Learning-Augmented Online Algorithms for Scheduling and Routing

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http://schedulingseminar.com

Scheduling under Uncertainty

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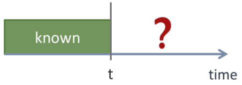
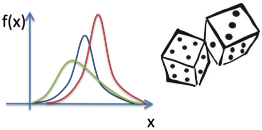
Different models for uncertain input

Online Information	Stochastic Information





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- explorable uncertainty
- etc.

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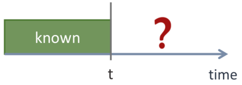

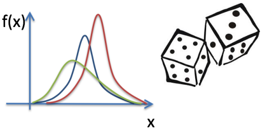

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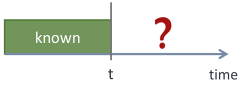

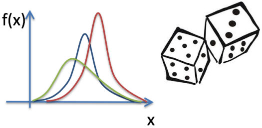

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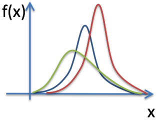

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Can error-prone predictions improve upon performance guarantees?

Learning-Augmented Algorithms

- Algorithm with access to prediction
- No assumption on the quality of the prediction

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Desired properties

- Consistency: better than worst case if the prediction errors are small
- Robustness: bounded worst-case for arbitrary predictions
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Beyond worst case: competitive ratio (online alg.), running time, etc.

Roadmap

Prediction models and learning-augmented algorithms for

1. **Online Scheduling**: uncertain processing times
2. **Online Routing**: uncertain job arrival times and locations

Roadmap

Prediction models and learning-augmented algorithms for

1. **Online Scheduling**: uncertain processing times
2. **Online Routing**: uncertain job arrival times and locations

Important questions

- What to predict? Can we beat worst case bounds?
- How to ensure robustness?
- What is a good error measure?

Online Scheduling

Joint work with Alexander Lindermayr, SPAA 2022.

(Non)-Clairvoyant Scheduling

Input: set of jobs with (unknown) processing requirements p_j

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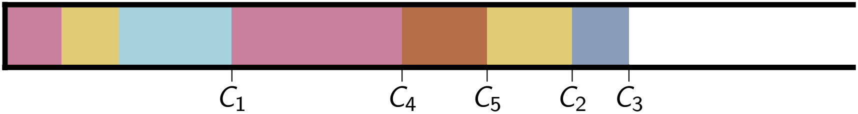

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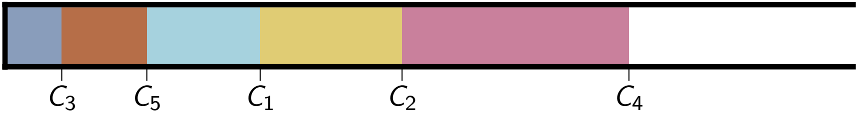

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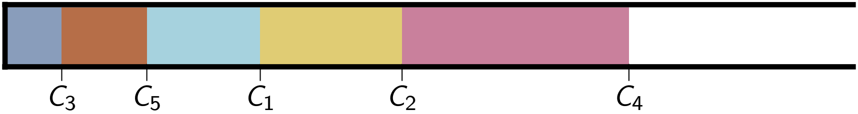

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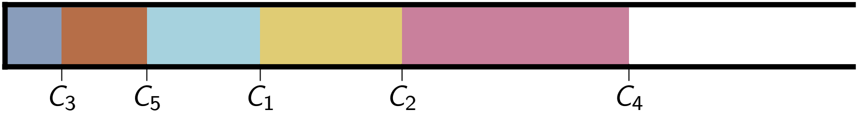

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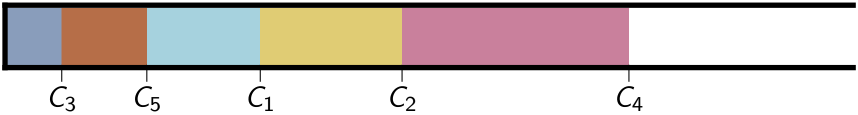

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We assume unknown processing times (non-clairvoyant scheduling).

Non-clairvoyant Scheduling

Competitive analysis (worst-case analysis)

An online algorithm is ρ-competitive if it achieves, for any input instance, a solution of cost within a factor ρ of the optimal cost:

$$\textsc{Alg}(I) \leq \rho \cdot \textsc{Opt}(I), \qquad \text{for any input } I.$$

Non-clairvoyant Scheduling

Round-Robin (RR) is 2-competitive for minimizing $\sum C_j$ on a single machine, and this is best-possible. [Motwani, Phillips, Torng 1994]

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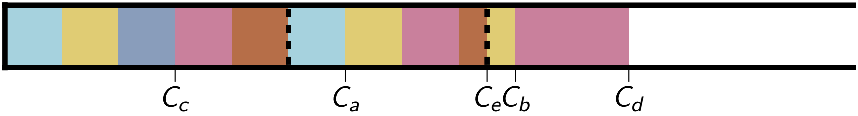

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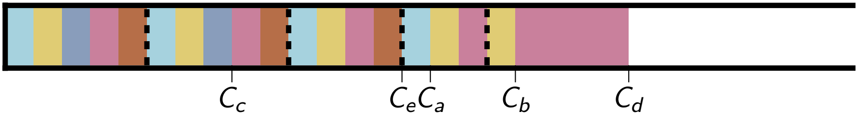

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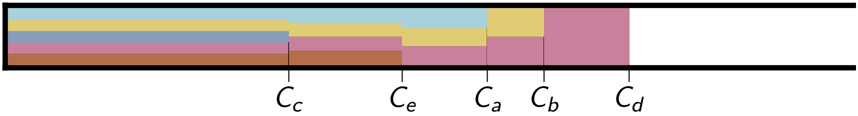

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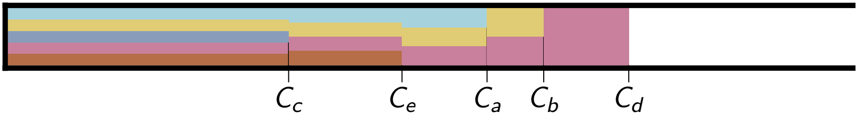

Further **Time-Sharing** algorithms for more general problems:

- Individual job weights: Weighted Round-Robin (2-competitive) [Kim, Chwa 2003]
- Identical machines: Weighted Dynamic Equipartition (2-comp.) [Beaumont, Bonichon, Eyraud-Dubois, Marchal 2012]
- Unrelated machines: Proportional Fairness (128-competitive) [Im, Kulkarni, Munagala 2018]

Length Predictions and ℓ_1-errors

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Predict job lengths y_j

[Kumar, Purohit, Svitkina 2018]
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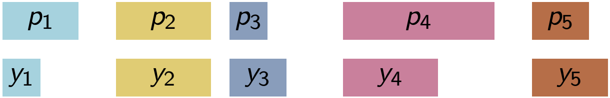

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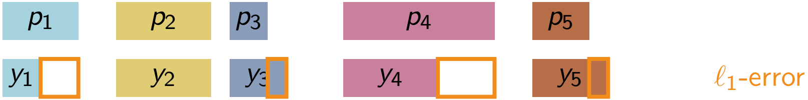

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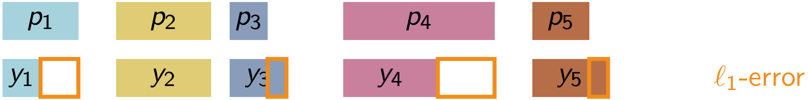

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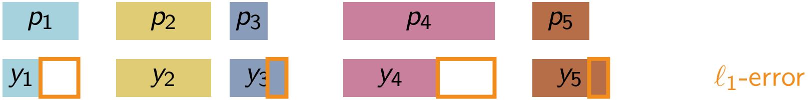

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SPF achieves scheduling cost $\mathrm{SPF}(y_j, p_j) \leq \textsc{Opt}(p_j) + n \cdot \ell_1$.

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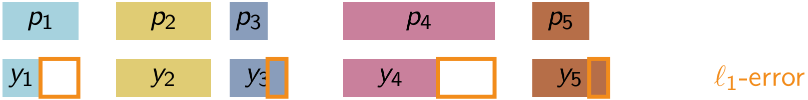

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Consistent but not robust (against bad predictions).

Preferential Time Sharing Framework (PTS)

[Kumar, Purohit, Svitkina 2018]

Input:

- prediction-clairvoyant alg. $\mathcal{A}^C$ ("follow the prediction") with some error-dependent competitive ratio
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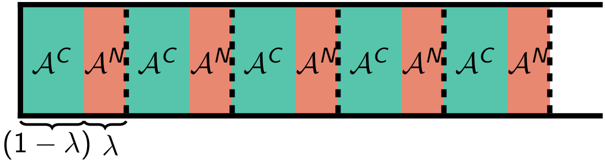

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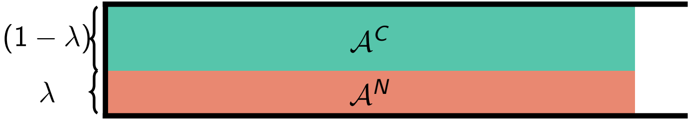

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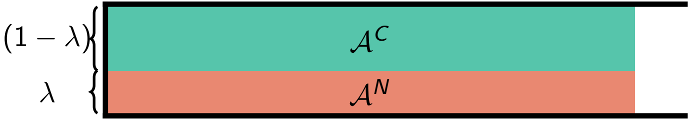

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Monotone algorithms: if p_j's shrink, completion times do not increase

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Theorem

PTS$(\lambda, \mathcal{A}^C, \mathcal{A}^N)$ has competitive ratio min $\left\{\frac{1}{1-\lambda}\left(\alpha + \frac{\eta}{\mathrm{OPT}}\right), \frac{\beta}{\lambda}\right\}$, if

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Corollary. PTS(λ, SPF, RR) achieves for non-clairvoyant $1|pmtn|\sum C_j$ with length predictions, for any $\lambda \in (0,1)$, a competitive ratio of

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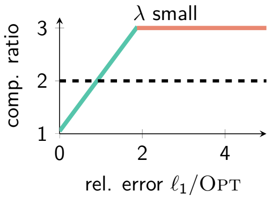

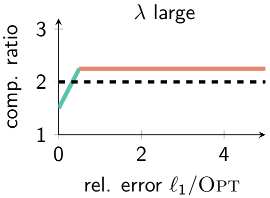

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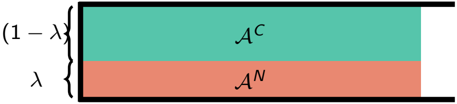

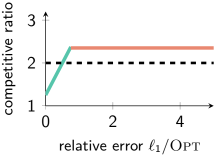

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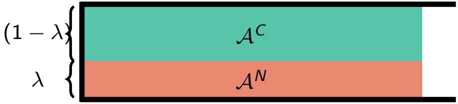

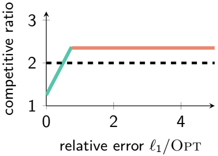

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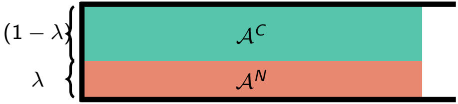

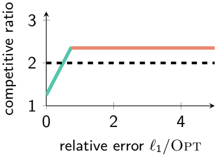

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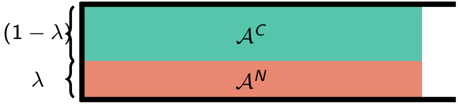

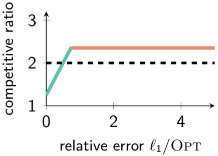

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Discussion ℓ_1-Error Measure

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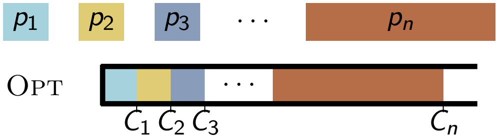

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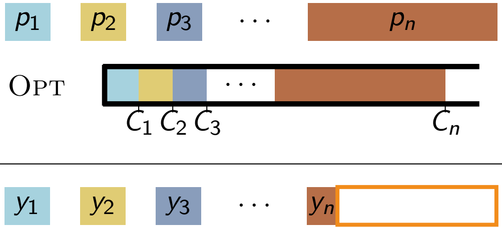

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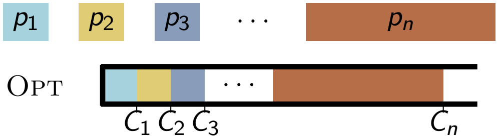

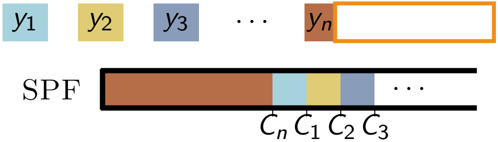

huge ℓ_1-error

$\mathrm{SPF} \approx \textsc{Opt} + n \cdot \ell_1$

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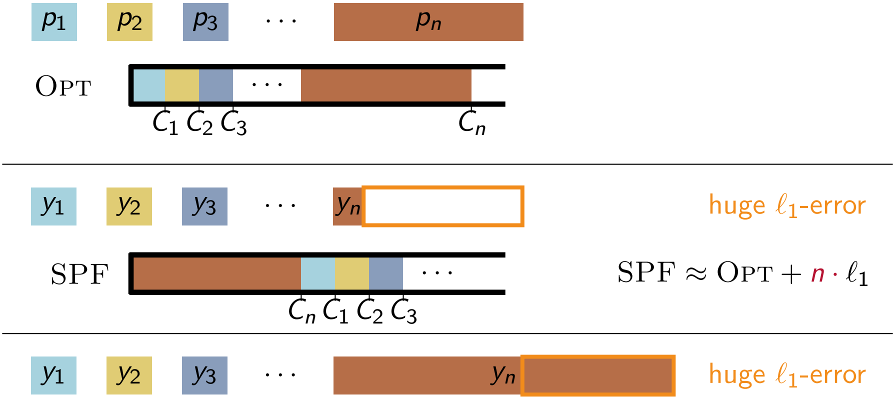

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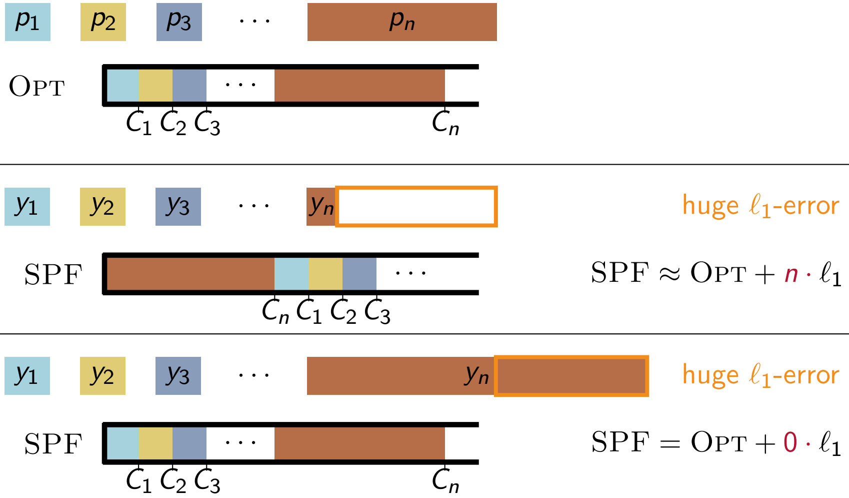
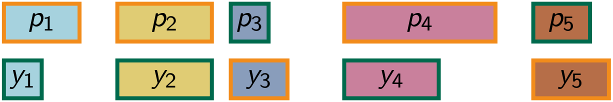


Length Predictions and ν-error

[Im, Kumar, Qaem, Purohit 2021]

Alternative error: $\nu = \textsc{Opt}(\{\max\{p_j, y_j\}\}_j) - \textsc{Opt}(\{\min\{p_j, y_j\}\}_j)$



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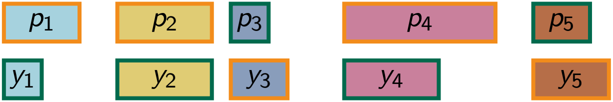

Error-tracker algorithm: $\min\left\{(1+\epsilon)\textsc{Opt} + \mathcal{O}\left(\frac{1}{\epsilon^3}\log\frac{1}{\epsilon}\right)\nu, \frac{2}{\epsilon}\textsc{Opt}\right\}$ for $1|pmtn|\sum C_j$

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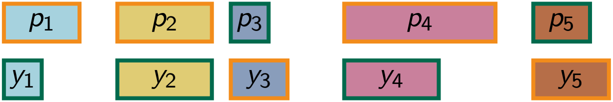

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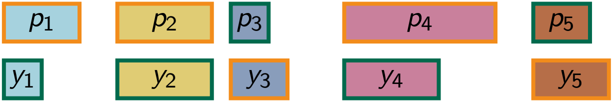

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Issue: Underestimates true difficulty of the following instance

$p_1 = y_1 = \ldots = p_{n-1} = y_{n-1} = 1$, but $p_n = n^2$ and $y_n = 0$.

$\nu = n^2 + n = \Theta(n^2)$ vs $\mathrm{SPF}(y_j, p_j) - \textsc{Opt}(p_j) = \Omega(n^3)$

Permutation Predictions

Permutation predictions: predict an order of jobs: $\hat{\sigma} : [n] \to [n]$

Motivation: knowing WSPT order is often sufficient for good approximations:

- optimal for $1|(pmtn)|\sum w_j C_j$ [Smith 1956]
- 2-competitive for $P|r_j, pmtn|\sum w_j C_j$ [**M**. & Schulz 2004]
- 5.83-competitive for $R|r_j, pmtn|\sum w_j C_j$ [Lindermayr & **M**. 2022]

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p_1 p_2 p_3 (W)SPT order $p_3 \leq p_1 \leq p_2$

$$\left.\begin{array}{lll} y_1 & y_2 & y_3 \\ y_1 & y_2 & y_3 \end{array}\right\}\text{Indicate correct order } y_3 \leq y_1 \leq y_2\text{, but } \ell_1, \nu > 0$$

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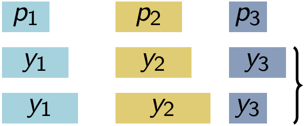

(W)SPT order $p_3 \leq p_1 \leq p_2$

Indicate correct order $y_3 \leq y_1 \leq y_2$, but $\ell_1, \nu > 0$

Error measure: quantifies effect of inversions $\mathcal{I}$ between $\hat{\sigma}$ and true WSPT order on list scheduling according to predicted order:

$$\eta^S = \sum_{(i,j) \in \mathcal{I}} (w_i p_j - w_j p_i)$$

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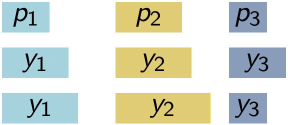

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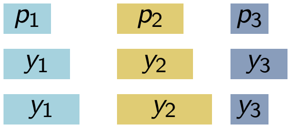

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- η^S captures structure instead of irrelevant numerical values.
- Permutation predictions are efficiently PAC-learnable w.r.t. η^S.

PTS and Permutation Predictions (1)

PTS for weighted jobs on a single machine $1|pmtn|\sum w_j C_j$

- prediction-clairvoyant $\mathcal{A}^C$: WSPT (monotone optimal) [Smith 1956]

Schedule jobs in WSPT order

$$\min\left\{ \frac{1}{1-\lambda} \cdot \left(\qquad\qquad \right), \frac{1}{\lambda} \cdot \quad \right\}$$

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- non-clairvoyant $\mathcal{A}^N$: WRR (monotone 2-competitive) [Kim, Chwa 2003]

$$\min\left\{\frac{1}{1-\lambda}\cdot\left(1+\frac{\eta^S}{\textsc{Opt}}\right),\frac{1}{\lambda}\cdot 2\right\}$$

PTS and Permutation Predictions (2)

PTS for m identical machines and release dates $P|r_j, pmtn|\sum w_j C_j$

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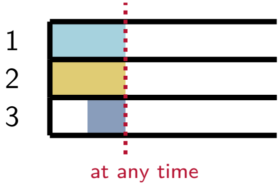
[**M**. and Schulz 2004]



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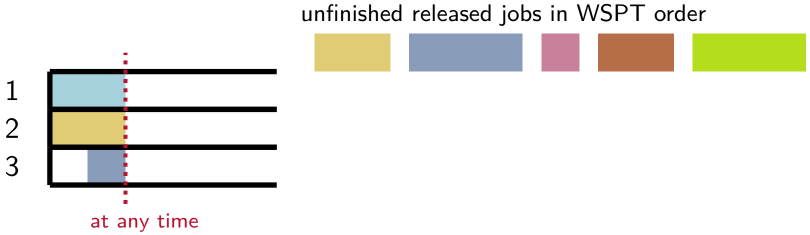

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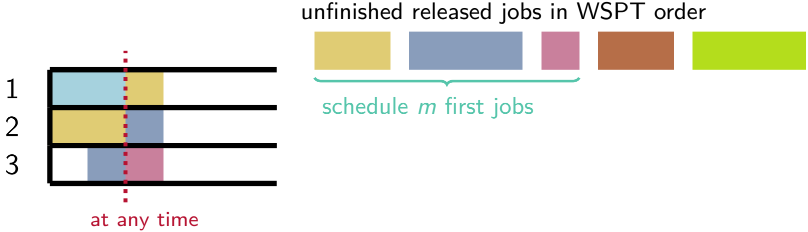

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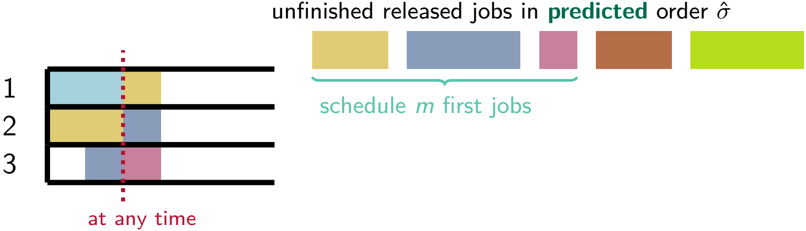

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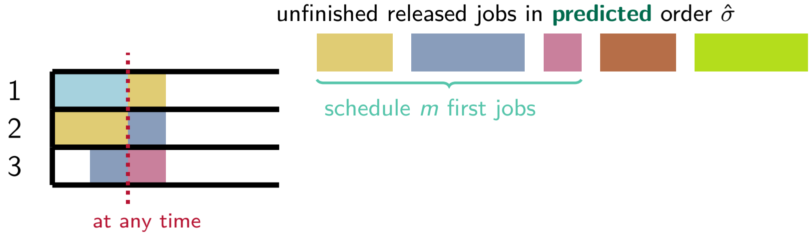

- non-clairvoyant $\mathcal{A}^N$: WDEQ (monotone 3-competitive)

[Beaumont, Bonichon, Eyraud-Dubois and Marchal 2012]

$$\min\left\{\frac{1}{1-\lambda}\cdot\left(2+\frac{\eta^S}{m\cdot\textsc{Opt}}\right), \frac{1}{\lambda}\cdot 3\right\}$$

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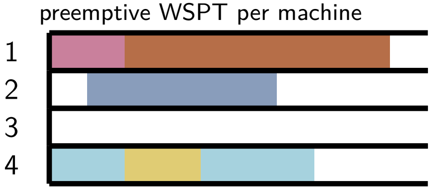

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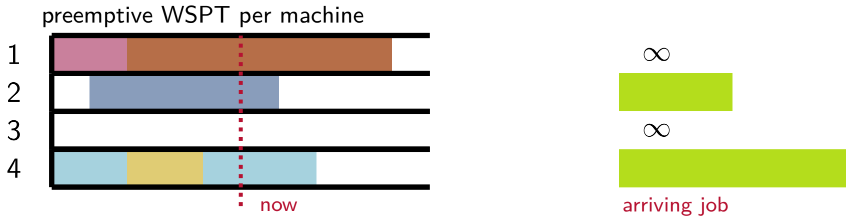

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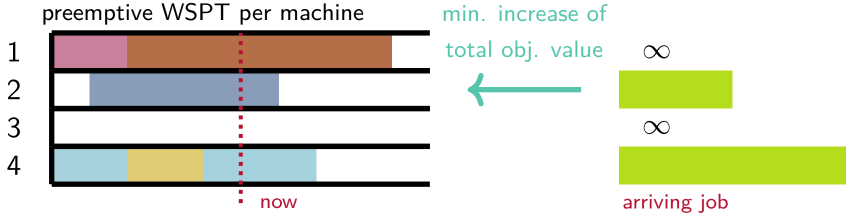

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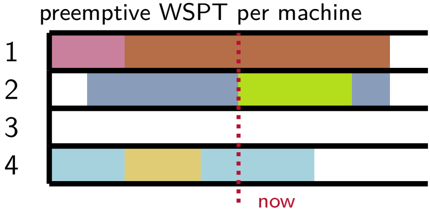

$$\min \left\{ \frac{1}{1-\lambda} \cdot \left(5.8284 + \frac{\eta^R}{\textsc{Opt}} \right), \frac{1}{\lambda} \cdot \qquad \right\}$$

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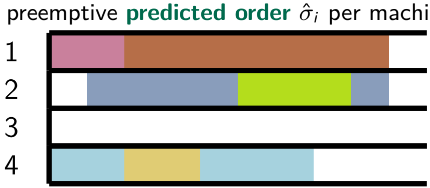

assign job j to

predicted machine i

i.e. $i \in [m]$ s.t. $j \in \hat{\sigma}_i$

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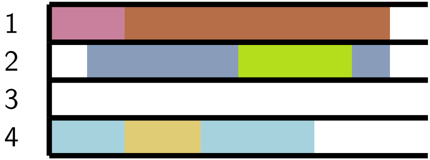
preemptive **predicted order** $\hat{\sigma}_i$ per machine i



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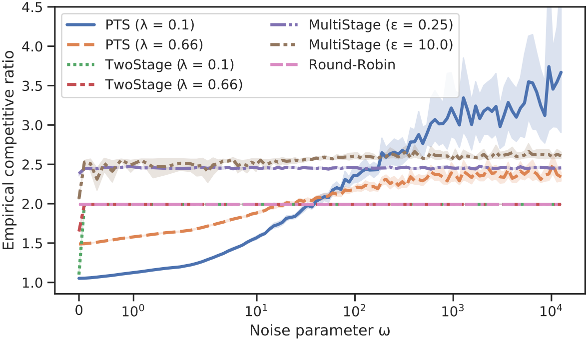
- non-clairvoyant $\mathcal{A}^N$: Proportional Fairness (mon. 128-competitive)

[Im, Kulkarni and Munagala 2018]

$$\min \left\{ \frac{1}{1-\lambda} \cdot \left(5.8284 + \frac{\eta^R}{\textsc{Opt}} \right), \frac{1}{\lambda} \cdot 128 \right\}$$

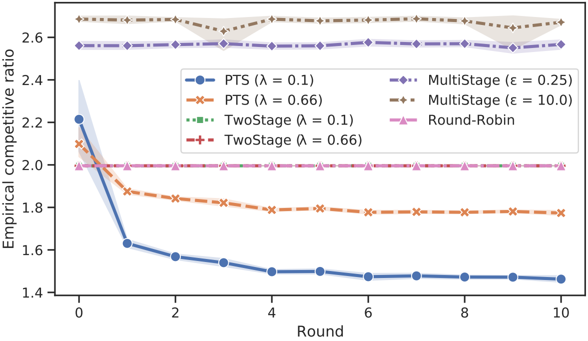
Sensitivity Experiments

- Single machine, unweighted jobs (PTS equals algorithm in [KPS18])
- Synthetic instances sampled from Pareto-distribution with shape 1.1
  Many small jobs and few very large jobs!



Online Learning Experiments

- learn prediction from previous instances
- already after one round performance improves substantially
- such instances can be learned fast in practice (detect long jobs)



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Non-clairvoyant scheduling with predictions

- Discussion of length and permutation prediction and error measures

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Other scheduling results involving predictions

- load balancing [Lattanzi, Lavastida, Moseley, Vassilvitskii SODA 2020]
- flowtime minimization [Azar, Leonardi, Touitou STOC 2021, SODA 2022]
- speed scaling [Bamas, Maggiori, Rohwedder, Svensson NeurIPS 2020], [Antoniadis, Ganje, Shahkarami SWAT 2022]

Online Routing Problems

more general online graph problems incl. network design

Joint work with Giulia Bernardini (Trieste), Alexander Lindermayr (Bremen), Alberto Marchetti-Spaccamela (Rome), Leen Stougie & Michelle Sweering (CWI).

The Online Traveling Salesperson Problem

Input: Requests (x, r) arrive online at time r at location x
Task: Determine a tour of minimum total length (makespan) visiting every request after its arrival and returning to the origin.

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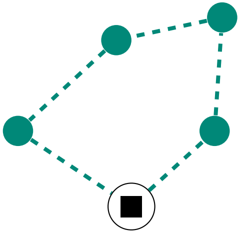

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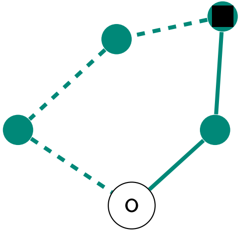

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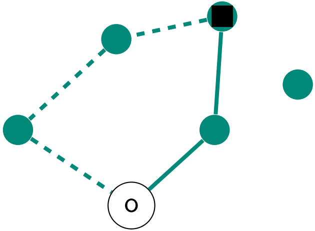

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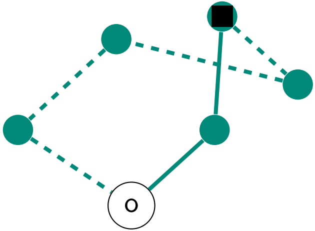

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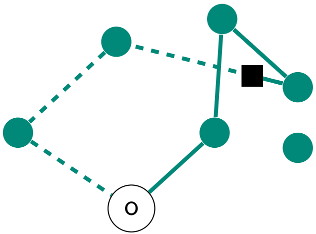

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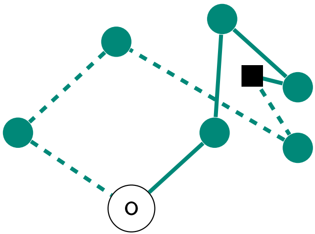

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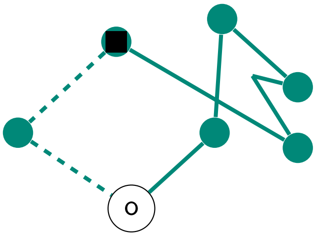

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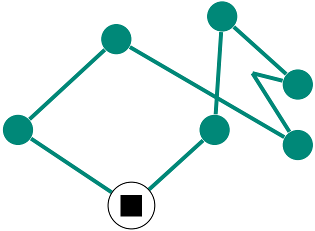

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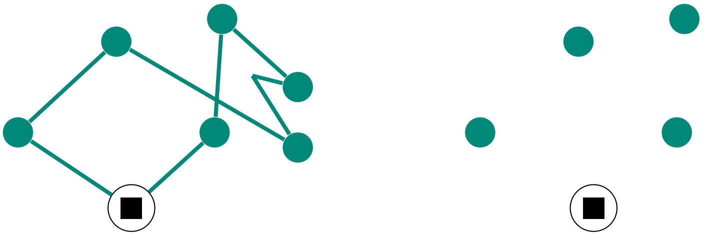

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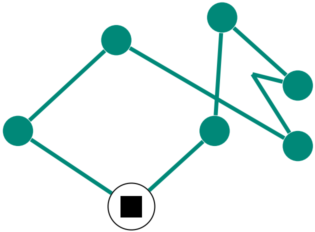

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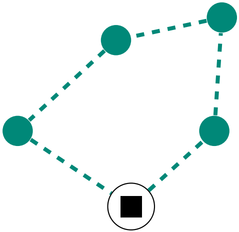

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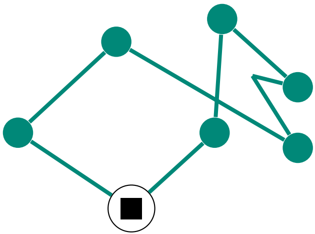

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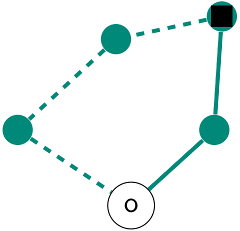

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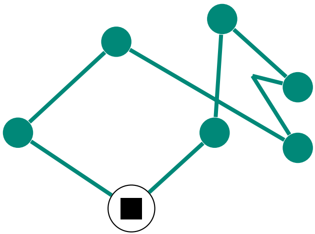

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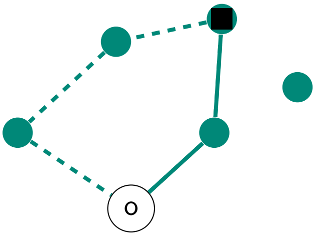

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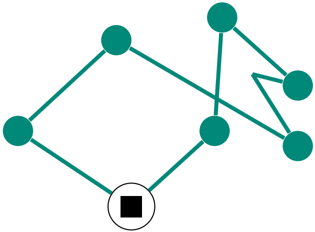

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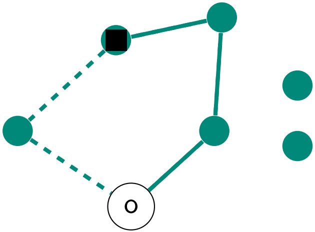

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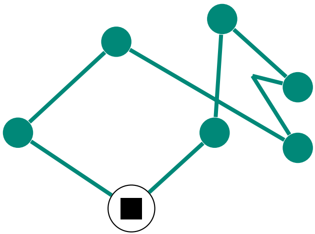

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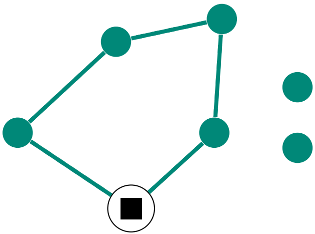

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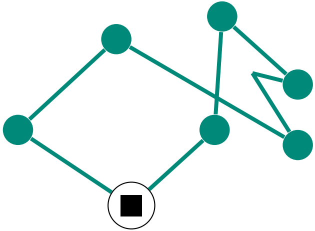

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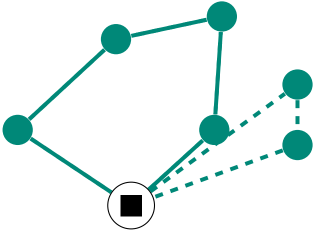

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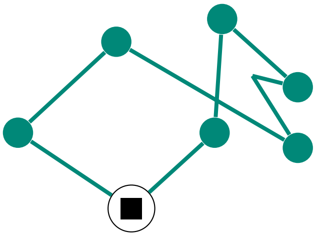

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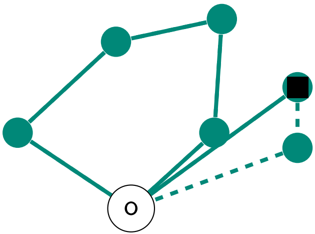

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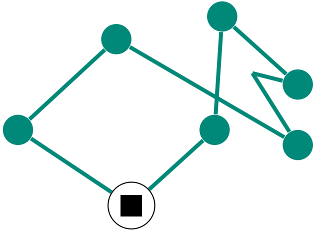

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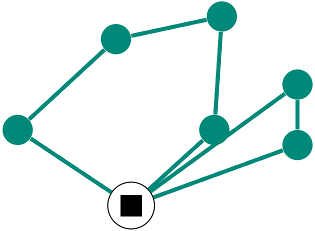

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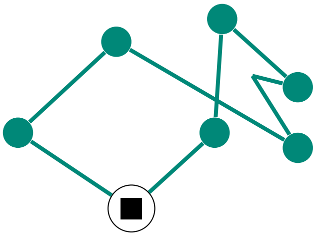

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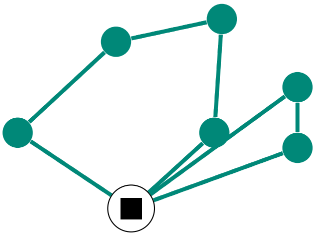

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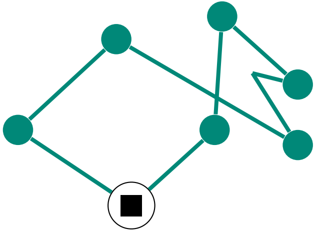

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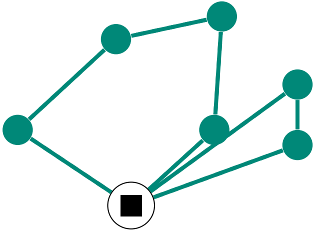

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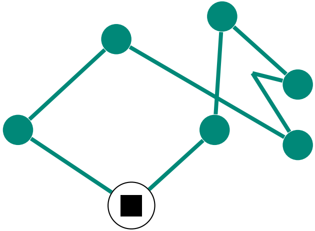

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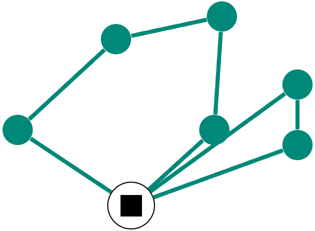

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Generalization: Online Dial-a-Ride (tour for transportation requests)

Graph Problems with Predictions and Roadmap

Other works: graph problems with predictions

- Network design [Azar, Panigrahi, Touitou, SODA 2022], [Moseley, Xu, AAAI 2022]
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Roadmap

1. Universal error measure for input predictions based on edge covers in suitably defined graphs
2. Algorithms with error-dependent competitive ratio
   - Online TSP (and Online Dial-a-Ride)
   - Online Steiner Tree (Online Facility Location, Steiner Forest)

Prediction Model

Prediction: request sequence (release dates and points/nodes)

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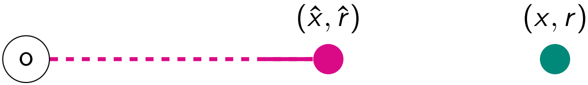

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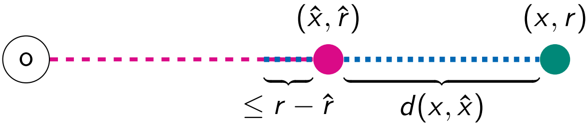

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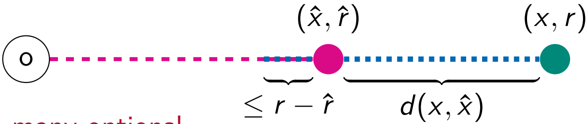
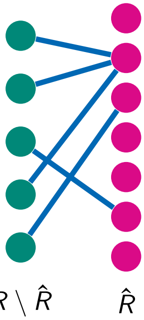


. . . might be many options!

Minimum-Cost Edge Cover

Idea: Model potential detours for serving unexpected requests as complete bipartite graph with edge cost $\gamma((x,r),(\hat{x},\hat{r})) = (r-\hat{r})_+ + d(x,\hat{x})$.



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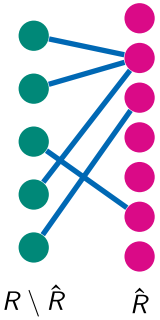

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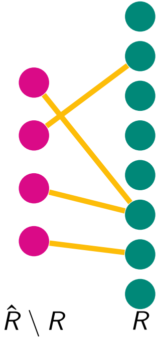

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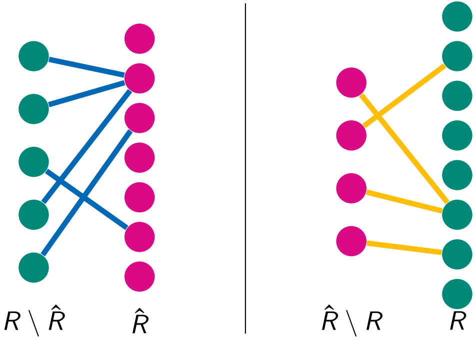

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Learning-Augmented Algorithms

Common approach: combine online & offline algorithms in a clever way

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Algorithm DelayedTrust

$\hat{T}$: optimal tour on prediction; $\hat{C}$: its length; $\alpha > 0$ confidence param.

(i) Follow blackbox online algorithm $\mathcal{A}$ as long as for time t holds $t \leq \alpha \cdot \hat{C} - d(p(t), 0)$. (**robustness**)

(ii) Move the server to the origin.

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Theorem

For every $\alpha > 0$ and ρ-competitive online algorithm $\mathcal{A}$, DelayedTrust has a competitive ratio of at most

$$\min\left\{(1+\alpha)\left(1+\frac{2\cdot\Lambda}{\mathrm{OPT}}\right), 1+\left(1+\frac{1}{\alpha}\right)\cdot\rho\right\}.$$

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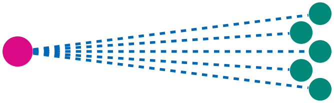

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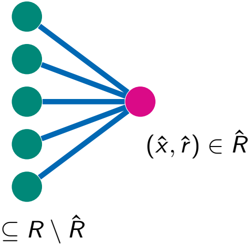

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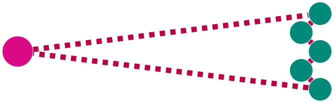

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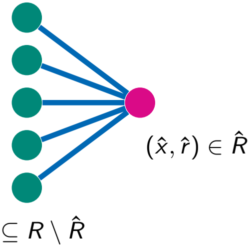

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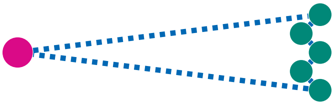

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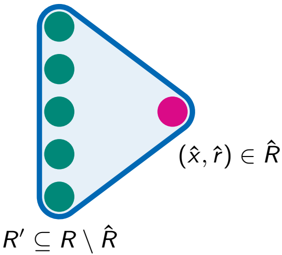

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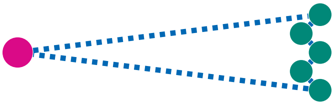

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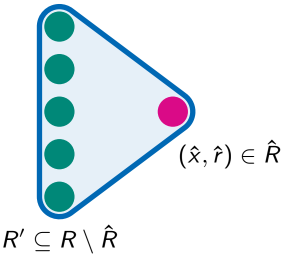

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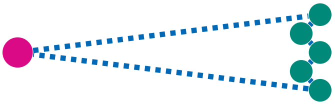

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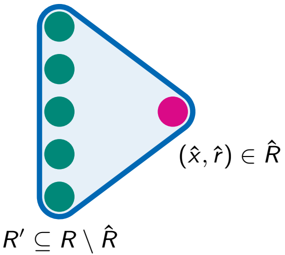

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Further Applications

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- Bounds for (new and old) learning-augmented algorithms
  - first framework for online TSP and Dial-a-Ride (online-time)
  - new bounds for known algorithm for network design (online-list)

[Azar, Panigrahi, Touitou SODA 2022]

Setting	Problem	Algorithm	Error-dependency
online-time	TSP	SmartTrust	$(1+\alpha)\cdot \mathrm{OPT} + 3 \cdot \Lambda_k$
	DARP	SmartTrust	$(1+\alpha)\cdot \mathrm{OPT} + 3 \cdot \Lambda_k$
online-list	Steiner Tree	APT	$\mathcal{O}(1)\cdot \mathrm{OPT} + \mathcal{O}(\log(k)) \cdot \Lambda_k$
	Steiner Forest	APT	$\mathcal{O}(1)\cdot \mathrm{OPT} + \mathcal{O}(k) \cdot \Lambda_k$
	Facility Location	APT	$\mathcal{O}(1)\cdot \mathrm{OPT} + \mathcal{O}(\log(k)) \cdot \Lambda_k$

Future & Discussion

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Thank you.