



**FRIEDRICH-SCHILLER-  
UNIVERSITÄT  
JENA**

Wirtschaftswissenschaftliche  
Fakultät

# **Scheduling in the e-commerce era: New scheduling problems in order fulfilment and warehousing**

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## **Scheduling Seminar**

**Chair of Operations Management**  
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[www.om.uni-jena.de](http://www.om.uni-jena.de)

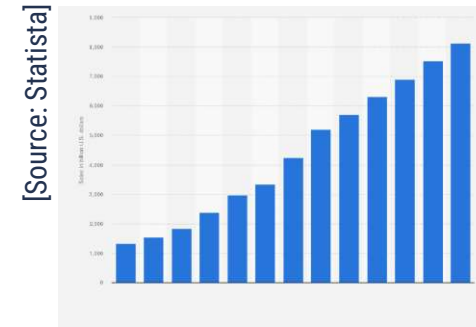
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# Evolution of e-commerce and warehousing, and the impact on scheduling research



Jane Snowball (then 72) ordered groceries via TV and phone line in 1984



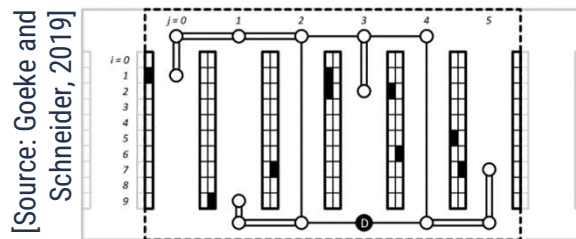
Tremendous growth of e-commerce sales



Traditional warehouse



E-commerce warehouse of German fashion retailer Zalando

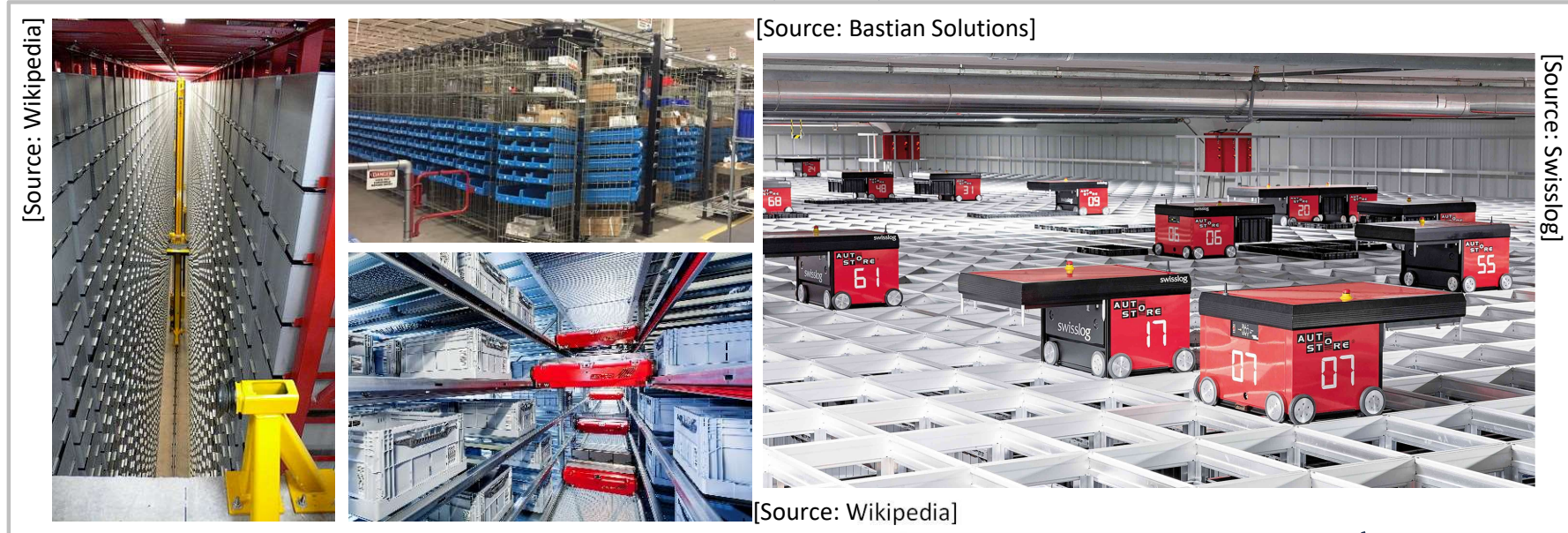


Picker-to-parts warehouse:  
Picker routing



Parts-to-picker warehouse:  
Order fulfillment scheduling

## Automated storage and retrieval system (ASRS)



## Picking workstation

Shipping area ←



[Source: Fundamental Technologies]



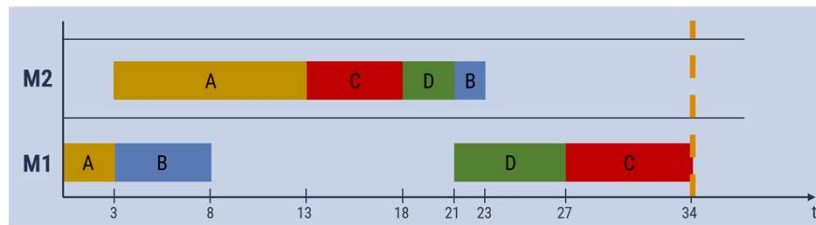
Conveyor system

- Peculiarities of order fulfillment scheduling
- Classification scheme
- Four examples:
  - ▶ Warehouse setup
  - ▶ Order fulfillment scheduling problem
  - ▶ Selected results
- Outlook



[Source: RightHand Robotics]

## Machine scheduling (MS)



## Order fulfillment scheduling (OFS)



vs.

### ■ Input-output process:

- ▶ MS: Input: Jobs → Output: Products
- ▶ OFS: Input: Bins with many SKUs (stock keeping units) → Output: customer orders

### ■ Relation among input and output:

- ▶ MS (single or parallel machine): 1:1 – one job → one product
- ▶ MS (Job shop or flow shop): 1:n – multiple jobs → one product
- ▶ OFS: n:m – each SKU bin can contribute to multiple orders and each order requires multiple SKU bins for completion

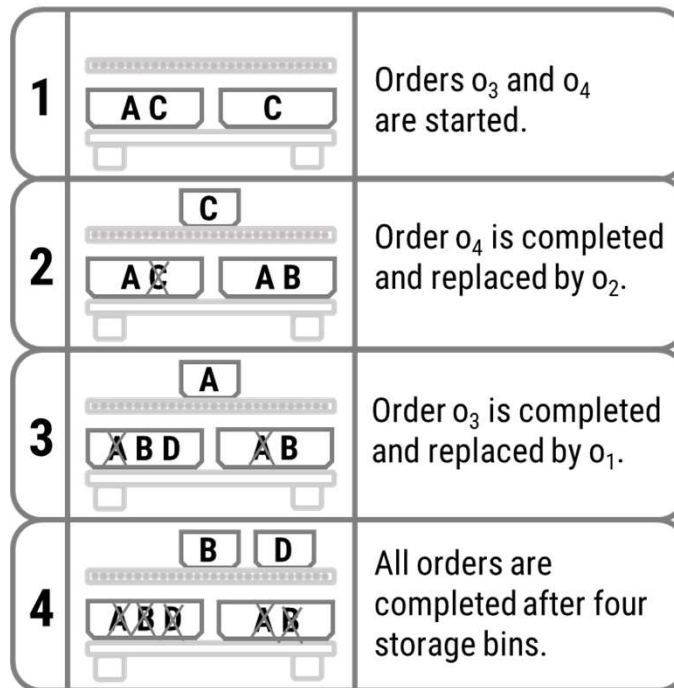
### ■ Batching:

- ▶ MS (default): one job per machine at a time
- ▶ OFS: Parallel batching: Multiple SKU bins and/or customer bins in parallel

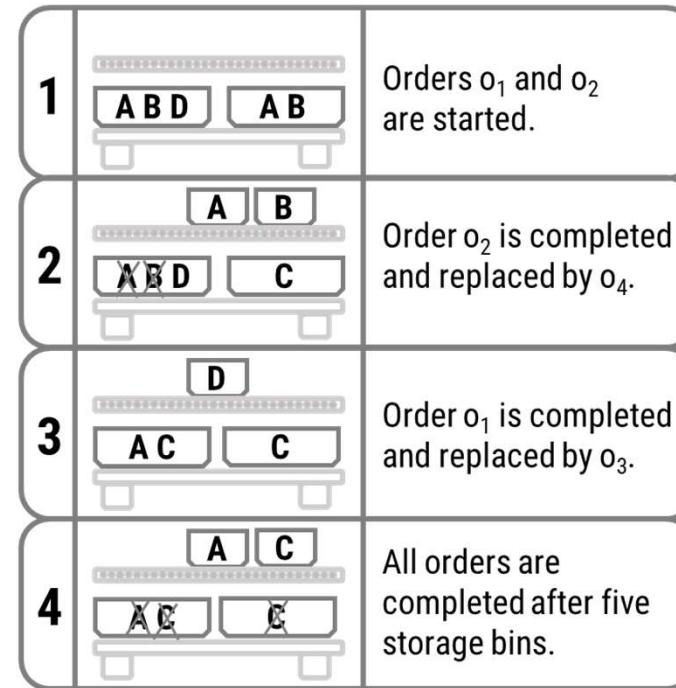
### ■ OFS: Synchronization problem of SKU bins with customer bins to improve order throughput

# Example for order fulfillment scheduling

[Source: Dematic]



**Solution (a)**



**Solution (b)**

**3-field notation:** [  $\alpha$  (Inbound stream) |  $\beta$  (Outbound stream) |  $\gamma$  (Objective) ]

$\alpha_1$ capacity for parallel SKU bins	◦ $k$	only a single SKU bin at a time $k > 1$ SKU bins in parallel
$\alpha_2$ bin composition	◦ mix	only one SKU per bin multiple SKUs per bin
$\alpha_3$ arrival sequence	◦ fix	part of optimization problem already fixed
$\alpha_4$ bin inventory	◦ pieces	all bins carry enough pieces limited number of pieces in the SKU bins

[Source: Dematic]



$\alpha_1 = \circ$ : single SKU bin

[Source: SSI Schäfer]



$\alpha_1 = k$ : multiple SKU bins in parallel  
access

**3-field notation:** [  $\alpha$  (Inbound stream) |  $\beta$  (Outbound stream) |  $\gamma$  (Objective) ]

$\alpha_1$ capacity for parallel SKU bins	○ $k$	only a single SKU bin at a time $k > 1$ SKU bins in parallel
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$\alpha_3$ arrival sequence	○ fix	part of optimization problem already fixed
$\alpha_4$ bin inventory	○ pieces	all bins carry enough pieces limited number of pieces in the SKU bins

[Source: Vanderlande]



$\alpha_2 = \circ$ : homogeneous SKU bins

[Source: Amazon]



$\alpha_2 = \text{mix}$ : heterogeneous inventory pods



## 3-field notation: [ $\alpha$ (Inbound stream) | $\beta$ (Outbound stream) | $\gamma$ (Objective) ]

$\beta_1$ capacity for parallel customer bins	1 ○	only a single customer bin at a time multiple customer bins in parallel
$\beta_2$ order composition	1-SKU ○	each order demands only a single SKU each order may demand multiple SKUs
$\beta_3$ processing sequence	○ fix	part of optimization problem already fixed
$\beta_4$ bin exchange	○ batch seq	random access to each customer bin batch-wise exchange of customer bins customer bins enter and leave concurrently
$\beta_5$ SKU availability	○ fast	new customer bin cannot get current SKU new customer bin can reach current SKU

[Source: TGW Logistics Group]



$\beta_1 = 1$ : Picking workstation with on active customer bin

[Source: Lightning Pick]



$\beta_1 = \infty$ : Put-to-light system with many customer bins

## 3-field notation: [ $\alpha$ (Inbound stream) | $\beta$ (Outbound stream) | $\gamma$ (Objective) ]

$\beta_1$ capacity for parallel customer bins	1 ○	only a single customer bin at a time multiple customer bins in parallel
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$\beta_4$ bin exchange	○ batch seq	random access to each customer bin batch-wise exchange of customer bins customer bins enter and leave concurrently
$\beta_5$ SKU availability	○ fast	new customer bin cannot get current SKU new customer bin can reach current SKU

[Source: Lightning Pick]



$\beta_5 = \circ$ : Manual bin exchange

[Source: Vanderlande]



$\beta_5 = \text{fast}$ : Picking workstation with automated bin exchange

**3-field notation:** [  $\alpha$  (Inbound stream) |  $\beta$  (Outbound stream) |  $\gamma$  (Objective) ]

- minimizing the total number of SKU bins
- ★ another objective

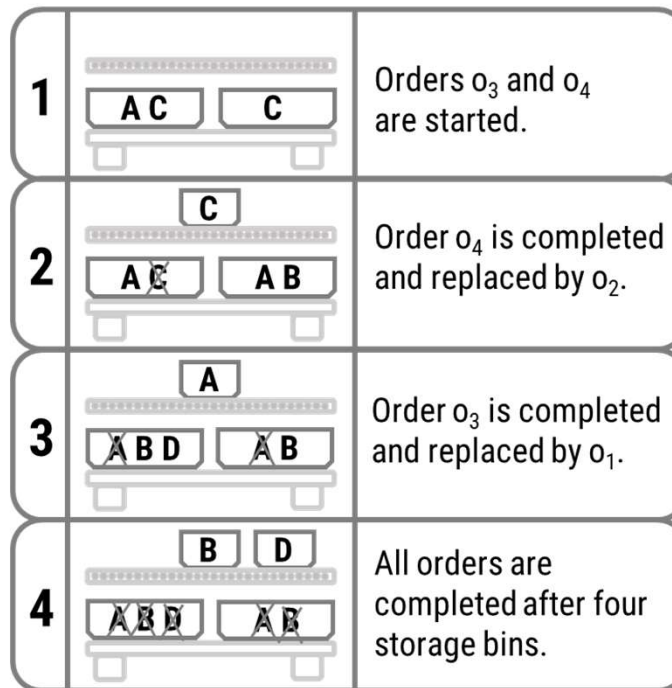
- All objectives of traditional machine scheduling are possible.
- Reduced setup times:
  - ▶ Setup time associated with each SKU bin exchange
    - ◆ Waiting time during bin switch
    - ◆ Orientation time (e.g., perceive new put-to-light signals)
- Relief of ASRS:
  - ▶ Fewer SKU bins to be delivered relief the bin supply system
  - ▶ Each bin change is a source of potential delay (e.g., delayed robot arrival)

## Summary of synchronization literature.

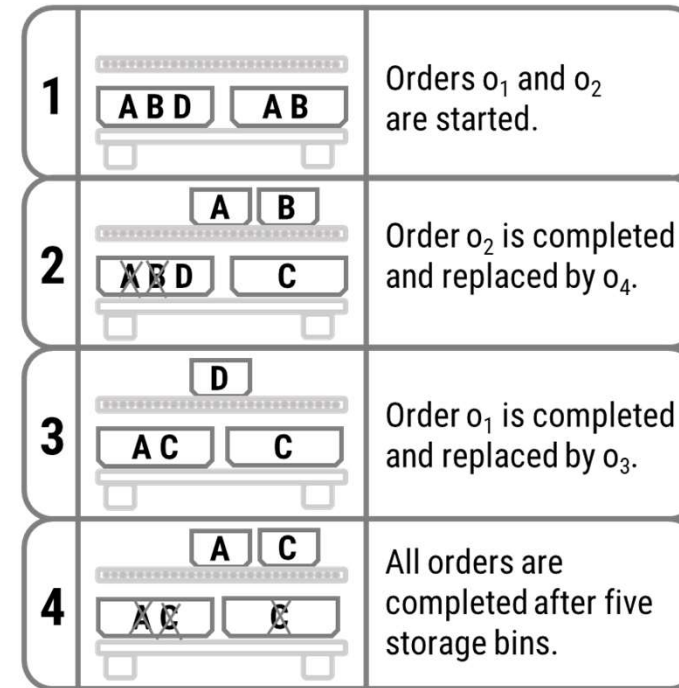
reference	tuple	Methods	application context
Asahiro et al. (2012)	[ 1-SKU,fix,fast ]	–	paint shop batching
Boysen et al. (2017)	[mix fast ]	MIP, HEU	shelf-lifting mobile robots
	[mix,fix fast ]	EX	
	[mix fix,fast ]	EX	
Chan et al. (2012)	[ 1-SKU,fix,fast ]	EX	paint shop batching
Füßler & Boysen (2017)	[ ]	MIP, HEU	inverse order picking
Füßler & Boysen (2019)	[ fast ]	MIP, HEU	ergonomic picking workstation
Nicolas, Yannick, & Ramzi (2018)	[mix batch ]	MIP	vertical lift module
Ouzidan, Sevaux, Olteanu, Pardo, & Duarte (2022)	[ fast ]	MIP, HEU	ergonomic picking workstation
Valle & Beasley (2020)	[mix,pieces *]	HEU	shelf-lifting mobile robots

Legend: MIP: mixed integer program, EX: exact procedure, HEU: heuristic.

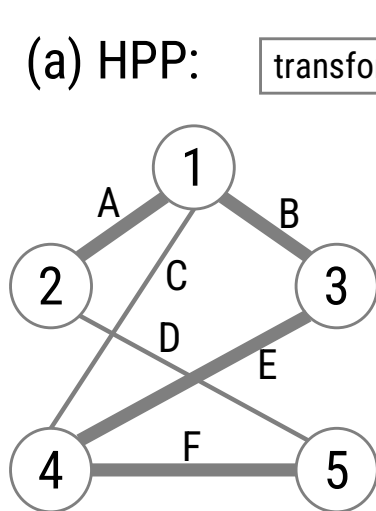
[Source: Dematic]



**Solution (a)**



**Solution (b)**



transformation

(b) [[fast]]:

$o_1 = \{A, B, C\}$

$o_2 = \{A, D\}$

$o_3 = \{B, E\}$

$o_4 = \{C, E, F\}$

$o_5 = \{D, F\}$

$C=1$

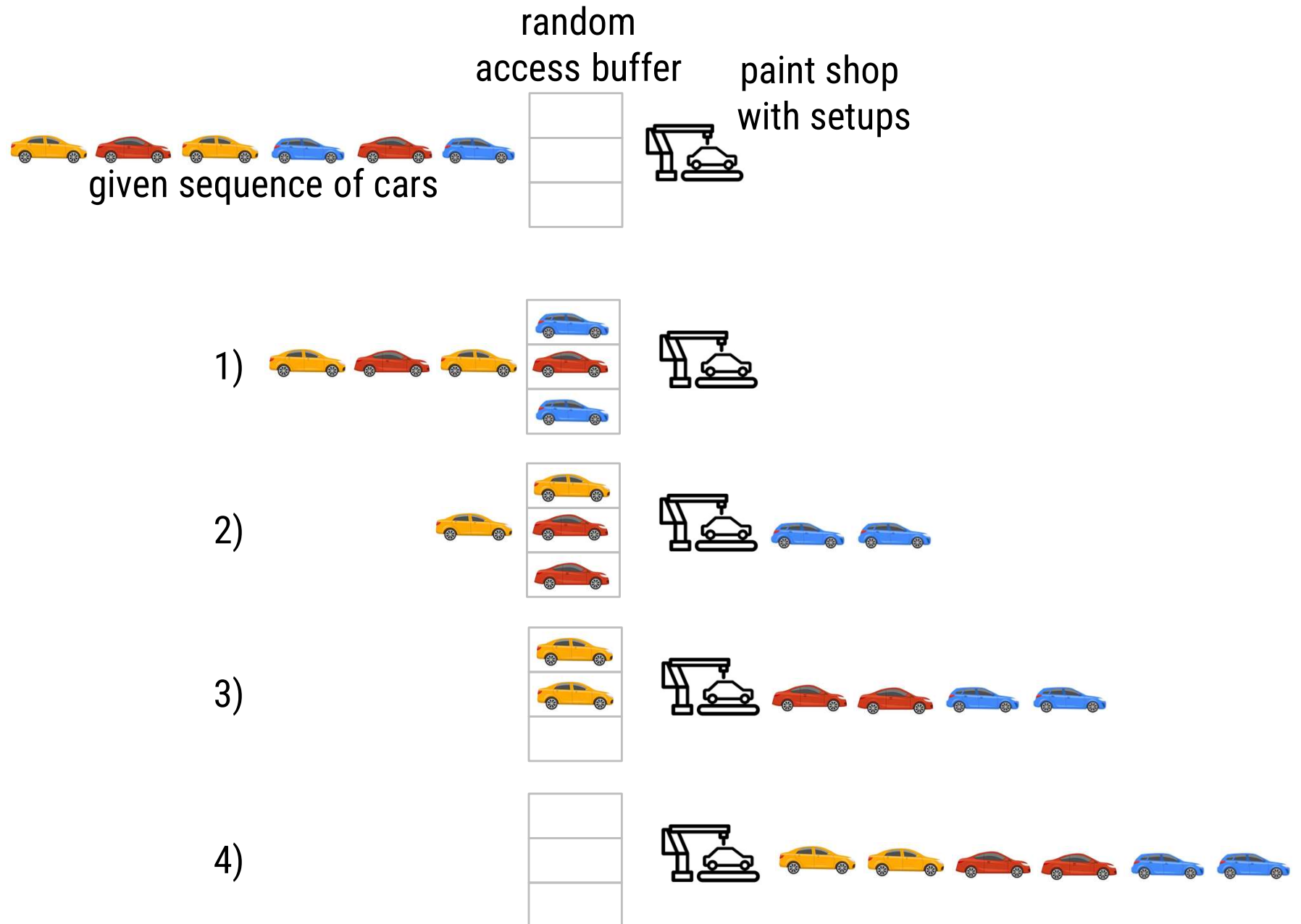
(c) solution:

order sequence

2 1 3 4 5

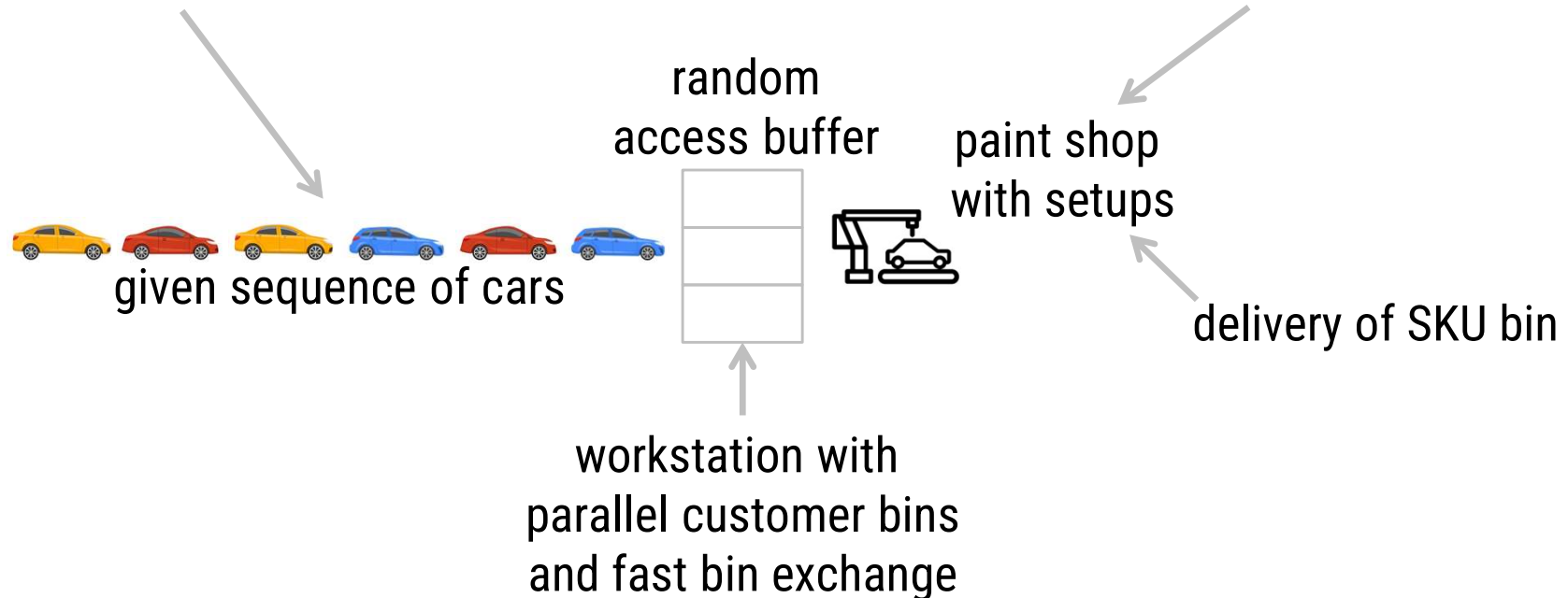
DA-ACB-BE-ECF-FD

SKU bin sequence



given sequence of 1-SKU  
customer orders

workstation with  
one SKU bin at a time



## ■ Previous research:

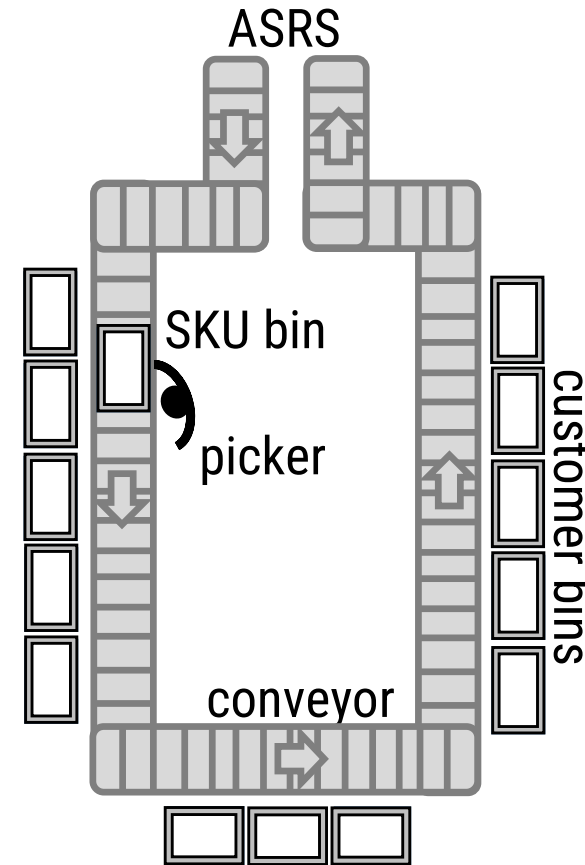
- ▶ Asahiro, Y., Kawahara, K., & Miyano, E. (2012). NP-hardness of the sorting buffer problem on the uniform metric. *Discrete Applied Mathematics*, 160(10-11), 1453-1464.
- ▶ Chan, H. L., Megow, N., Sitters, R., & van Stee, R. (2012). A note on sorting buffers offline. *Theoretical Computer Science*, 423, 11-18.
- ▶ Adamaszek, A., Renault, M. P., Rosén, A., & van Stee, R. (2017). Reordering buffer management with advice. *Journal of Scheduling*, 20, 423-442.



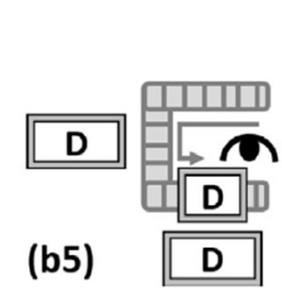
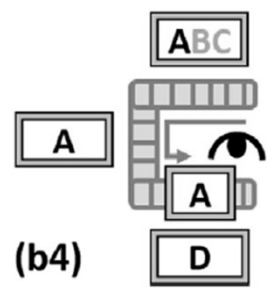
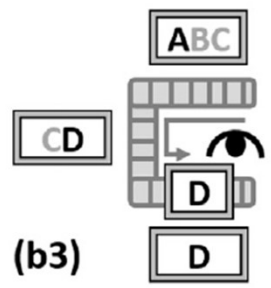
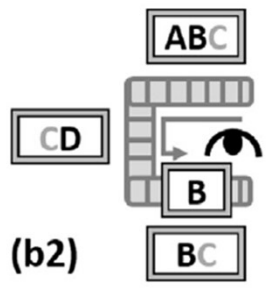
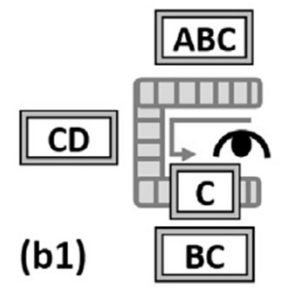
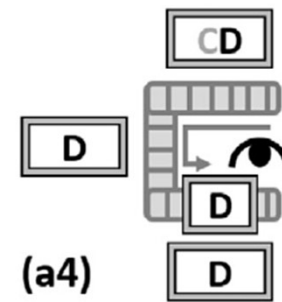
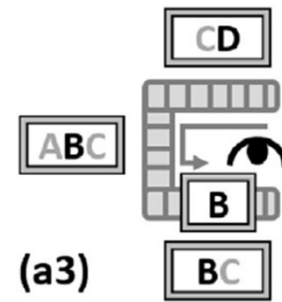
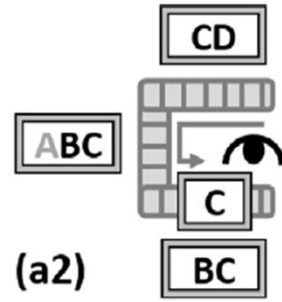
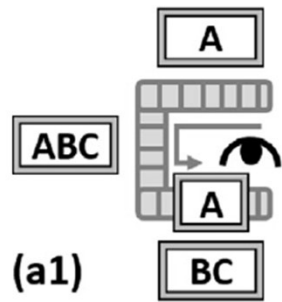
[Source: Lightning Pick]




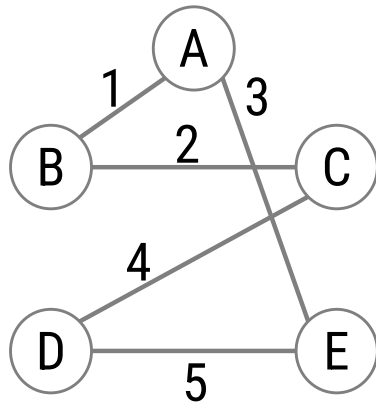
(a) Put-to-light system Lightning Pick at apparel retailer Charlotte Russe



(b) System setup



(a) MCLA:  transformation



$k=2$

(b) [[|]]:

$o_1 = \{A, B\}$

$o_2 = \{B, C\}$

$o_3 = \{A, E\}$

$o_4 = \{C, D\}$

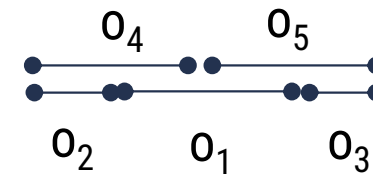
$o_5 = \{D, E\}$

$k=C=2$

(c) solution:

Vertex labels

$C^1 \quad B^2 \quad D^3 \quad A^4 \quad E^5$



(a) Robots lift shelves...  
(here CarryPick of Swisslog)

[Source: Swisslog]



(b) and deliver them to  
picking stations

[Source: Amazon]



## **order selection and assignment**

(selects the next orders from the pool and assigns them to pick stations)



## **order fulfillment scheduling (OFS)**

(determines the assignment of orders to batches and their processing sequence at a pick station and assigns racks to satisfy the demanded SKUs)



## **rack assignment problem**

(assigns each stopover of racks a storage position)

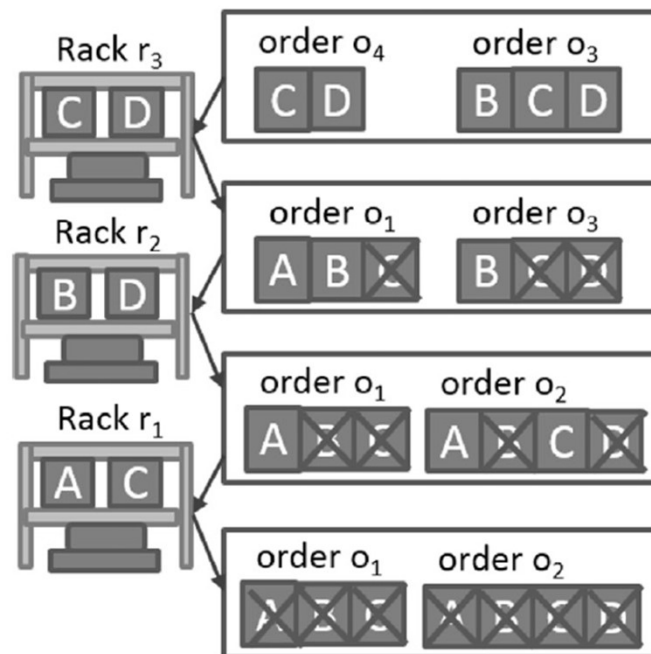


## **robot assignment and path planning**

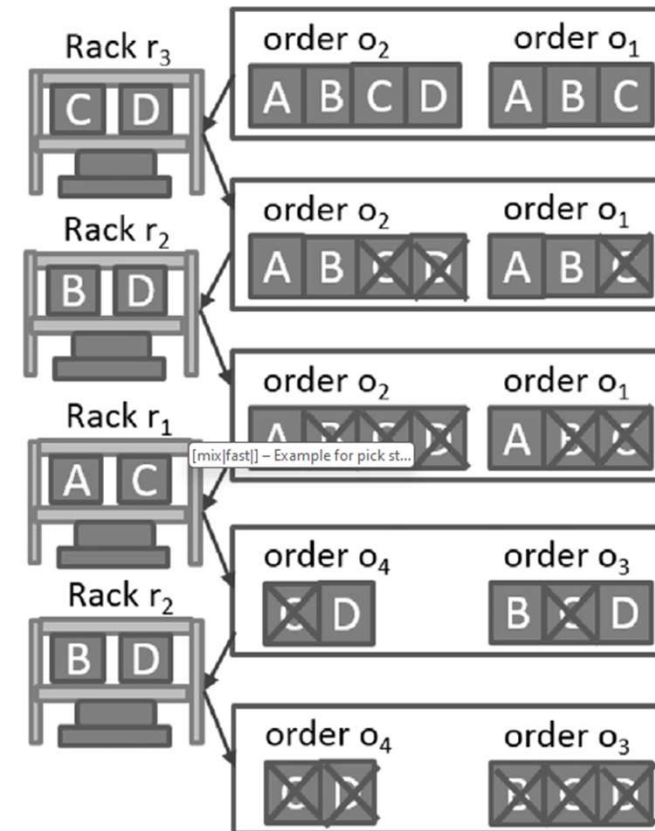
(assigns a robot to each movement of a rack and coordinates their travel paths on the shop floor)

# [mix|fast] – Example for pick station scheduling (PSS)

- Set  $S = \{A, B, C, D\}$  an SKUs
- $n = 4$  orders:  $o_1 = \{A, B, C\}$ ,  $o_2 = \{A, B, C, D\}$ ,  $o_3 = \{B, C, D\}$ ,  $o_4 = \{C, D\}$
- $m = 3$  racks:  $r_1 = \{A, C\}$ ,  $r_2 = \{B, D\}$ ,  $r_3 = \{C, D\}$
- Capacity  $C = 2$



Order sequence =  $\langle o_4, o_3, o_1, o_2 \rangle$   
 Rack sequence =  $\langle r_3, r_2, r_1 \rangle$



Order sequence =  $\langle o_2, o_1, o_4, o_3 \rangle$   
 Rack sequence =  $\langle r_3, r_2, r_1, r_2 \rangle$

## ■ Decomposition

- ▶ Solve rack sequencing for given order sequence – str. NP-hard
- ▶ Solve order sequencing for given rack sequence – str. NP-hard

[Source: Amazon]



Strongly NP-hard synchronization problems.

	class of synchronization problems	# SP	transformation from	reference
1	[o; o; o; o; o; o; o; o; o; o]	1	min-cut linear arrangement	(Füßler & Boysen, 2017)
2	[o; -; o; - o; o; o; o; o]	4	[o; o; o; o; o; o; o; o]	Lemma 2
3	[o; -; o; - 1; o; o; -; fast o]	12	Hamilton path	Theorem 1
4	[o; -; o; - o; o; -; fast o]	24	Hamilton path	Corollary 1
5	[k; -; o; - o; o; -; fast o]	24	[o; -; o; - o; o; -; fast o]	Corollary 11
6	[o; o; o; o; o; 1-SKU; fix; o; fast o]	1	sorting buffer problem	(Asahiro et al., 2012; Chan et al., 2012)
7	[o; -; o; - o; -; fix; o; fast o]	8	[o; o; o; o; o; 1-SKU; fix; o; fast o]	Lemma 2
8	[o; -; -; pieces o; -; o; -; o o]	48	3-Partition	Theorem 2
9	[o; -; -; pieces o; o; o; -; fast o]	12	3-Partition	Theorem 2
10	[k; -; -; pieces o; o; o; -; fast o]	12	[o; -; -; pieces o; o; o; -; fast o]	Corollary 11
11	[o; -; o; pieces 1; -; -; o o]	24	3-Partition	Theorem 2
12	[o; -; fix; pieces 1; -; o; -; fast o]	12	3-Partition	Corollary 5
13	[k; -; fix; pieces 1; o; o; -; fast o]	6	[o; -; fix; pieces 1; -; o; -; fast o]	Corollary 11
14	[o; -; fix; pieces o; -; o; batch; - o]	8	3-Partition	Corollary 6
15	[k; -; fix; pieces o; o; o; batch; fast o]	2	[o; -; fix; pieces o; o; o; batch; fast o]	Corollary 11
16	[o; -; fix; pieces o; -; o; seq; - o]	8	3-Partition	Corollary 6
17	[k; -; fix; pieces o; o; o; seq; fast o]	2	[o; -; fix; pieces o; o; o; seq; fast o]	Corollary 11
18	[o; -; o; pieces o; -; -; fix; -; - o]	48	3-Partition	Corollary 7
19	[k; -; o; pieces o; o; fix; -; -; fast o]	12	[o; -; o; pieces o; o; fix; -; -; fast o]	Corollary 11
20	[o; -; fix; pieces o; o; fix; o; - o]	4	3-Partition	Corollary 8
21	[k; -; fix; pieces o; o; fix; o; fast o]	2	[o; -; fix; pieces o; o; fix; o; fast o]	Corollary 11
22	[o; mix; o; o o; 1-SKU; -; -; - o]	12	set covering	(Boysen et al., 2017), Theorem 3
23	[k; mix; o; o o; 1-SKU; -; -; -; fast o]	6	set covering	(Boysen et al., 2017), Theorem 3
24	[o; mix; o; - o; -; -; -; - o]	48	[o; mix; o; o o; 1-SKU; -; -; - o]	Lemma 2
25	[k; mix; o; - o; -; -; -; -; fast o]	24	[k; mix; o; o o; 1-SKU; -; -; -; fast o]	Lemma 2
26	[o; mix; o; - 1; -; o; -; fast o]	24	set covering	Corollary 9
27	[k; mix; o; - 1; -; -; -; -; fast o]	12	set covering	Corollary 9
28	[o; mix; o; - o; -; -; -; - o]	96	set covering	Corollary 10
29	[o; mix; fix; o 1; o; o; o; fast o]	1	interval scheduling	(Boysen et al., 2017)
30	[o; mix; fix; - o; o; -; -; fast o]	12	[o; mix; fix; o 1; o; o; o; fast o]	Lemmas 1 and 2
31	[k; mix; fix; - o; o; -; -; fast o]	12	[o; mix; fix; - o; o; -; -; fast o]	Corollary 11

281 out of 576 problems are shown to be strongly NP-hard.

156 out of 576 problems are solvable in polynomial time.

Synchronization problems solvable to optimality in polynomial time.

	class of synchronization problems	# SP	valid only if	reference
1	[o; -; fix; o o; 1-SKU; o; -; fast o]	12		Lemma 5
2	[o; o; fix; o o; 1-SKU; o; -; o o]	6		Lemma 5
3	[o; o; o; o o; 1-SKU; o; -; fast o]	6		Lemma 5
4	[o; o; o; pieces o; 1-SKU; o; -; fast o]	6		Lemma 5
5	[o; o; o; o 1; 1-SKU; fix; -; - o]	6		Lemma 6
6	[o; o; o; o 1; -; -; -; o o]	12		Lemma 7
7	[o; -; fix; o o; -; fix; -; - o]	48		Lemma 8
8	[o; -; -; o 1; 1-SKU; -; -; o o]	24		Lemma 9
9	[o; mix; fix; o o; 1-SKU; o; -; o o]	6		Lemma 10
10	[o; mix; o; o 1; 1-SKU; fix; -; -; fast o]	3		Theorem 4
11	[o; -; o; o o; -; -; fix; o; fast o]	16	(a) and (b) and (c)	Corollary 12
12	[o; -; -; o; o o; -; -; fix; seq; fast o]	16	(a) and (b) and (c)	Corollary 13
13	[o; -; -; o; o o; -; -; fix; batch; fast o]	16	(a) and (b) and (c)	Corollary 14
14	[o; -; -; fix; o o; -; -; fix; -; -; fast o]	48	(a) and (b) and (c)	Corollary 15
15	[o; -; -; o o; -; -; fix; -; -; fast o]	96	(a) and (b) and (c)	Theorem 5
(a)	the SKU bin capacity $k$ is limited by a constant			
(b)	the customer bin capacity is limited by a constant			
(c)	the maximum number of SKUs required by a customer bin is limited by a constant			



Synchronization gains in number of SKU bins deliveries in % for different workstation setups related to default case [||] depending on different demand structures (EQ and ABC) and customer bin capacities  $\beta_1$ .

case	extension	EQ			ABC		
		$\beta_1 = 1$	$\beta_1 = 3$	$\beta_1 = 5$	$\beta_1 = 1$	$\beta_1 = 3$	$\beta_1 = 5$
[  ]	–	0.00	-27.55	-35.92	0.00	-42.50	-56.61
[3  ]	parallel SKU bins	-24.43	-32.60	-38.05	-48.27	-49.41	-59.86
[5  ]	parallel SKU bins	-33.63	-34.45	-38.77	-61.26	-61.26	-62.10
[mix  ]	mix of SKUs per bin	-2.83	-41.60	-52.26	-2.63	-50.24	-64.14
[ fix ]	given order sequence	0.00	-17.46	-27.17	0.00	-26.54	-45.51
[ batch ]	bin exchange	0.00	-8.51	-14.70	0.00	-25.00	-35.95
[ seq ]	bin exchange	0.00	-14.04	-25.31	0.00	-32.82	-44.42
[ fast ]	bin exchange	-11.54	-30.43	-36.61	-20.78	-49.21	-58.16



## ■ Should we have more bins?

- ▶ Yes, more bin capacity greatly reduces the SKU bin deliveries.
- ▶ The positive effect is especially strong for ABC orders.
- ▶ The positive effect quickly diminishes, so that more than five is barely worth the effort.
- ▶ Negative effect: More picker movement along the pick face.

## ■ Should be increase the capacity for SKU bins or customer bins?

- ▶ It does not matter.

Synchronization gains in number of SKU bins deliveries in % for different workstation setups related to default case [||] depending on different demand structures (EQ and ABC) and customer bin capacities  $\beta_1$ .

case	extension	EQ			ABC		
		$\beta_1 = 1$	$\beta_1 = 3$	$\beta_1 = 5$	$\beta_1 = 1$	$\beta_1 = 3$	$\beta_1 = 5$
[  ]	-	0.00	-27.55	-35.92	0.00	-42.50	-56.61
[3  ]	parallel SKU bins	-24.43	-37.60	-38.95	-48.27	-49.41	-51.86
[5  ]	parallel SKU bins	-33.63	-34.45	-38.77	-61.26	-61.26	-62.10
[mix  ]	mix of SKUs per bin	-2.83	-41.60	-52.26	-2.63	-50.24	-64.14
[ fix ]	given order sequence	0.00	-17.46	-27.17	0.00	-26.54	-45.51
[ batch ]	bin exchange	0.00	-8.51	-14.70	0.00	-25.00	-35.95
[ seq ]	bin exchange	0.00	-14.04	-25.31	0.00	-32.82	-44.42
[ fast ]	bin exchange	-11.54	-30.43	-36.61	-20.78	-49.21	-58.16

[Source: Vanderlande]



vs.



## Mixed SKU bins?

### ■ Should we mix the SKU bins?

- ▶ Not necessarily, the positive effect is rather small.
- ▶ Negative effect: More search effort for the picker to find the right SKU.
- ▶ Support in warehouses: Picture of SKU on display or laser beam onto right compartment.

Synchronization gains in number of SKU bins deliveries in % for different workstation setups related to default case [||] depending on different demand structures (EQ and ABC) and customer bin capacities  $\beta_1$ .

case	extension	EQ			ABC		
		$\beta_1 = 1$	$\beta_1 = 3$	$\beta_1 = 5$	$\beta_1 = 1$	$\beta_1 = 3$	$\beta_1 = 5$
[  ]	-	0.00	-27.55	-35.92	0.00	-42.50	-56.61
[3  ]	parallel SKU bins	-24.43	-37.60	-38.95	-48.27	-49.41	-51.86
[5  ]	parallel SKU bins	-31.63	-37.45	-38.17	-61.26	-61.26	-61.0
[mix  ]	mix of SKUs per bin	-21.1	-41.60	-51.66	-21.8	-51.24	-61.4
[ fix ]	given order sequence	0.0	-11.46	-21.17	0.0	-21.64	-41.61
[ batch ]	bin exchange	0.0	-8.1	-11.0	0.0	-21.0	-31.05
[ seq ]	bin exchange	0.0	-14.04	-25.41	0.0	-32.82	-42.42
[ fast ]	bin exchange	-11.54	-30.43	-36.61	-20.78	-49.21	-58.16



vs.



## Fast customer bin switches ?

- Should we invest into an automated mechanism to switch completed customer bins fast?
  - ▶ Yes, but only if a parallelization of multiple (SKU or customer) bins is not possible.

- Order fulfillment problems appear
  - ▶ in many different parts-to-picker systems
  - ▶ with slight variation.
  - ▶ There is not much work on these problems,
  - ▶ especially from a general perspective.



# Outlook: Within 5-10 years, we have the fully-automated e-commerce fulfillment factory

[Source: Mecalux]



Picking robot with vacuum gripper

Robotized sorting



[Source: Tompkins Robotics]



[Source: Industrial Automation]

Automated packing

**Robots and machinery need advice!**  
**We need more research on warehouse scheduling!**

[Source: Amazon]



## ■ Literature:

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- ▶ Boysen, N., Briskorn, D., & Emde, S. (2017). Parts-to-picker based order processing in a rack-moving mobile robots environment. *European Journal of Operational Research*, 262(2), 550-562.
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