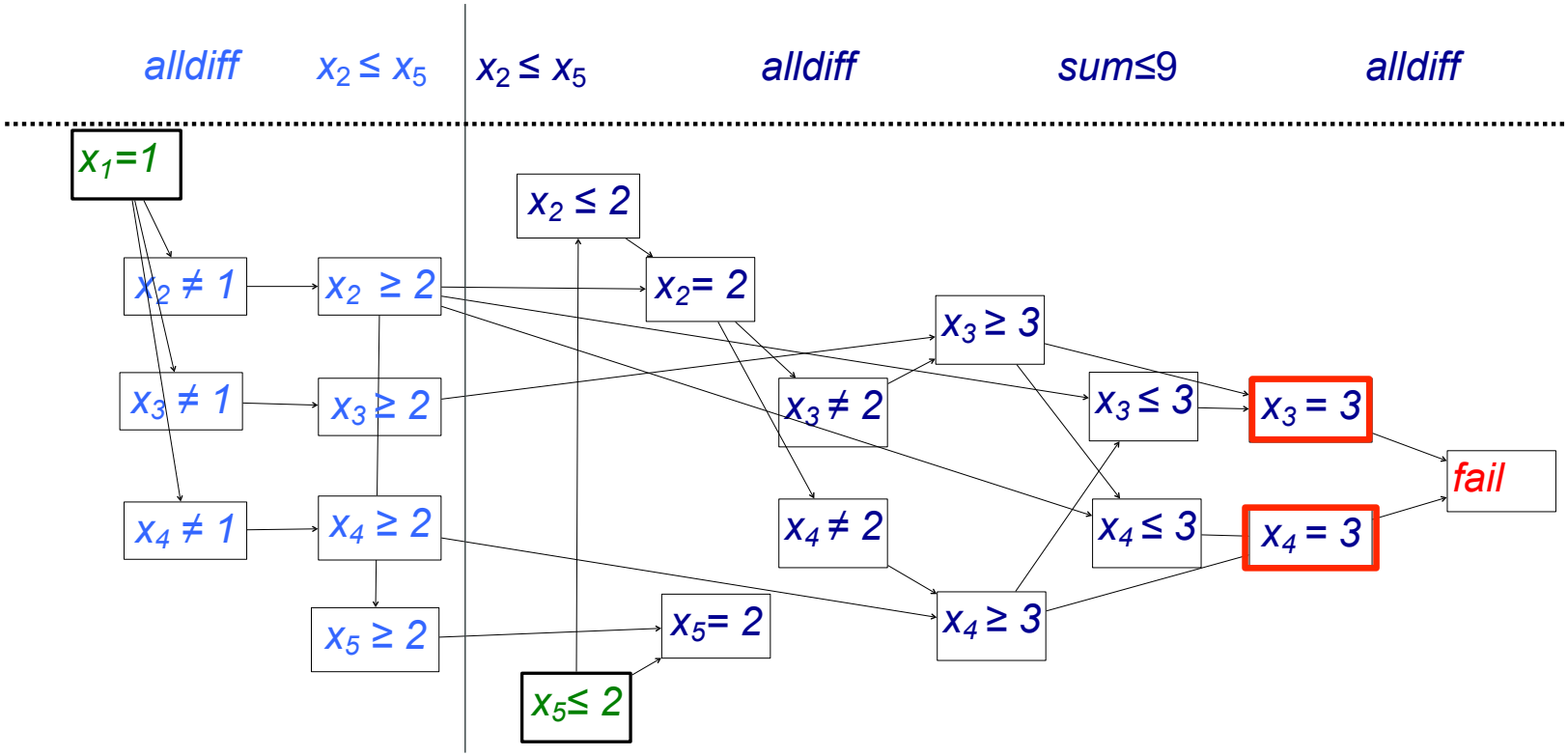

$x_3=3 \wedge x_4=3 \rightarrow$ *false*

Nogood Learning

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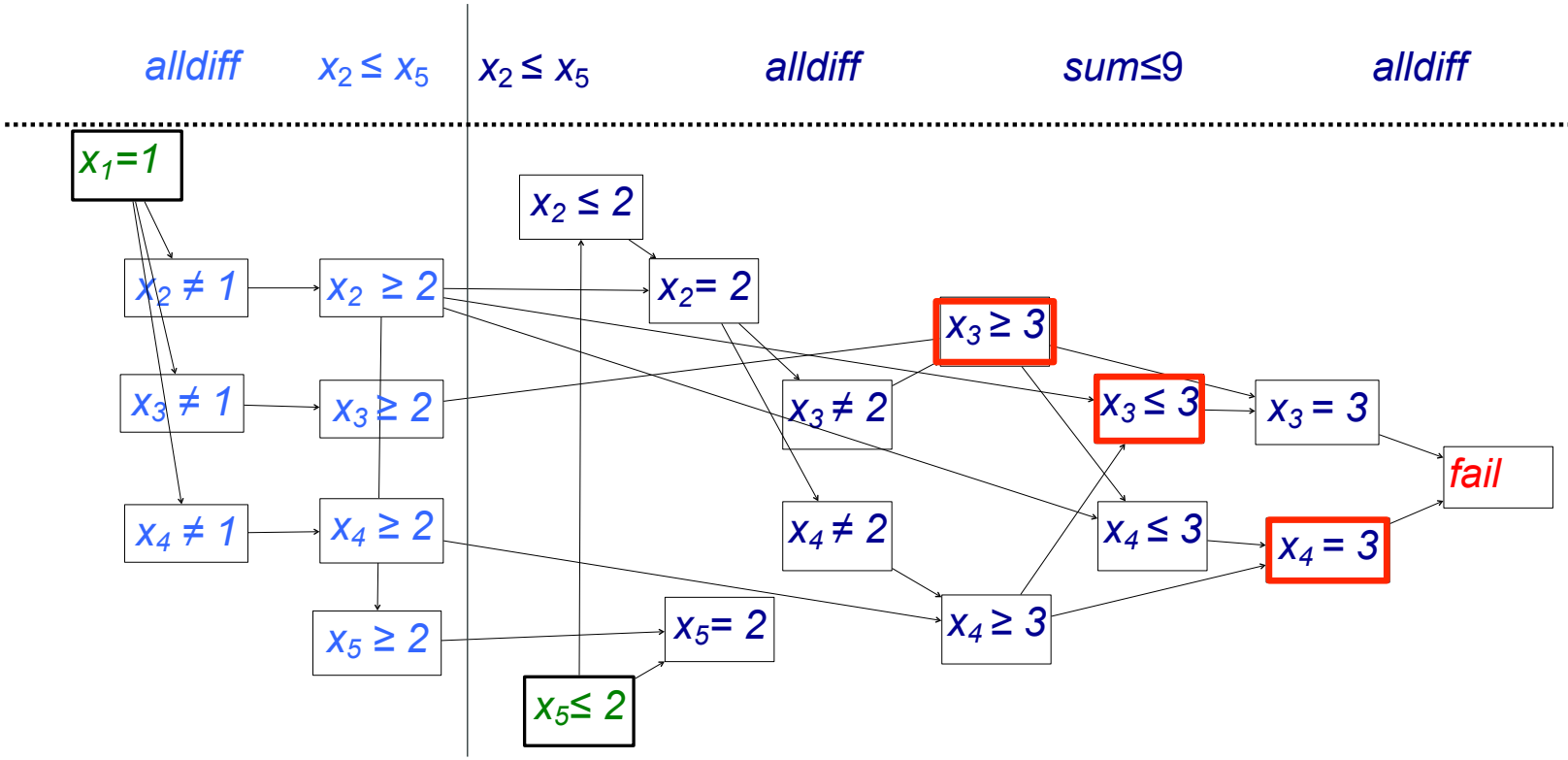

$x_3 \ge 3 \wedge x_3 \le 3 \wedge x_4=3 \rightarrow$ *false*

Nogood Learning

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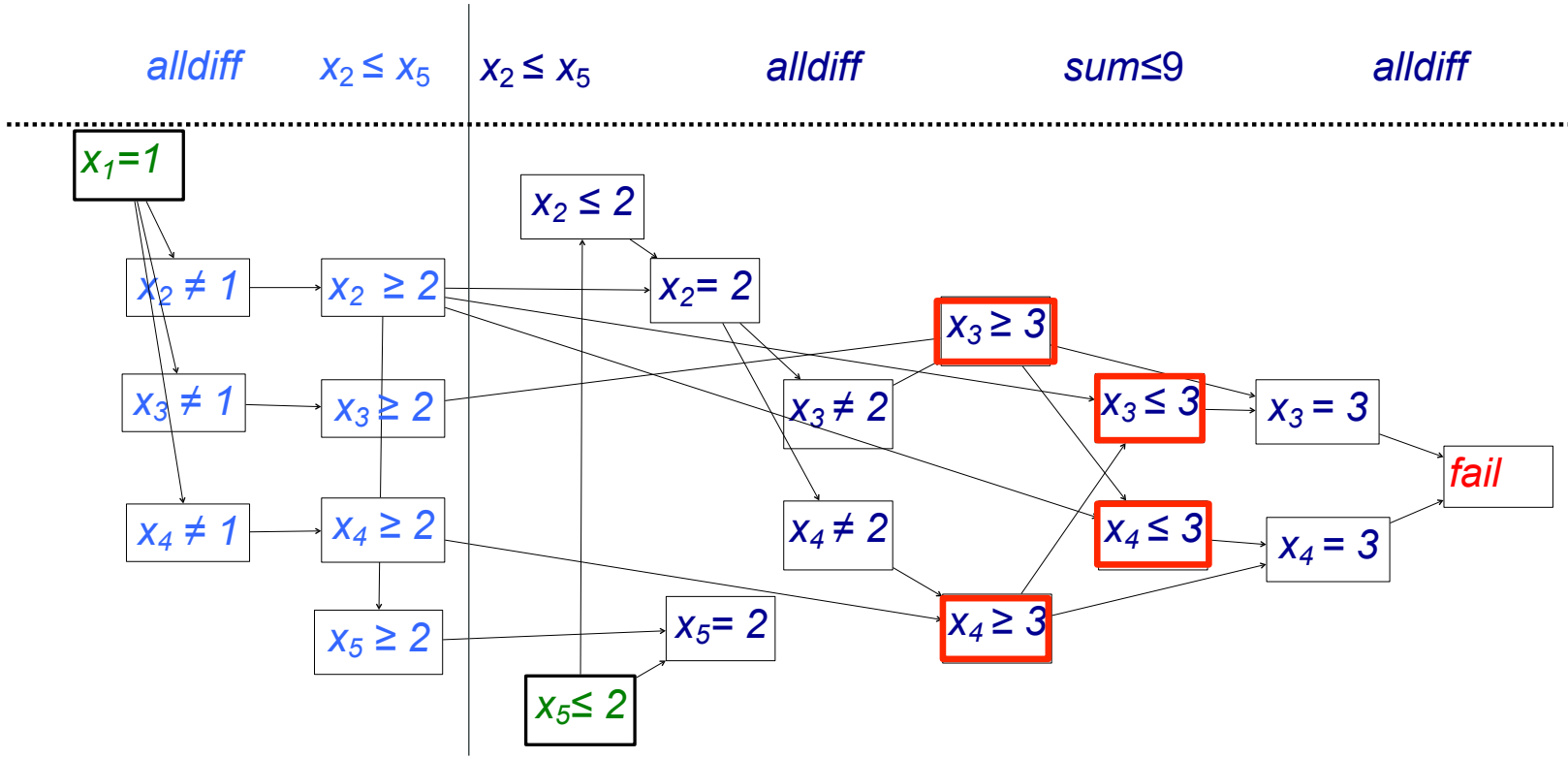

$\{x_3 \ge 3, x_4 \ge 3, x_3 \le 3, x_4 \le 3\} \rightarrow$ *false*

Nogood Learning

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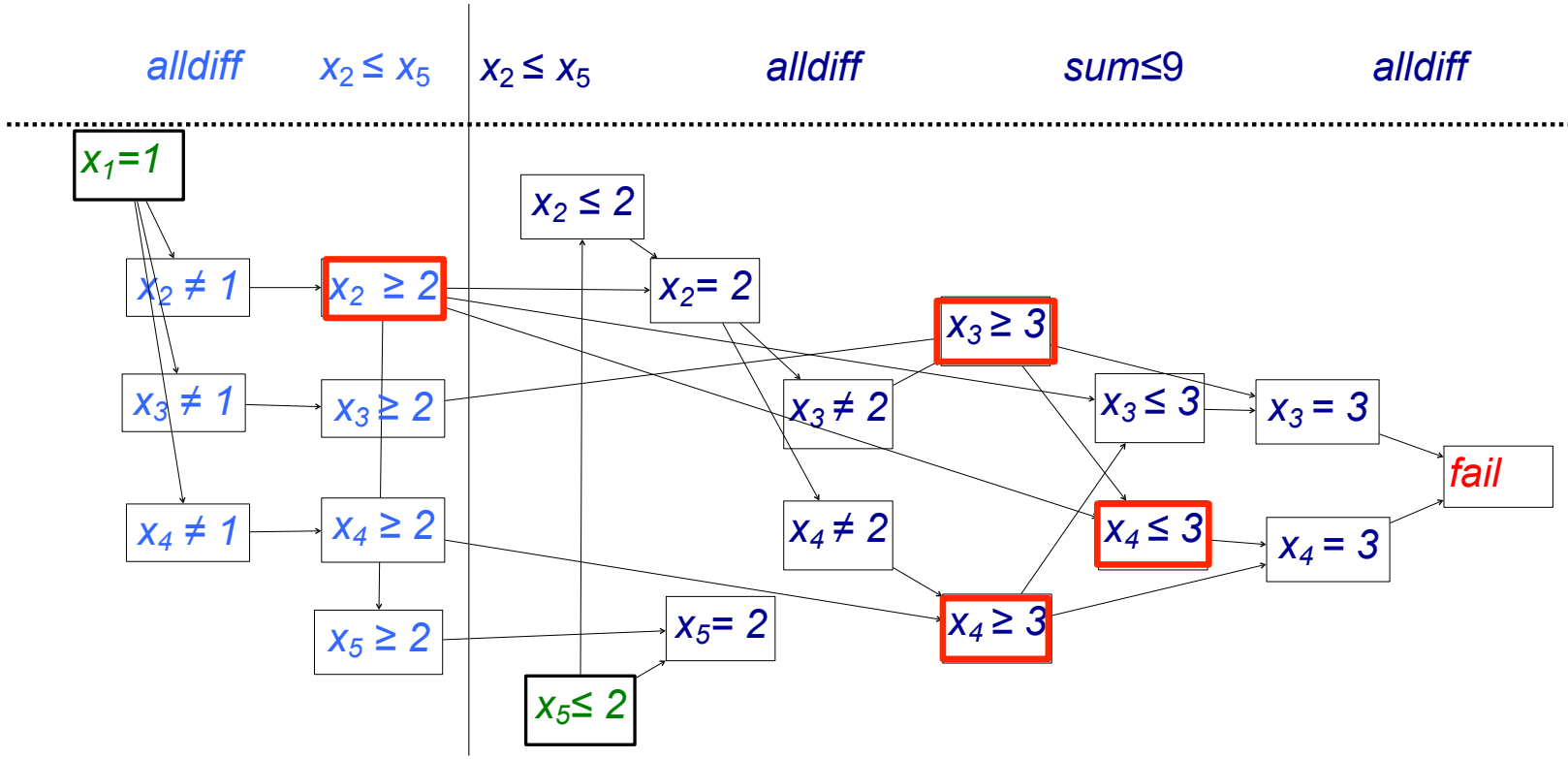

$\{x_2 \ge 2, x_3 \ge 3, x_4 \ge 3, x_3 \le 3\}$ ➔ *false*

Nogood Learning

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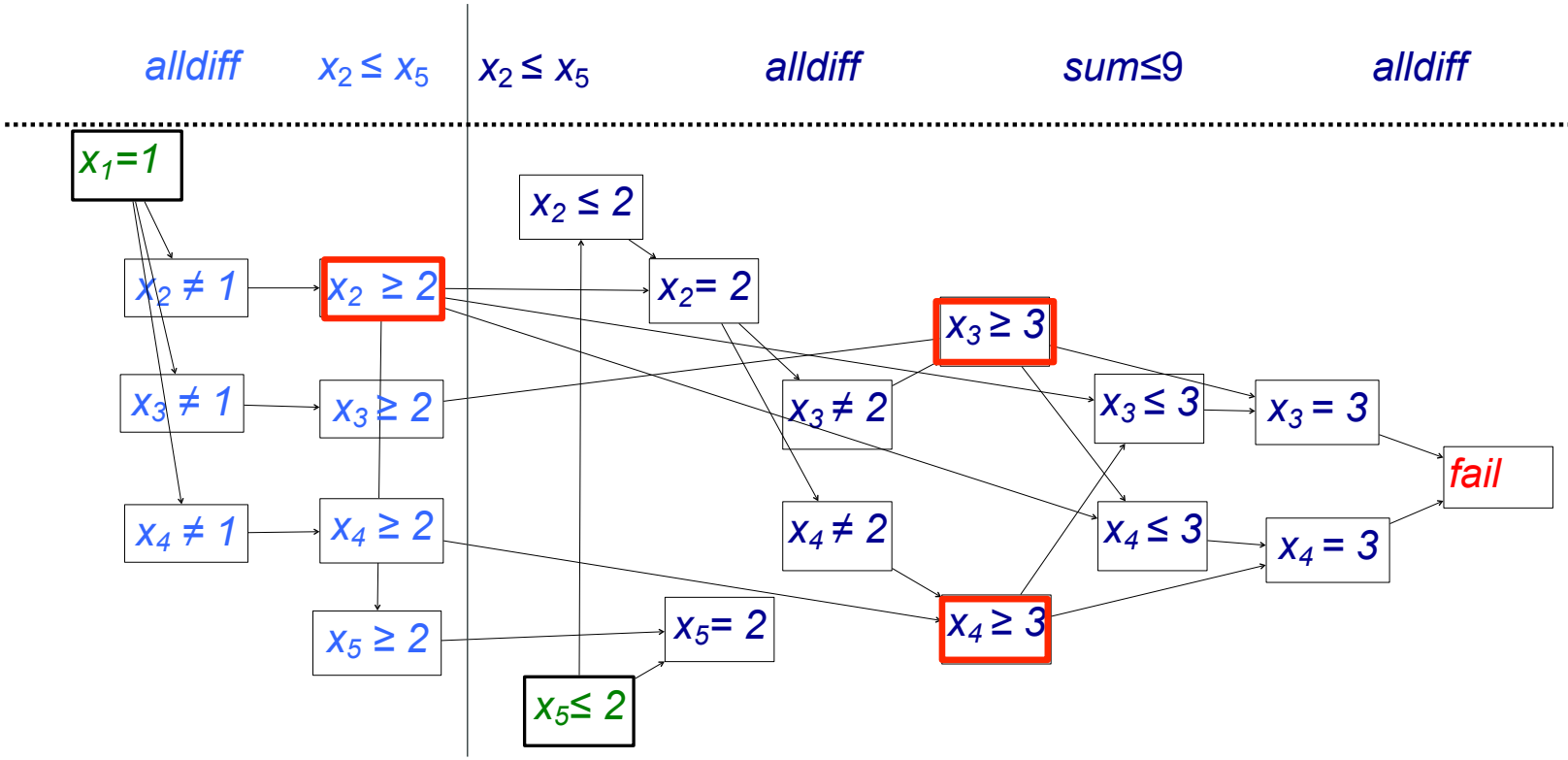

$\{x_2 \ge 2, x_3 \ge 3, x_4 \ge 3\}$ ➔ *false*

Nogood Learning

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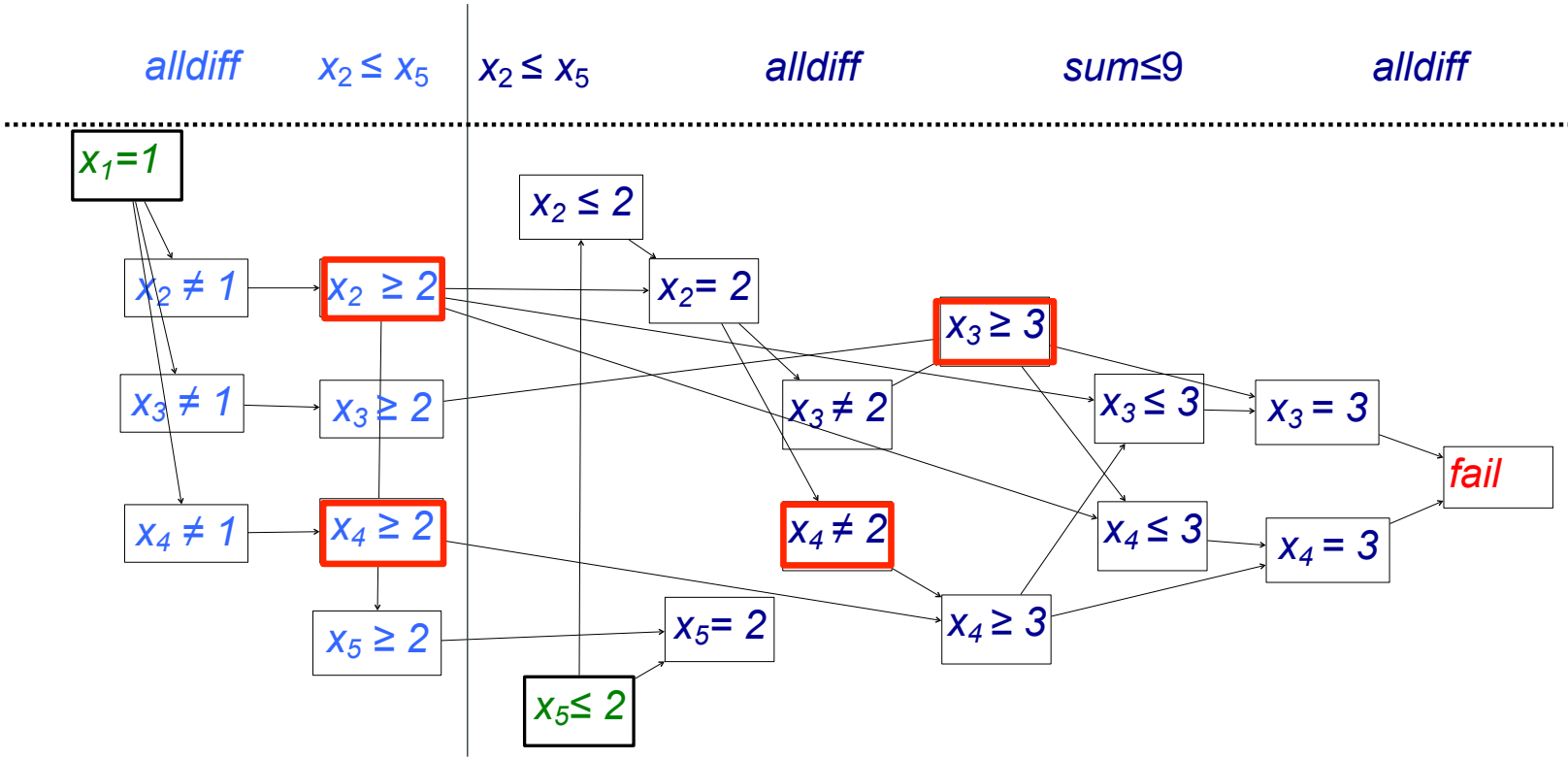

$\{x_2 \ge 2, x_4 \ge 2, x_4 \neq 2, x_3 \ge 3\}$ ➔ *false*

Nogood Learning

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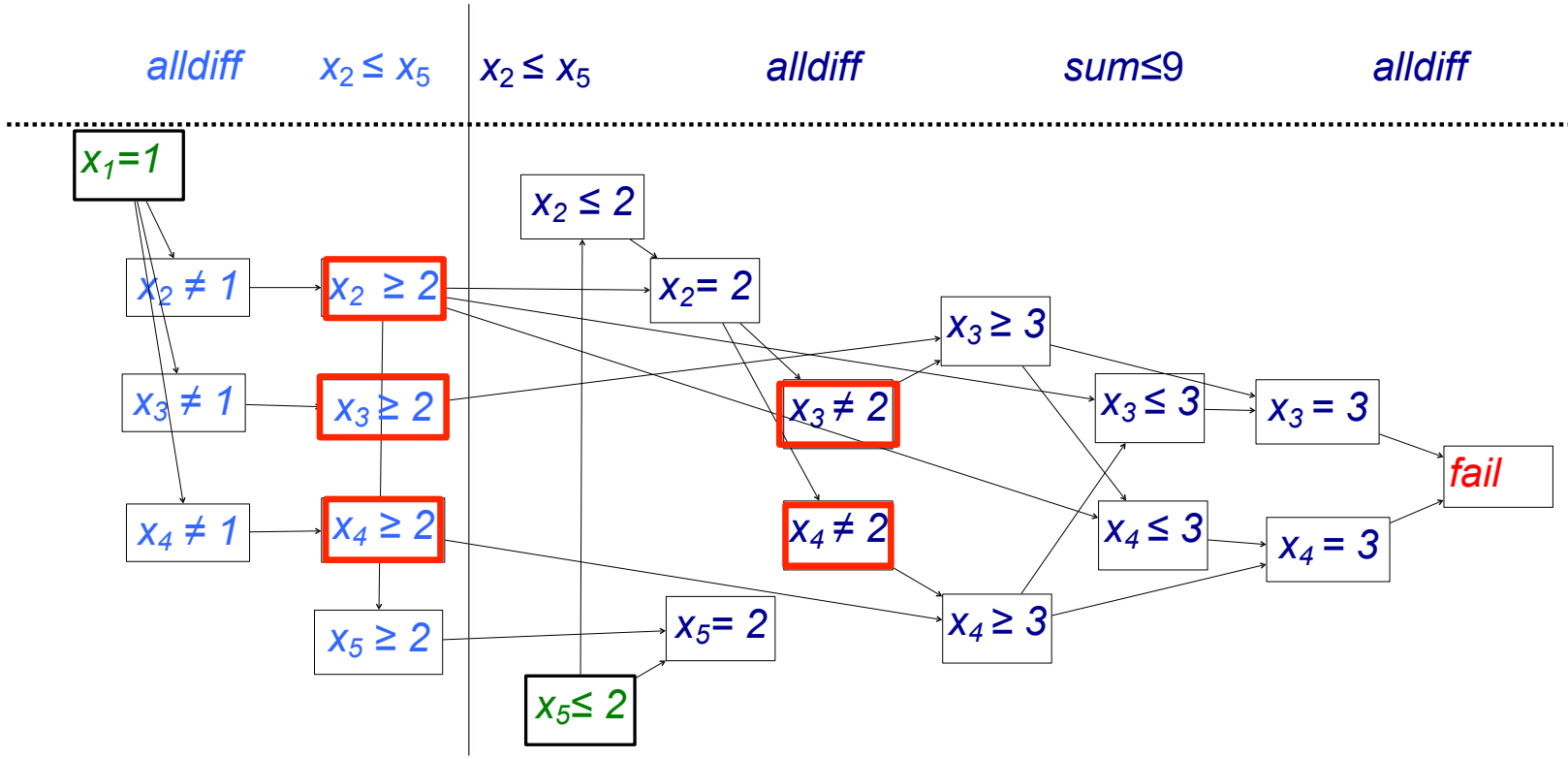
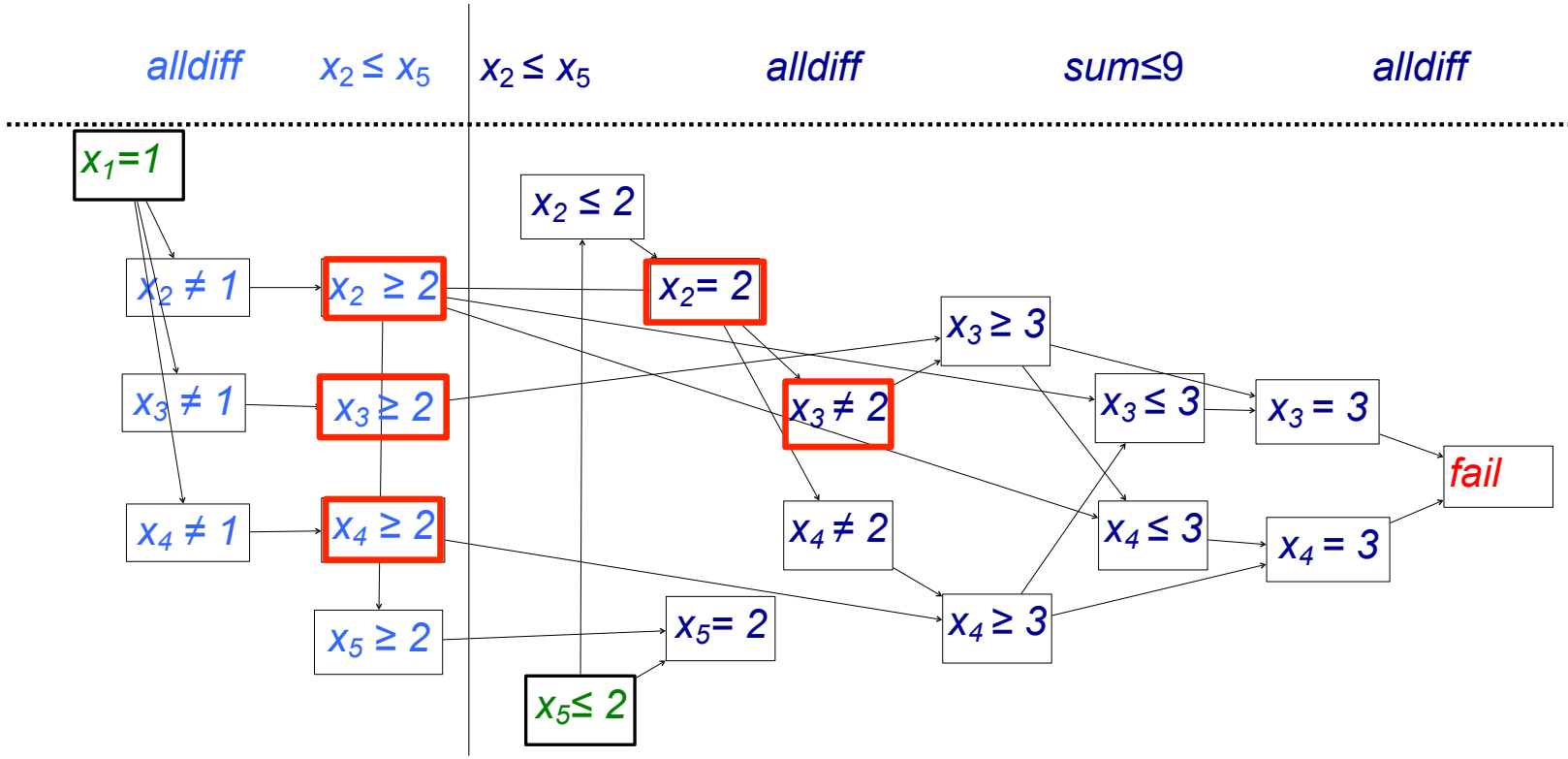


$\{x_2 \ge 2, x_3 \ge 2, x_4 \ge 2, x_3 \neq 2, x_4 \neq 2\}$ ➔ *false*

Nogood Learning



$\{x_2 \geq 2, x_3 \geq 2, x_4 \geq 2, x_2 = 2, x_3 \neq 2\}$ → *false*

Nogood Learning

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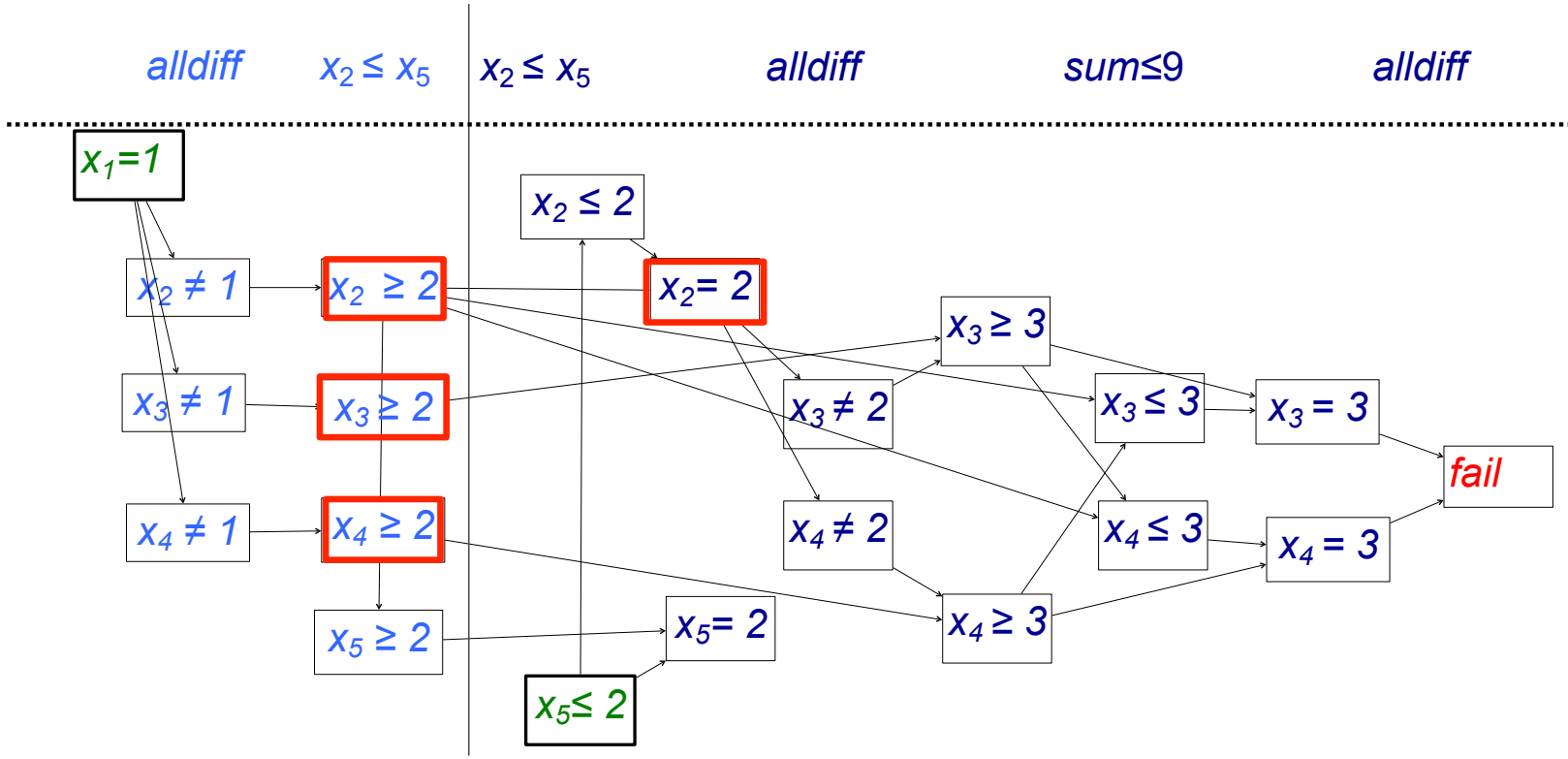

$\{x_2 \ge 2, x_3 \ge 2, x_4 \ge 2, x_2 = 2\}$ ➔ *false*

Nogood Learning

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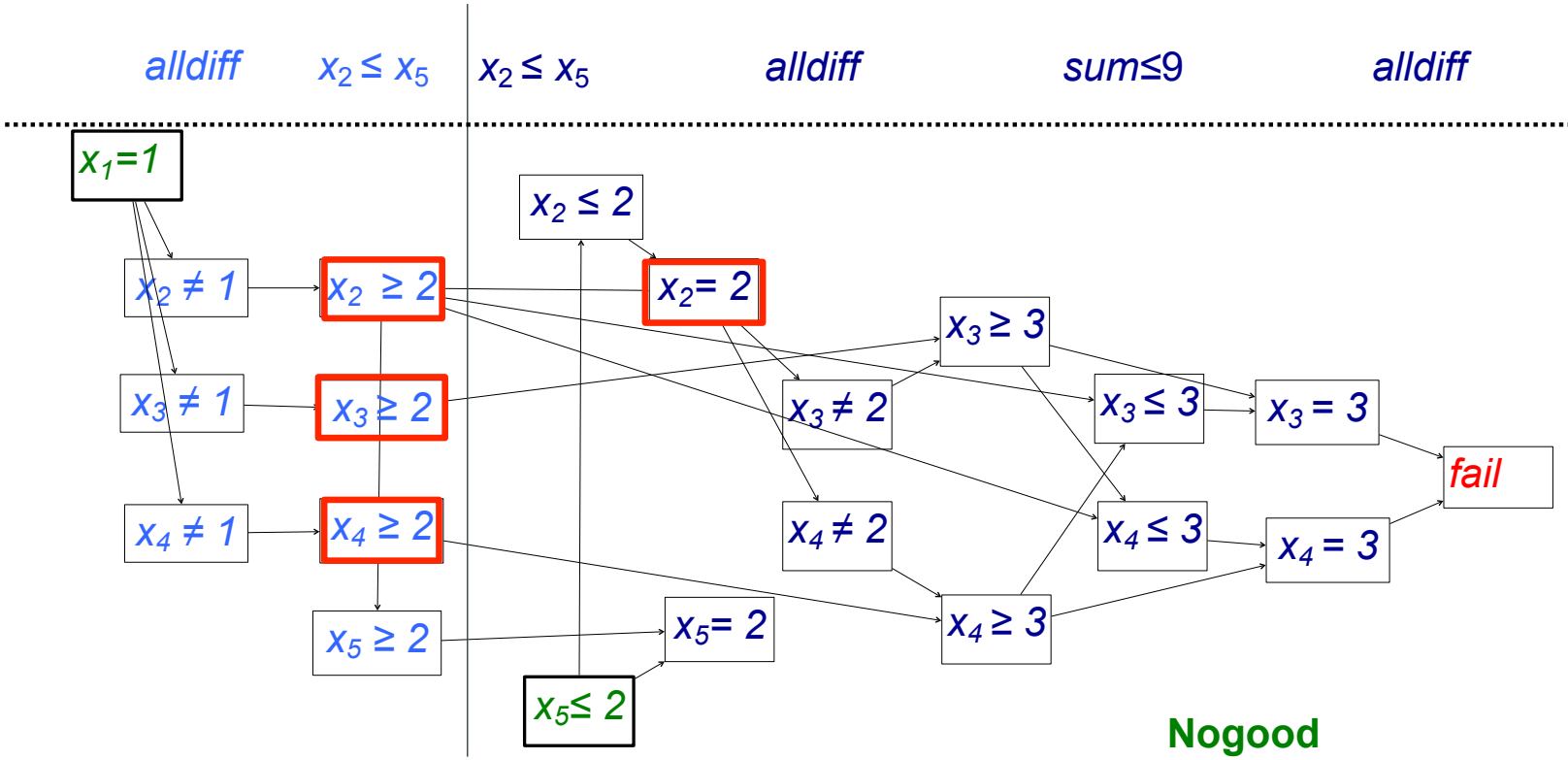

Nogood

$\{x_2 \ge 2, x_3 \ge 2, x_4 \ge 2, x_2 = 2\} \rightarrow$ *false*

$x_3 \le 1 \vee x_4 \le 1 \vee x_2 \neq 2$

The Role of Constraints

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- Traditionally
  - feasibility checking
  - domain filtering
- With learning
  - explaining its failures
  - explaining its inferences
- Key goal in learning
  - make the explanations as general/reusable as possible

Explaining the Cumulative Constraint

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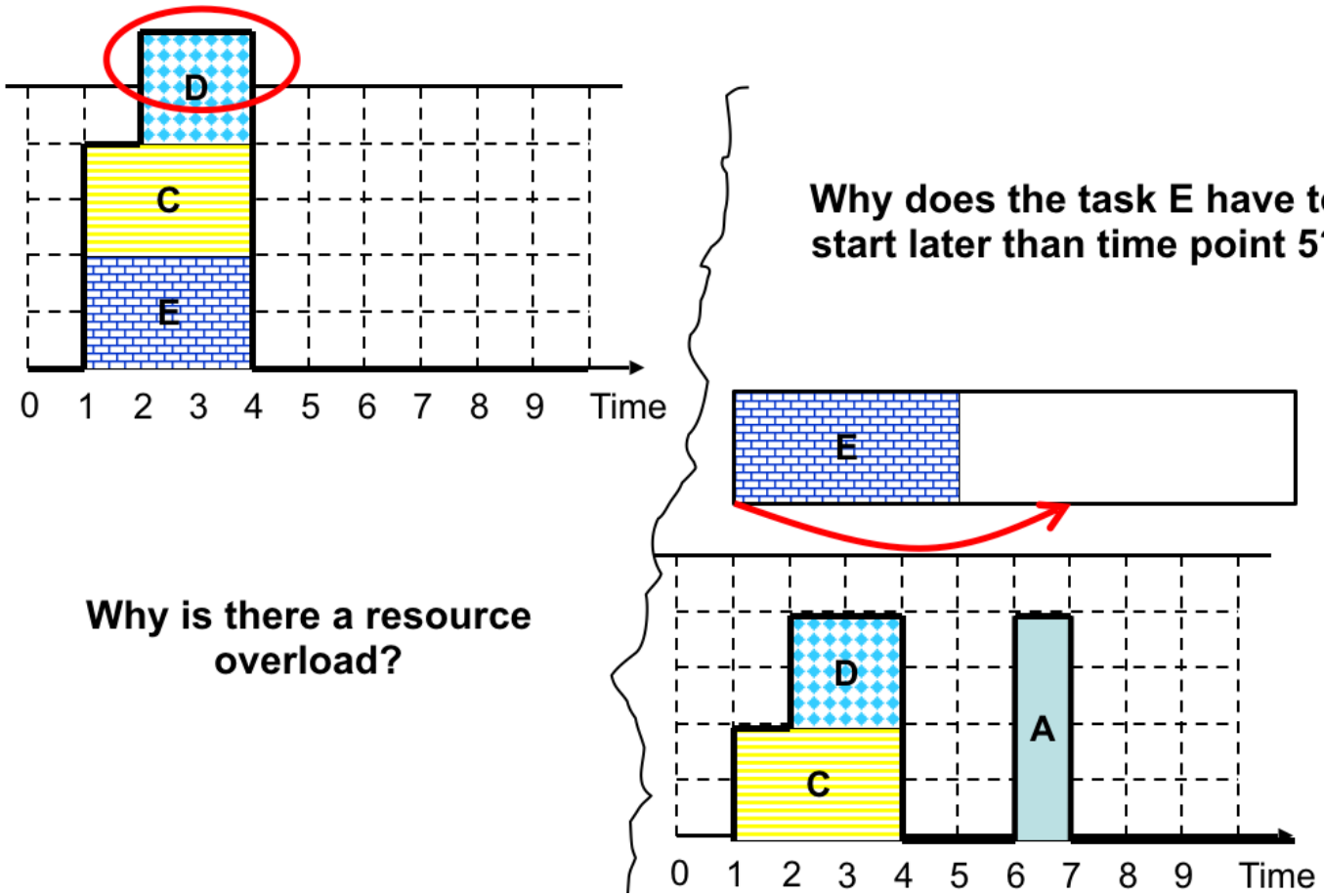

Explaining the Cumulative Constraint

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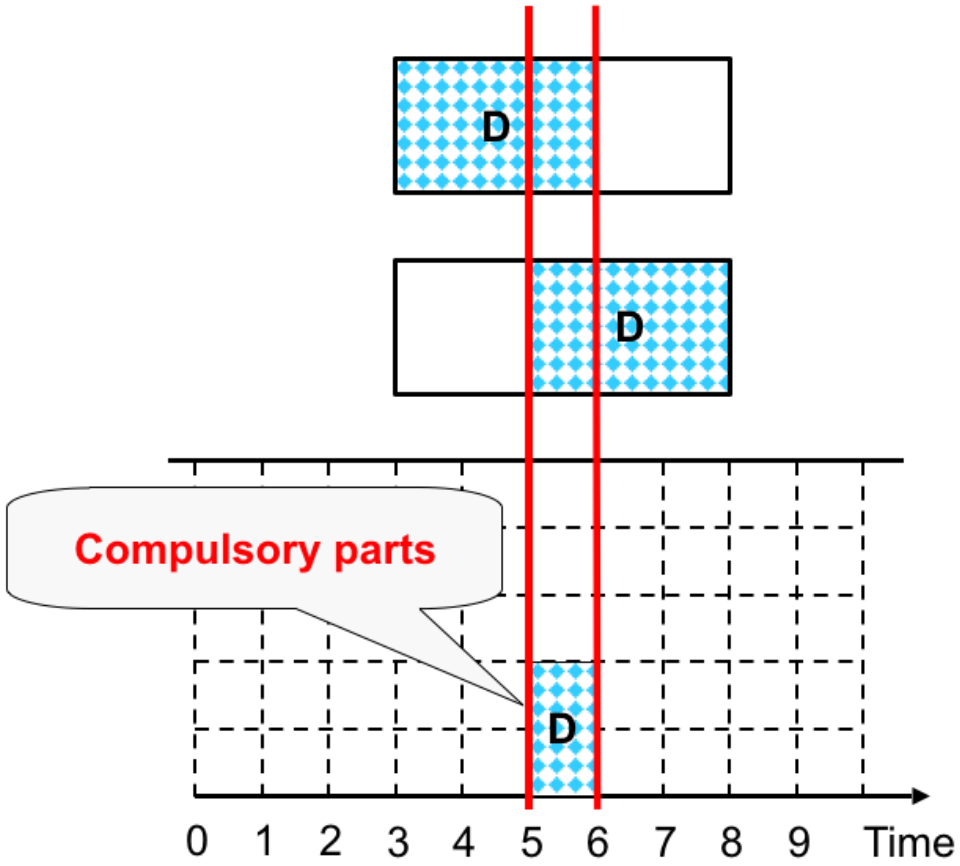

Explaining Failures

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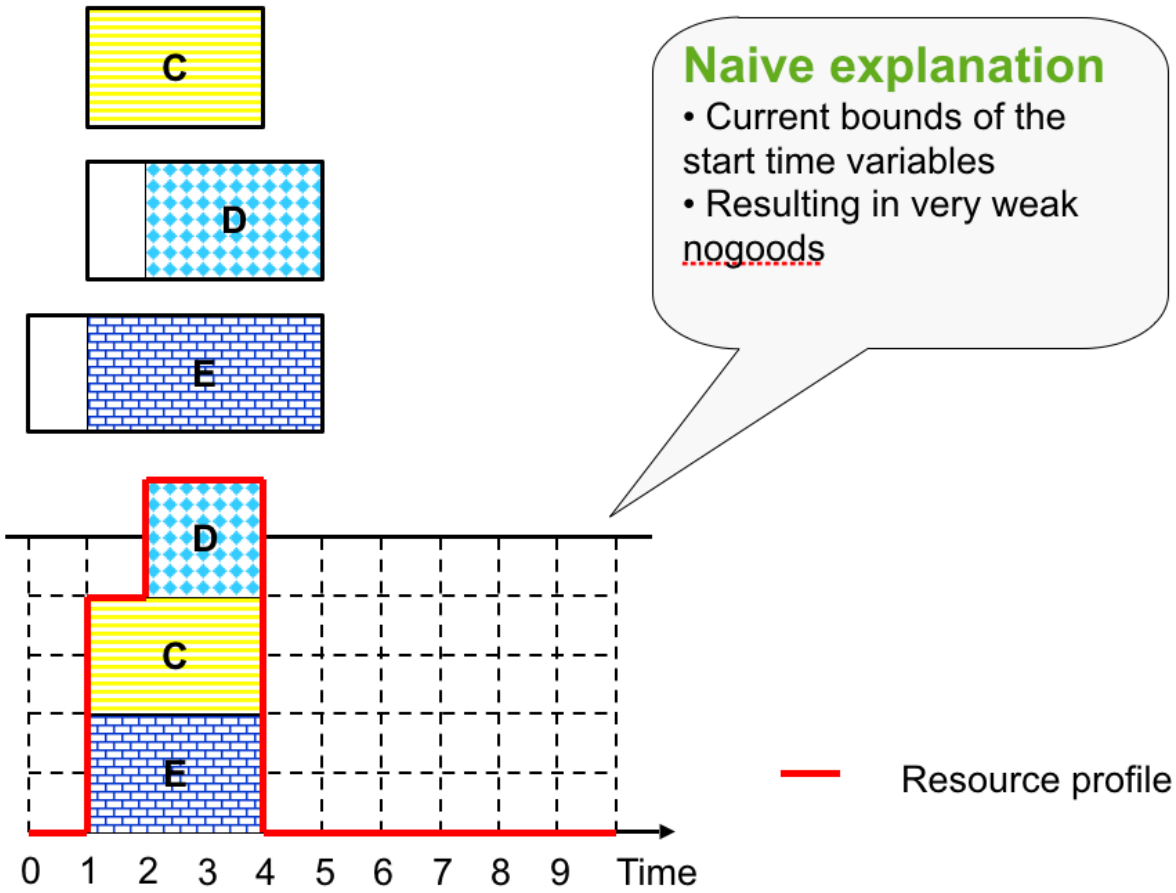

Explaining Failures

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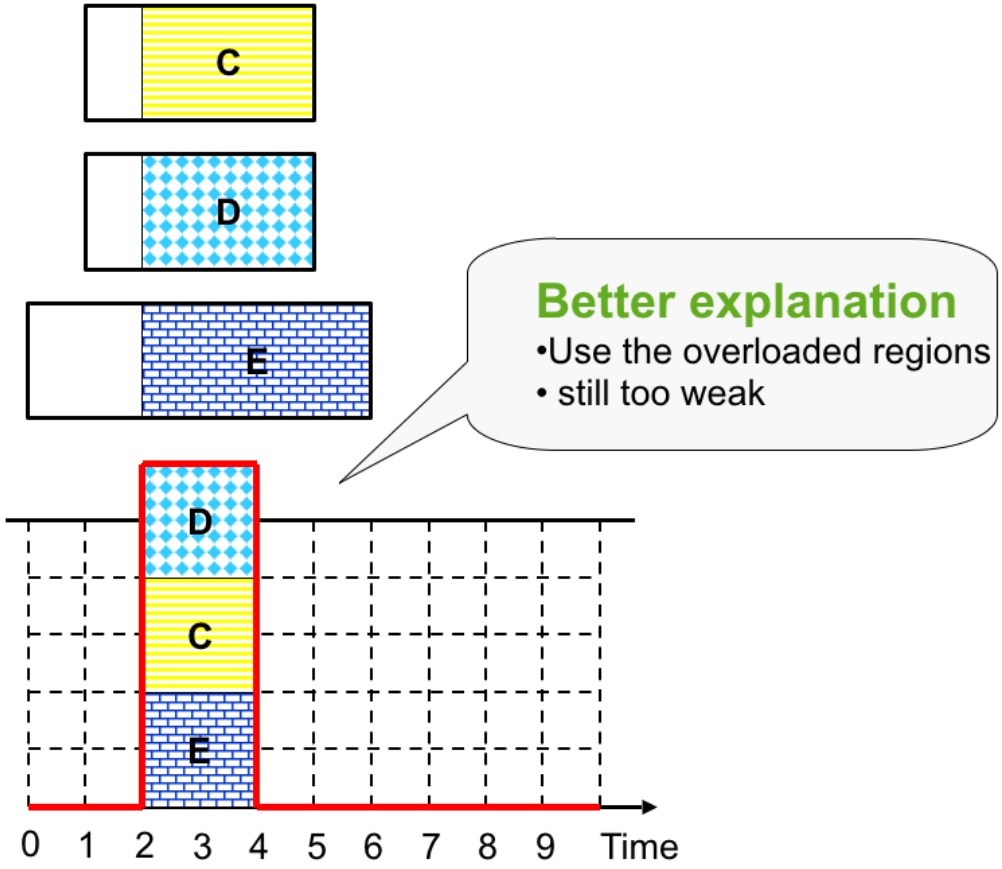

Explaining Failures

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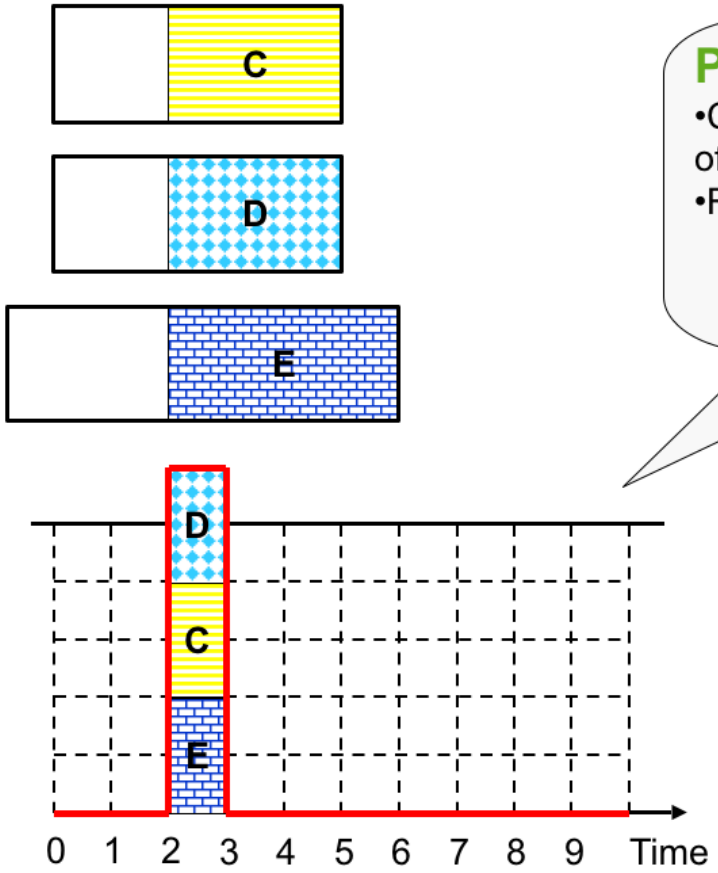

Point-wise explanation

- Consideration only single units of time in the resource overload
- Resulting in strong nogoods

Explaining Inferences

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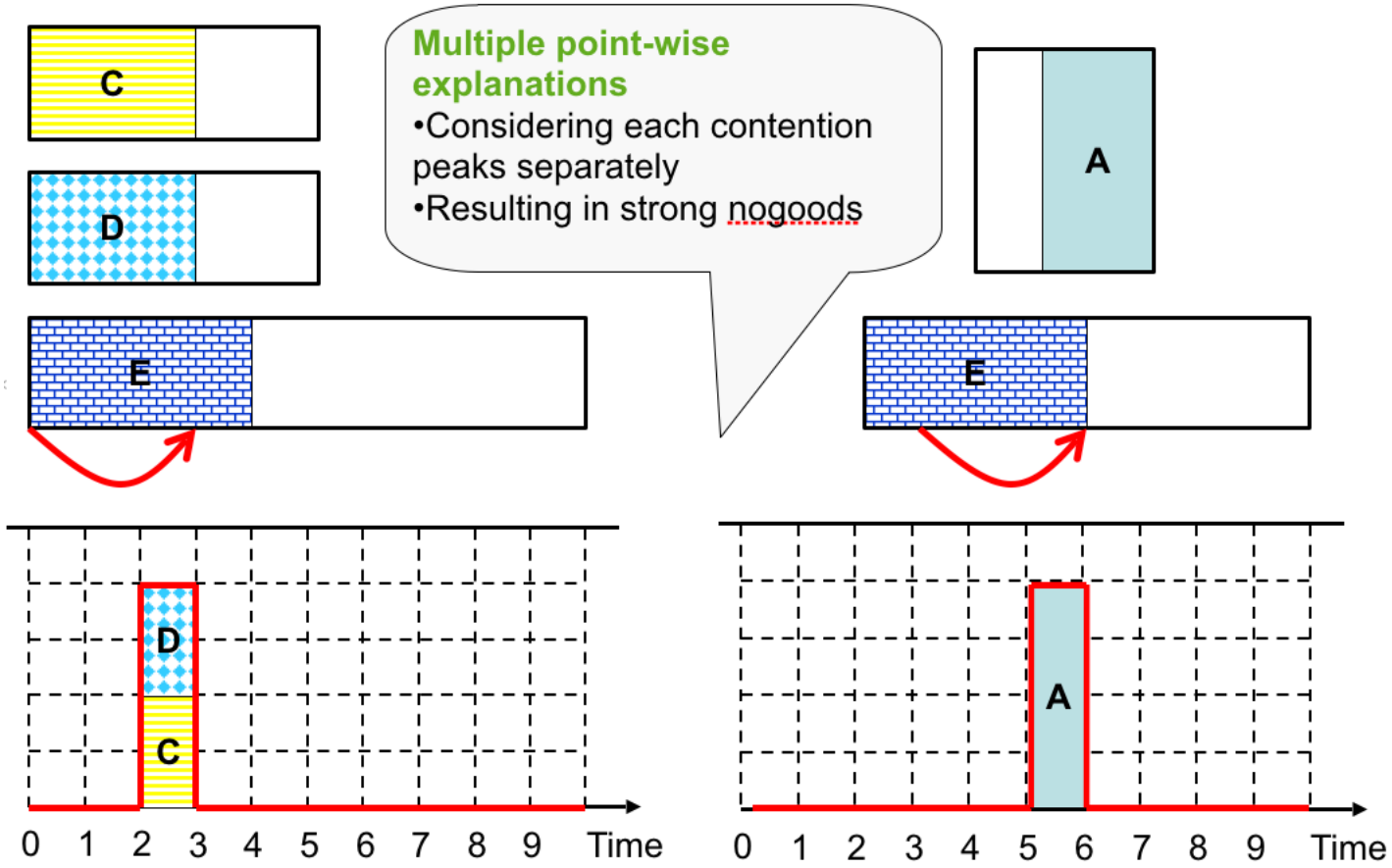

The Value of Learning

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- Scheduling
  - Resource Constrained Project Scheduling Problems (RCPSP)
    - (probably) the most studied scheduling problems
    - closed 71 open problems
    - Solves more problems in 18s then previous state of the art in 1800s
  - RCPSP/Max (more complex precedence constraints)
    - closed 578 open instances of 631
    - recreates or betters all best known solutions by any method on 2340 instances except 3
  - RCPSP/DC (discounted cashflow)
    - Always finds solution on 19440 instances, optimal in all but 152 (versus 832 in previous state of the art)
    - the state of the art complete method for this problem

Scheduling in the Last Decade

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What's
NEXT

Outline

AI Institute for Advances in Optimization

- Brief Overview to constraint programming
- Scheduling with constraint programming
- Hybridizing constraint programming and MIP
- Learning-based constraint programming
- Learning Optimization Proxies

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PRELIMINARY PREPRINT VERSION: DO NOT CITE
The AAAI Digital Library will contain the published
version some time after the conference.

Fast Approximations for Job Shop Scheduling: A Lagrangian Dual Deep Learning Method

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[1] Syracuse University
[2] Georgia Institute of Technology
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Optimization Proxies

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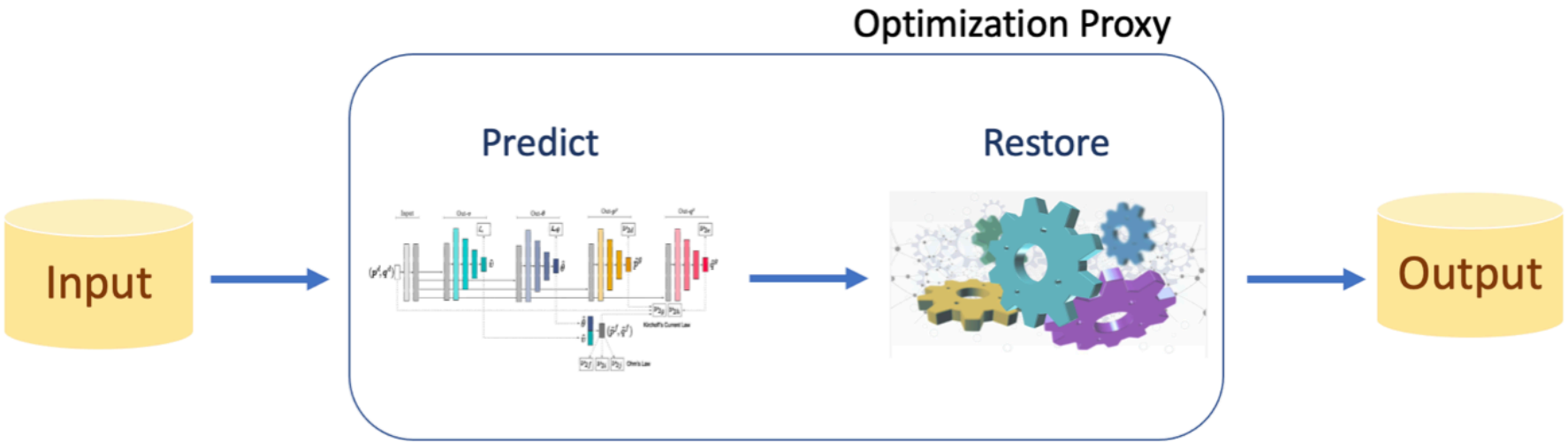
- Role of Optimization Proxies
  - learning the input/output relationship of an optimization problem



Optimal Power Flow

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Model 1 (AC Optimal Power Flow (AC-OPF)).

$$\mathcal{O}(\dot{\boldsymbol{p}}^d, \dot{\boldsymbol{q}}^d) = \arg\min_{\boldsymbol{p}^g,\, \boldsymbol{v}} \sum_{i\in\mathcal{N}} \text{cost}(p_i^g) \tag{1}$$

subject to

$$\dot{v}_i^{\min} \leqslant v_i \leqslant \dot{v}_i^{\max} \qquad \forall i \in \mathcal{N}, \tag{2a}$$

$$-\dot{\theta}_{ij}^{\Delta} \leqslant \theta_i - \theta_j \leqslant \dot{\theta}_{ij}^{\Delta} \qquad \forall (ij) \in \mathcal{E}; \tag{2b}$$

$$\dot{p}_i^{g\min} \leqslant p_i^g \leqslant \dot{p}_i^{g\max} \qquad \forall i \in \mathcal{N}, \tag{3a}$$

$$\dot{q}_i^{g\min} \leqslant q_i^g \leqslant \dot{q}_i^{g\max} \qquad \forall i \in \mathcal{N}; \tag{3b}$$

$$(p_{ij}^f)^2 + (q_{ij}^f)^2 \leqslant \dot{S}_{ij}^{f\max} \qquad \forall (ij) \in \mathcal{E}; \tag{4}$$

$$p_{ij}^f = \dot{g}_{ij} v_i^2 - v_i v_j (\dot{b}_{ij} \sin(\theta_i - \theta_j) + \dot{g}_{ij} \cos(\theta_i - \theta_j)) \qquad \forall (ij) \in \mathcal{E}; \tag{5a}$$

$$q_{ij}^f = -\dot{b}_{ij} v_i^2 - v_i v_j (\dot{g}_{ij} \sin(\theta_i - \theta_j) - \dot{b}_{ij} \cos(\theta_i - \theta_j)) \qquad \forall (ij) \in \mathcal{E}; \tag{5b}$$

$$p_i^g - \dot{p}_i^d = \sum_{(ij)\in\mathcal{E}} p_{ij}^f \qquad \forall i \in \mathcal{N}; \tag{6a}$$

$$q_i^g - \dot{q}_i^d = \sum_{(ij)\in\mathcal{E}} q_{ij}^f \qquad \forall i \in \mathcal{N}; \tag{6b}$$

output: $(\boldsymbol{p}^g, \boldsymbol{v})$ – The system operational parameters.

Learning Optimization Proxies

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- Challenges
  - physical, engineering, and business constraints
- Challenging learning problems
  - empire risk minimization under constraints
- Data augmentation
  - data is not an issue since we have an optimization models
- Nice synergies
  - between optimization and learning

Scheduling Proxies

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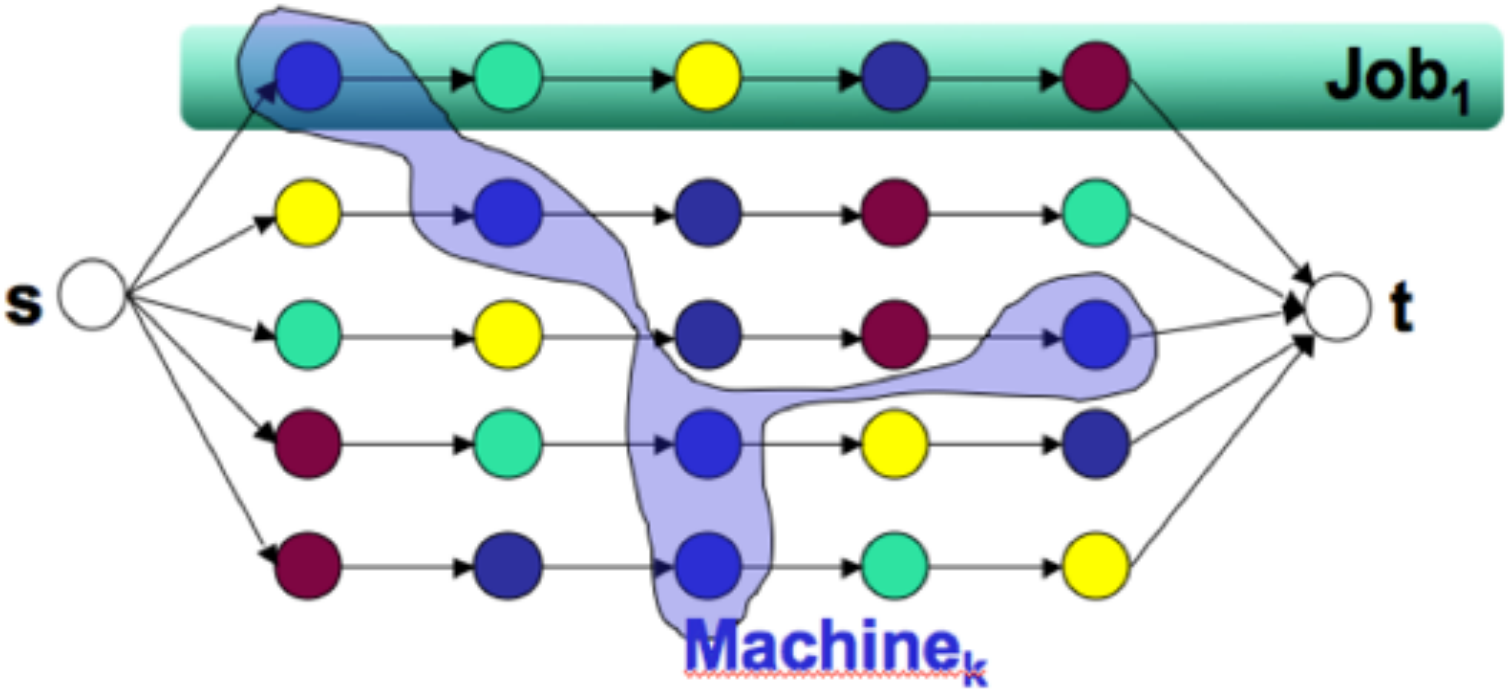

Predict

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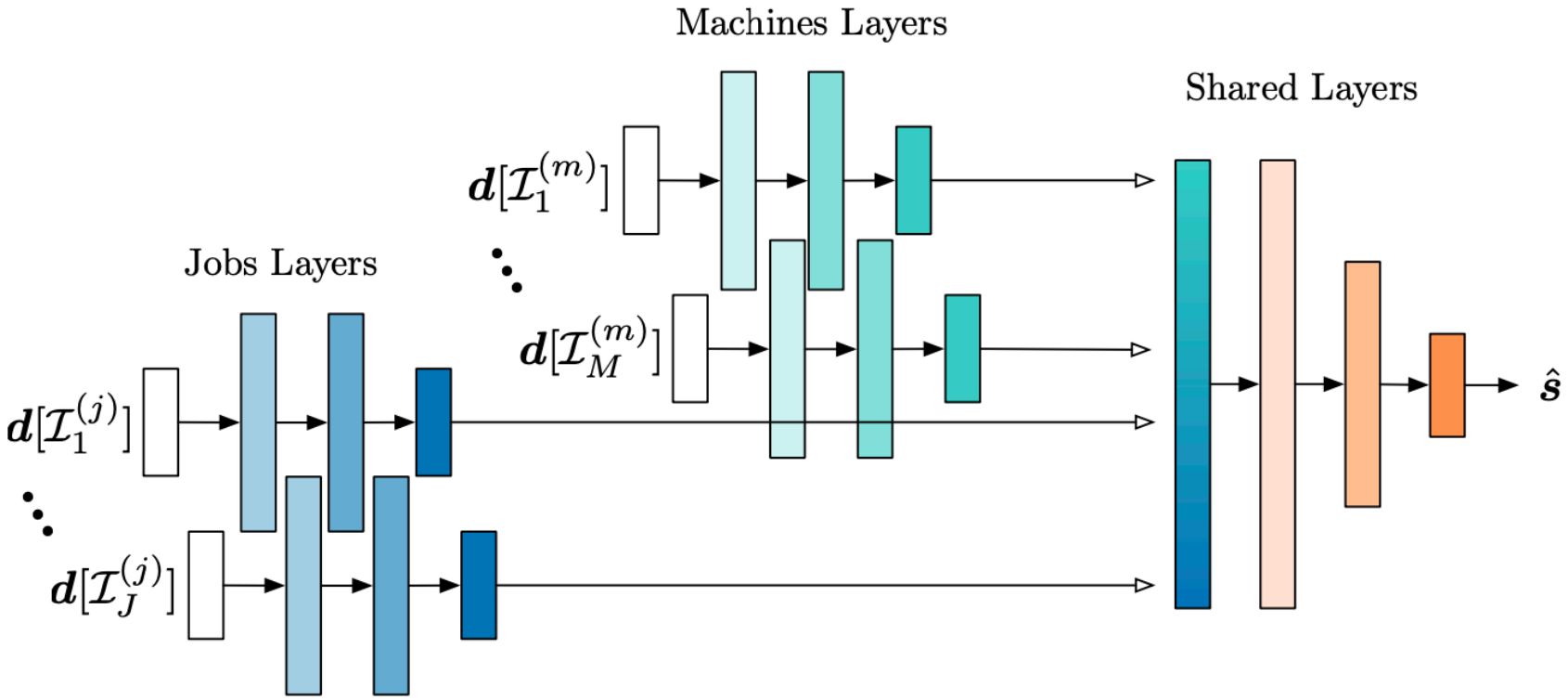

Repair

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Model 2: Recovering a Feasible Solution to the JSP.

$$\Pi(\boldsymbol{s}) = \text{argmin}_{\boldsymbol{s}} \; u$$

subject to: (2a), (2b)

$$s_t^j \geq s_{t'}^{j'} + d_{t'}^{j'} \quad \forall j, j' \in [J], t, t' \in [T] \text{ s.t. } (j,t) \preceq_{\hat{\boldsymbol{s}}} (j',t') \tag{8a}$$

$$s_t^j \geq 0 \quad \forall j \in [J], t \in [T] \tag{8b}$$

Algorithm 2: *The JSP Greedy Recovery.*

Input: $\{\hat{s}_t^j\}_{t\in[T],j\in[J]}$: predicted start times

$Q \leftarrow$ enqueue$((j,1)), \quad \forall j \in [J]$
while Q *is empty* **do**
  $(j,t) \leftarrow$ dequeue(Q)
  Schedule task (j,t) with start time $\hat{s}_t^j$
  if $t < T$ **then**
    $\hat{s}_{t+1}^j \leftarrow \max(\hat{s}_{t+1}^j, \hat{s}_t^j + d_t^j)$
    $Q \leftarrow$ enqueue(j,t)
  end
  foreach (t', j') *s.t.* $t \neq t'$ & $\sigma_{t'}^{j'} = \sigma_t^j$ & $\hat{s}_{t'}^{j'} \geq \hat{s}_t^j$ **do**
    $\hat{s}_{t'}^{j'} \leftarrow \max(\hat{s}_{t'}^{j'}, \hat{s}_t^j + d_t^j)$
  end
end

Experimental Setting

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- Slowdown of a machine
  - from 1% to 50%
- Training/Testing instances
  - 5,000
- Optimization solvers
  - CP Optimizer with 1800 seconds
- Heuristics
  - Shortest Processing Time (SPT), Least Work Remaining (LWR), Most Work Remaining (MWR), Least Operations Remaining (LOR), and Most Operations Remaining (MOR)

Learning Scheduling Problems

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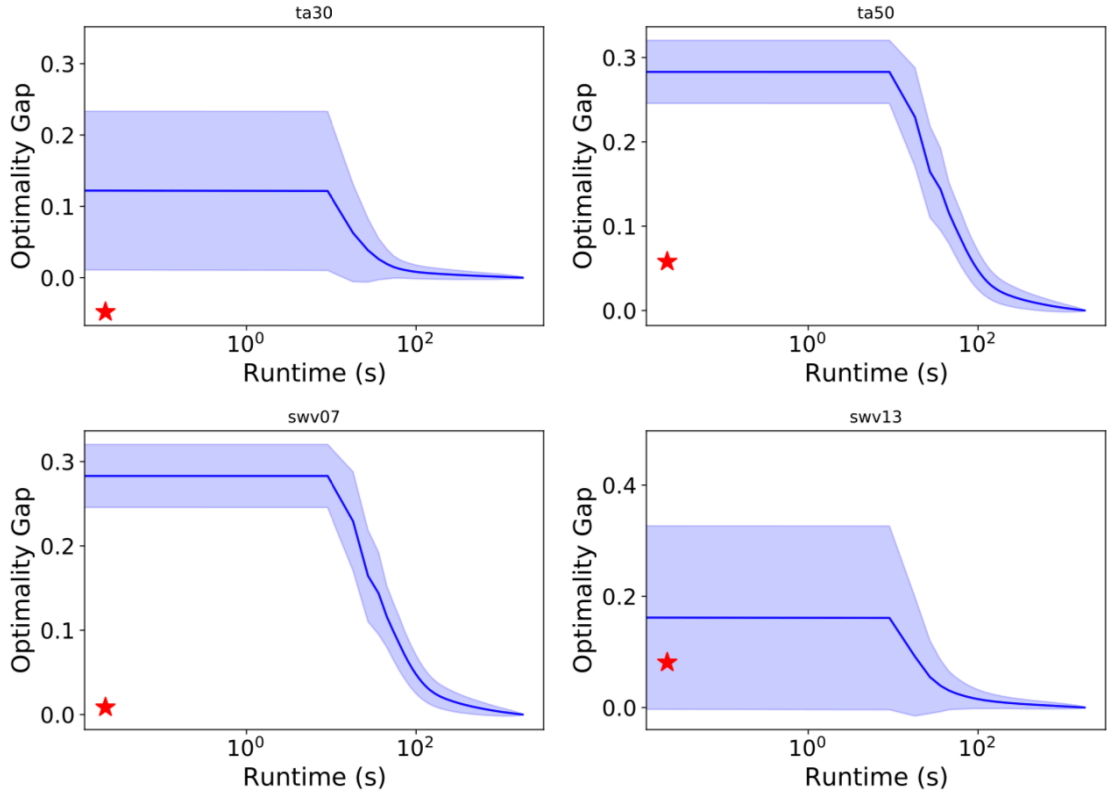
- Comparing SOTA solvers with deep learning + feasibility restoration



Learning Scheduling Problems

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- Comparing SOTA solvers with deep learning + feasibility restoration

Instance	Size	Prediction Err(×10) ↓		Constraint Viol(×10^2) ↓		Opt. Gap Heuristics (%) ↓					Opt. Gap DNNs (%) ↓		Time SoTA Eq. (s) ↑	
	$J \times M$	FC	JSP-DNN	FC	JSP-DNN	SPT	LWR	MWR	LOR	MOR	FC	JSP-DNN	FC	JSP-DNN
yn02	20×20	2.770	**0.138**	1.134	**0.122**	628	837	40	934	40	12.80 ± 5.4	**-0.045 ± 0.9**	10.20	**1800+**
ta25	20×20	1.607	**0.361**	0.631	**0.244**	593	877	59	787	46	13.61 ± 3.13	**-0.143 ± 0.8**	11.02	**1800+**
ta30	30×15	4.338	**1.196**	1.483	**0.357**	558	910	63	856	46	15.01 ± 2.63	**-0.48 ± 5.18**	9.06	**1800+**
ta40	30×20	7.880	**3.341**	1.863	**0.104**	492	794	57	836	25	23.11 ± 7.33	**3.19 ± 1.88**	8.40	**12.04**
ta50	50×10	4.580	**1.322**	1.223	**0.225**	789	789	53	1116	43	18.30 ± 5.22	**5.85± 2.72**	8.02	**90.30**
swv03	20×15	9.473	**2.683**	2.777	**0.850**	203	212	75	190	50	28.61 ± 14.27	**7.62 ± 2.51**	4.04	**36.36**
swv05	20×10	6.586	**2.950**	2.325	**0.626**	183	192	80	177	66	20.78 ± 10.54	**6.34 ± 1.82**	7.24	**18.18**
swv07	20×10	4.587	**0.681**	1.222	**0.223**	299	295	68	352	43	10.69 ± 6.83	**0.01± 4.75**	26.0	**254.5**
swv09	20×15	5.678	**3.462**	2.132	**0.211**	322	270	69	285	75	22.12 ± 8.52	**5.42 ± 1.21**	6.48	**28.32**
swv11	50×10	7.958	**3.244**	2.711	**0.282**	237	231	94	263	73	23.18 ± 2.27	**4.80 ± 4.47**	7.02	**92.00**
swv13	50×10	23.21	**3.557**	1.615	**0.323**	225	203	114	218	79	22.79 ± 16.21	**8.11 ± 4.20**	7.08	**24.08**

Table 2: Accuracy metrics compared between FC and JSP-DNN (left sub-table) and accuracy of simple heuristics vs CP-Optimizer at 1800s (right sub-table). Best results shown in bold.

Outline

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Thank you for listening

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