Tabu search for the time-dependent vehicle routing problem with time windows on a road network

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Online scheduling seminar
November 24, 2021

Acknowledgements

- Maha Gmira (CIRRELT and Polytechnique Montréal, now Euromed University of Fes, Morocco)
- Jean-Yves Potvin (CIRRELT and DIRO, U. de Montréal)
- Andrea Lodi (CIRRELT and Polytechnique Montréal, now Cornell University)

Outline

Introduction

Vehicle Routing Problems

- Introduced by Dantzig and Ramser in 1959
- One of the most studied problem in the area of logistics.
- The basic problem involves delivering given quantities of some product to a given set of customers using a fleet of vehicles with limited capacities.
- The objective is to determine a set of minimum-cost routes to satisfy customer demands.

Vehicle Routing Problems

Many variants involving different constraints or parameters:

- Introduction of travel and service times with route duration or time window constraints
- Multiple depots
- Multiple types of vehicles
- …

VRP with Time Windows

- Solomon (1987)
- Each customer must be visited within an interval [earliest service time, latest service time].
- Usually, vehicles may wait at customer locations if they arrive before their earliest service time.
- “Vehicle routing and scheduling”
- Probably, the VRP variant that has received the most attention.

VRP with Time Windows

- "Soft" vs. "hard" time windows
- Multiple time windows are sometimes allowed.
- Wide variety of solution methods:
  - Simple heuristics (e.g., insertion heuristics, local search)
  - Metaheuristics (Tabu search, ALNS, GAs, HGS, ...)
  - Exact methods
    - Column generation and Branch-and-Price
    - Branch-and-Price

Time-dependent VRPTW

Time-dependent VRP

- Many VRP applications are encountered in urban environments.
- Travel times between locations may vary wildly during a day due to traffic congestion.
- In practice, one may wish to exploit variations in travel times during the day to build more efficient routes to visit customers.

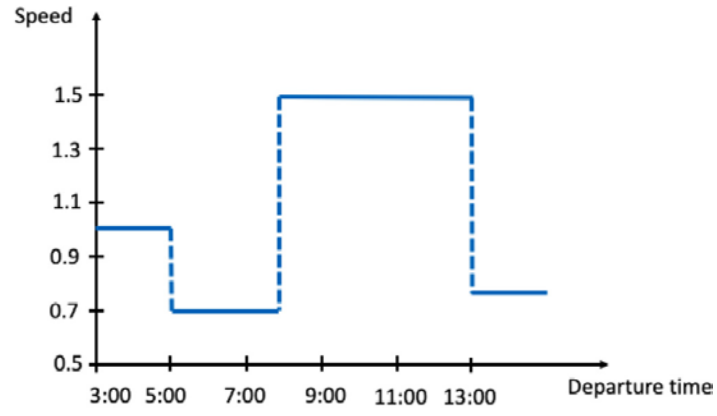
Time-dependent VRP

- Different approaches to model travel time variations
- Malandraki and Daskin (1992)
  - TDVRP without time windows
  - The planning horizon is divided into intervals over which travel times are constant.
  - Unfortunately, this model does not satisfy the FIFO (first in, first out) property
    → strange situations may occur (it is sometimes advantageous to delay departures from locations).

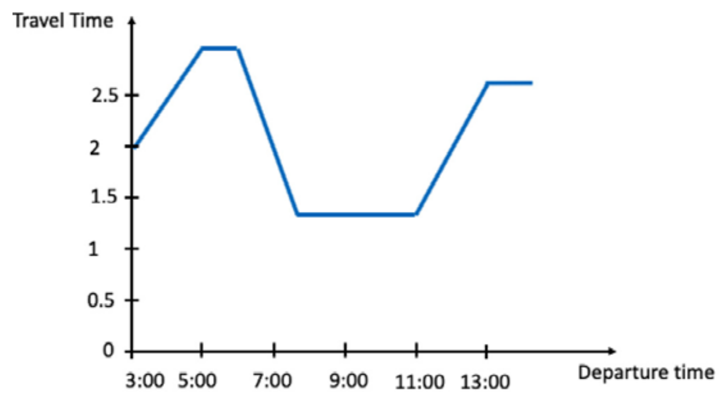
Time-dependent VRP

- Ichoua, Gendreau and Potvin (2003)
  - TDVRP with soft time windows
  - The planning horizon is divided into intervals over which travel speeds are constant.
  - This IGP model satisfies the FIFO property
    → one should always depart from a location as soon as possible.
  - Computational results show the importance of accounting for travel time (speed) variations.

The IGP model



(a) Speed function



(b) Travel time function

Time-dependent VRPTW

- The IGP model is the most often used in the TDVRP literature:
  - Donati et al. (2008): TDVRP with hard time windows
  - Figliozzi (2012): TDVRP with hard and soft TWs
  - Dabia et al. (2013): exact solution method
  - ... (see Gendreau, Ghiani, and Guerriero, 2015)
- Other notable contributions:
  - Haghani and Jung (2005) , Jung and Haghani (2001)
    → continuous travel time functions (dynamic TDVRPTW)
  - Fleischmann et al. (2004): general framework for TDVRPs

Time-dependent VRPTW on road networks

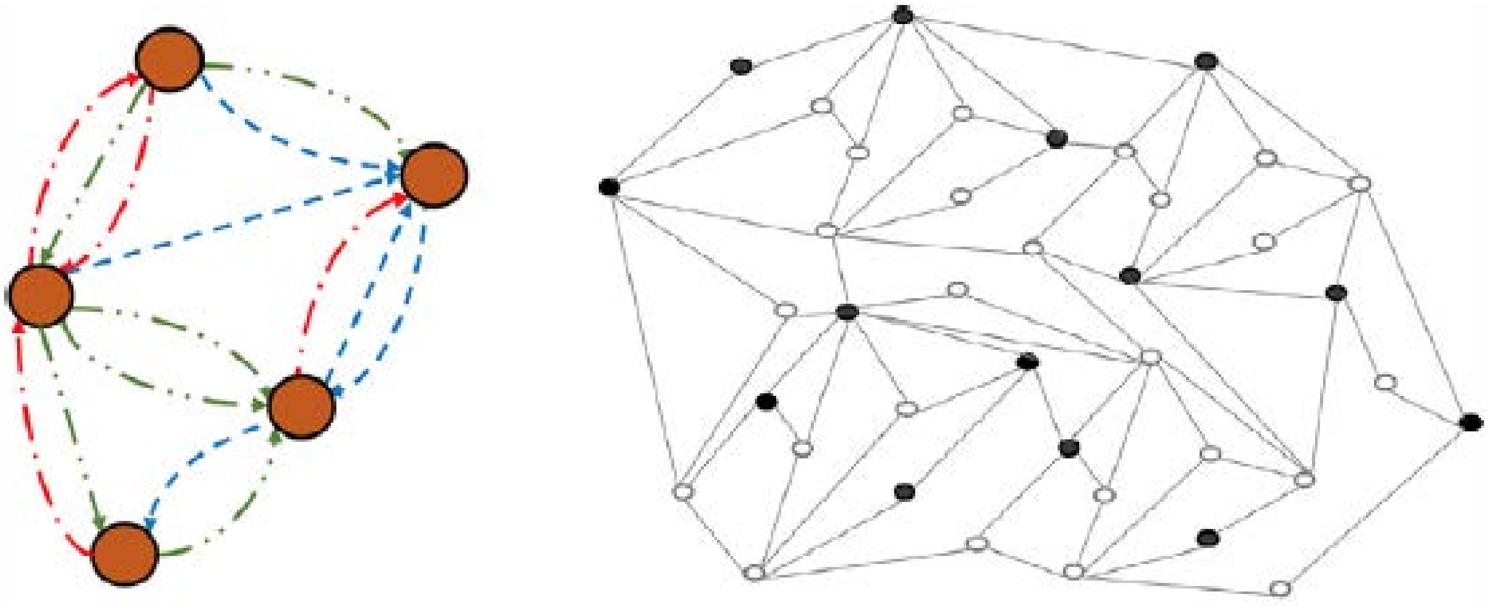
Time-dependency issues

- Most of the TDVRP literature deals with customer-to-customer graphs:
  - While travel times may evolve during the day, paths used remain the same!
  - In practice, in a time-dependent environment, one would usually see changes in the shortest paths used to travel from one location to another!
  - One should consider multiple paths between locations.

TDVRP with multiple paths

- Two options to consider multiple paths between locations:
  - Consider a multi-graph with a set of paths identified between any pair of customers (or the depot);
  - Work directly on the underlying road (street) network.

Road network representations



(a) Multigraph (b) Road network graph

Fig. 2. Road network representations.

TDVRP on customer-based multigraphs

- Garaix, Artigues, Feillet, and Josselin (2010)
  - Dial-a-ride application
- Wang and Lee (2014)
  - TDVRPTW
- Setak, Habibi, Karimi, and Abedzadeh (2015)
  - TDVRP with competing channels
- Lai, Caliskan Demirag, and Leung (2016)
  - Time-constrained heterogeneous VRP
- Ben Ticha, Absi, Feillet, and Quilliot (2017)
  - TDVRPTW

TDVRP on road networks

- Mancini (2014)
  - TDVRPTW on a real network with two categories of arcs: main roads with time-dependent travel speeds, and small streets with constant speeds
- Huang, Zhao, van Woensel, and Gross (2017)
  - TDVRP with path flexibility under stochastic and deterministic conditions
- Ben Ticha (2017) and Ben Ticha, Absi, Feillet, Quilliot, and van Woensel (2019)
  - Exact method (branch-and-price) for the TDVRPTW

Our objectives

Objectives

- Scientific:
  - Advance the state of knowledge on TDVRPTW and related problems (revisit the work done in Ichoua et al. 2003).
- Practical:
  - Optimize time-dependent delivery routes in an urban setting;
  - Produce more realistic delivery routes by accounting for the time of the day to identify the shortest path in a road network to get from one customer to the next;
  - Solve heuristically large routing problems;
  - Develop an efficient solution method to integrate it into real-time settings.

PROBLEM DESCRIPTION

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1. Time-dependent road network
2. Time-dependent shortest path problem
3. Time-dependent vehicle routing problem with time windows on a road network

PROBLEM DESCRIPTION
1. Time-dependent road network

The road network is a directed graph $G = (V, E)$, where:

- $V = \{0, 1, 2, ..., n\}$, set of vertices corresponding to road junctions
- E, set of arcs road segments between junctions
- d_{ij} distance associated with arc (i, j)
- v_{ij} time-dependent speed function
- c_{ij} the cost of traversing arc (i, j) at time t

Definition

A path p in the road network G from a source node $s \in V$ to a destination node $s' \in V$ is defined as a sequence of consecutive arcs $(i_1, i_2), (i_2, i_3), ..., (i_{j-1}, i_j)$ with $i_1 = s$ and $i_j = s'$. Alternatively, the path can be viewed as the sequence of nodes $s = i_1, i_2, i_3, ..., i_{j-1}, i_j = s'$.

Problem

The *TDSPP* consists in identifying a minimum-cost path p from a source node $i_1 = s$ to a destination node $i_j = s'$, given a departure time t_0. The cost of path p at time t_0, $cp_p(t_0)$, is defined recursively as follows:

PROBLEM DESCRIPTION
2. Time-dependent shortest path problem

$$cp_{i_1,i_2}(t_0) = c_{i_1,i_2}(t_0), \tag{1}$$

$$cp_{i_1,\ldots,i_j}(t_0) = cp_{i_1,i_2,\ldots i_{j-1}}(t_0) + c_{i_{j-1},i_j}(t_0 + tp_{i_1,\ldots,i_{j-1}}(t_0)), \tag{2}$$

where

$$tp_{i_1,i_2}(t_0) = tt_{i_1,i_2}(t_0), \tag{3}$$

$$tp_{i_1,\ldots,i_j}(t_0) = tp_{i_1,i_2,\ldots i_{j-1}}(t_0) + tt_{i_{j-1},i_j}(t_0 + tp_{i_1,\ldots,i_{j-1}}(t_0)). \tag{4}$$

Note that:

- $tp_p(t_0)$ is the travel time of path p at departure time t_0
- $tt_{ij}(t) = \frac{d_{ij}}{v_{ij}(t)}$ is the time-dependent travel time along arc (i, j) at time t

PROBLEM DESCRIPTION
3. $TDVRPTW_{RN}$

Set of customers $C \subset V$

- demand q_i
- time window for the start of service $tw_i = [a_i, b_i]$
- service or dwell time s_i

Set of vehicles K

- vehicle cannot arrive at customer i after b_i but can arrive before a_i,
- vehicle waits until time a_i to start the service
- vehicles are located at the depot
- capacity Q
- time window at the depot $[a_0, b_0]$ defines the beginning and end of the time horizon

Objective

- minimize total route duration (travel time + waiting time + service time)
- all customers must be served
- one route per vehicle that starts and ends at the depot and serve all customers at minimum cost
- constraints must be satisfied: capacity, time windows at customers and at depot

SOLVING APPROACH

SOLVING APPROACH

1. Time-dependent Dijkstra
2. Tabu search
    2.1 Initial solution
    2.2 Neighborhood structure
    2.3 Tabu list
    2.4 Aspiration criterion
    2.5 Diversification strategy

SOLVING APPROACH
1. Time-dependent Dijkstra

- A time-dependent variant of Dijkstra's algorithm is implemented to identify the minimum cost path between a source customer s and a target customer s' in the road network G at a given departure time t_0
- The arrival time at each non-permanently labeled successor is calculated using the speed function associated with that arc (see next slide)

SOLVING APPROACH
1. Time-dependent Dijkstra

Algorithm 1 Arrival time at a successor of a node

Input: road network $G = (V, A)$, arc (i, j), departure time t
Output: arrival time at j
$period \leftarrow 0$
while $(t \geq ub(period))$ **do** $period \leftarrow period + 1$ ▷ find time period of t
$d^- \leftarrow 0$
$d^+ \leftarrow (ub(period) - t) \times v_{ij}(t)$
while $d^+ \leq d_{ij}$ **do**
    $t \leftarrow ub(period)$
    $d^- \leftarrow d^+$
    $period \leftarrow period + 1$
    $d^+ \leftarrow d^- + (ub(period) - t) \times v_{ij}(t)$
$t \leftarrow t + (d_{ij} - d^-)/v_{ij}(t)$

SOLVING APPROACH
2. Tabu search

2.1. Initial solution

- greedy insertion heuristic with random selection of customer at each iteration
- feasibility of an insertion is assessed in constant time and its cost is evaluated exactly by propagating its impact along the route

2.2. Neighborhood structure

- the neighborhood structure is based on CROSS exchanges

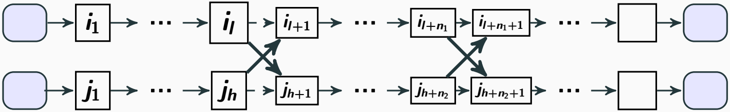

Figure 1: CROSS exchange

- the neighborhood is explored in a systematic way by considering all possible exchanges of sequences for every pair of routes in the current solution
- the feasibility of a neighborhood solution as well as its approximate cost are evaluated in constant time

2.3. Tabu list

- the inverse of a move is declared tabu for a number of iterations

2.4. Aspiration criterion

- revoke tabu status of an exchange if it leads to a neighborhood solution which is better than the best known solution

SOLVING APPROACH

2. Tabu search

2.5. Diversification strategy

- minimization of the total distance, for a certain number of iterations if stagnation is observed
- the shortest paths in the road network between two customers are based on distance while the time-dependent travel times are only taken into account to guarantee feasibility
- after a certain number of iterations with the distance objective, the original objective (duration) is reinstated until stagnation occurs again

CONSTANT TIME EVALUATION FRAMEWORK

CONSTANT TIME EVALUATION FRAMEWORK

1. Feasibility
   1.1 Dominant shortest-path structure
   1.2 Bounds on departure times
   1.3 CROSS exchange feasibility
2. Approximate cost

CONSTANT TIME EVALUATION FRAMEWORK

1. Feasibility

1.1. Dominant shortest-path structure

1. generate shortest paths for each pair i, j of customers by applying the time-dependent Dijkstra's algorithm with different departure times from i.
2. calculate a piecewise linear function associated with each path from i to j by combining the speed functions of all arcs that make the path.
3. gather the functions of all paths between customers i and j to create a so-called dominant shortest path structure for the pair i, j by considering the crosspoints among these paths and by selecting the best path between each pair of consecutive crosspoints.

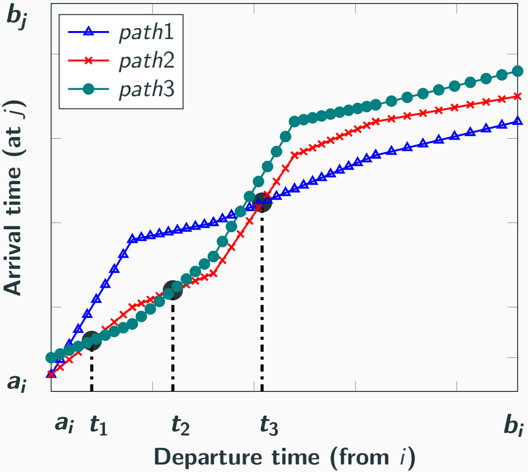

Figure 2: Piecewise linear arrival time functions for different paths between customers i and j

CONSTANT TIME EVALUATION FRAMEWORK

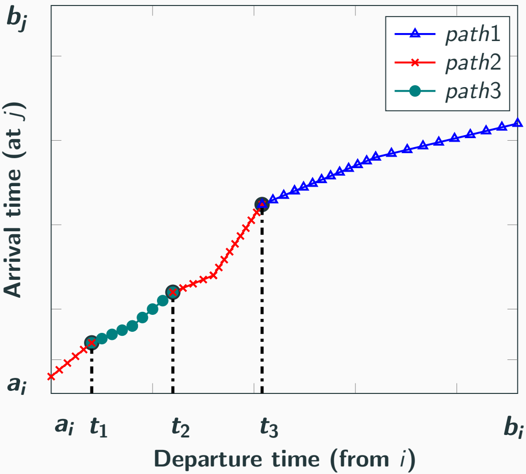

Figure 3: Dominant shortest path structure between i and j

1. Feasibility

1.2. Bounds on departure times

Algorithm 2 Latest departure times

Input: road network $G = (V, A)$; route $i_0, i_1, ...i_{nr}, i_{nr+1}$, end of time horizon $ld_{i_{nr+1}}$ at end depot i_{nr+1}
Output: bounds on departure times at each node
for $k = nr, nr - 1, ..., 0$ **do**
    calculate ld_{i_k} from $ld_{i_{k+1}}$ by moving backward along the path p used to go from i_k to i_{k+1} in current solution
$ld_{i_k} = min\{ld_{i_k}, b_{i_k}\}$
if best path from i_k to i_{k+1} at time ld_{i_k} is not p **then**
    find best path $p' \neq p$ and departure time ld_{i_k} to reach i_{k+1}, using dominant shortest path structure

CONSTANT TIME EVALUATION FRAMEWORK

1. Feasibility

1.2. Bounds on departure times

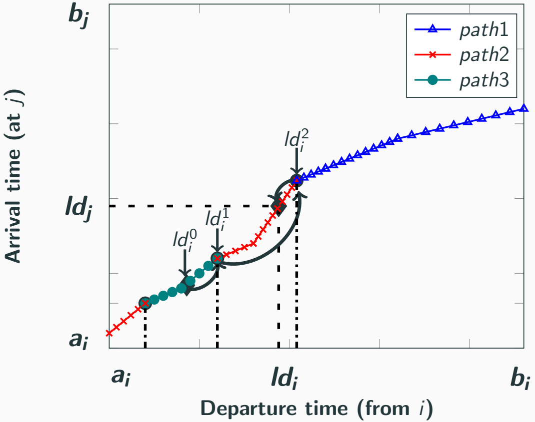

Figure 4: Latest departure time update

1. **Feasibility**

1.3. CROSS exchange feasibility

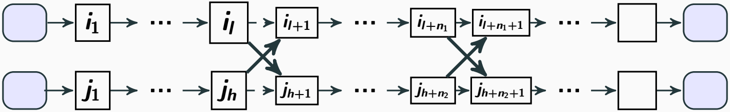

Figure 5: CROSS exchange

CONSTANT TIME EVALUATION FRAMEWORK

2. Approximate cost

1. penalty p_i: the delay incurred in the service start time of the next customer if the departure time at i is delayed by one time unit
2. once the impact of a CROSS exchange is propagated up to i_{l+n_1+1} (j_{h+n_2+1}), the additional cost of the two new routes in the neighborhood solution is approximated by multiplying the delay $\Delta_{i_{l+n_1+1}}$ ($\Delta_{j_{h+n_2+1}}$) by the penalty $p_{i_{l+n_1+1}}$ ($p_{j_{h+n_2+1}}$). That is, the additional cost associated with this neighborhood solution is $\Delta_{i_{l+n_1+1}} \times p_{i_{l+n_1+1}} + \Delta_{j_{h+n_2+1}} \times p_{j_{h+n_2+1}}$.
3. keep the n_{approx} best solutions in the neighborhood, based on this approximation.
4. each one of these n_{approx} solutions is evaluated exactly

COMPUTATIONAL EXPERIMENTS

COMPUTATIONAL EXPERIMENTS

1. Test instances
2. Comparison with optimal solutions
    2.1 Parameter tuning
    2.2 Results on Ben Ticha et al. instances
    2.3 Impact of diversification

1. Test instances

- Instances generated in [1] using a procedure previously reported in [2] for creating sparse graphs
- **3** graphs are available with $n = 50, 100$ and 200 nodes
- **3** different sets of random static travel times are associated with the set of arcs in each graph, based on different levels of correlation, namely, non-correlated (NC), weakly correlated (WC) and strongly correlated (SC)
- **5** time periods (time horizon)
- **21** different categories of test instances are obtained by considering a random selection of the depot node combined with a random selection of a given number of customers for each graph
- each category is duplicated by considering either narrow or wide time windows

2.1 Parameter tuning

An Exhaustive evaluation of 144 possible combinations of parameter settings. The following values were considered, where n_c denotes the number of customers:

- $tab = n_c/6,\ n_c/3,\ n_c/2,\ n_c$
- $it_{max} = 5n_c,\ 10n_c,\ 15n_c$
- $it_{cons} = n_c/2,\ n_c,\ 2n_c$
- $it_{div} = n_c/10,\ n_c/5,\ n_c/2,\ n_c$

2.2 Results on Ben Ticha et al. instances

$$Gap_O = \frac{Dist(TS) - Dist(Opt)}{Dist(Opt)} \times 100 \tag{5}$$

$$Impr = \frac{Dist(Init) - Dist(TS)}{Dist(Init)} \times 100 \tag{6}$$

2.2 Results on Ben Ticha et al. instances

Instance	# nodes	# arcs	# cust.	Corr.	*TS* Time(s)	*BP* Time(s)	Impr. (%)	$\mathrm{Gap_O}$ (%)
NEWLET	50	134	16	NC	0.640	95.6	3.020	0.761
				WC	0.967	1.7	1.712	0.751
				SC	0.296	1.0	1.846	0.781
			33	NC	2.813	3.5	5.610	0.822
				WC	2.945	14.4	3.860	0.816
				SC	2.694	8.9	2.823	0.850
	100	286	25	NC	2.719	1.1	6.713	0.874
				WC	1.935	1.4	2.528	0.922
				SC	1.429	1.5	5.727	0.930
			33	NC	2.488	1.0	1.704	0.968
				WC	2.982	2.0	1.492	0.964
				SC	3.264	49.6	2.019	0.942
			50	NC	5.251	4.0	2.898	0.992
				WC	5.464	4.0	1.826	0.982
				SC	7.151	201.3	1.961	0.962
	200	580	25	NC	2.393	4.0	3.071	0.948
				WC	2.731	4.0	2.986	0.883
				SC	2.406	3.9	1.833	0.991
			50	NC	17.931	13.0	1.547	0.929
				WC	27.182	7089.2	1.848	0.967
				SC	18.274	18.7	6.918	0.908

2.2 Results on Ben Ticha et al. instances

Instance	# nodes	# arcs	# cust.	Corr.	*TS* Time(s)	*BP* Time(s)	Impr. (%)	$\mathrm{Gap}_{\mathrm{O}}$ (%)
NEWLET	50	134	16	NC	1.561	15.0	6.772	0.751
				WC	1.526	2.8	7.556	0.716
				SC	1.463	1.7	7.083	0.743
			33	NC	4.769	7125.2	6.952	0.652
				WC	4.871	7200	6.693	–
				SC	2.163	7200	6.606	–
	100	286	25	NC	4.737	28.2	3.712	0.693
				WC	2.130	2.4	3.792	0.857
				SC	3.916	26.2	5.820	0.862
			33	NC	7.491	2229.9	6.681	0.882
				WC	5.766	5.1	4.570	0.901
				SC	5.865	24.4	4.548	0.909
			50	NC	13.051	6.1	3.774	0.945
				WC	17.366	56.7	2.769	0.845
				SC	11.774	1532.5	5.702	0.905
	200	580	25	NC	9.079	7199.2	4.693	0.822
				WC	8.481	96.6	6.914	0.966
				SC	9.316	27.4	5.628	0.901
			50	NC	63.811	7200	4.338	–
				WC	53.245	4364.3	2.511	0.974
				SC	43.602	7200	3.853	–

2.3 Impact of diversification

Table 1: *TS* optimal gaps with and without diversification - Narrow time windows

# nodes	# arcs	# cust.	Without diversification	With diversification
50	134	16	1.2560	0.7643
		33	1.3223	0.8293
100	286	25	2.0117	0.9088
		33	1.3730	0.9578
		50	1.1363	0.9784
200	580	25	1.2749	0.9406
		50	1.2009	0.9345

2.3 Impact of diversification

Table 2: *TS* optimal gaps with and without diversification - Wide time windows

# nodes	# arcs	# cust.	Without diversification	With diversification
50	134	16	1.0202	0.7366
		33	1.1461	0.6520
100	286	25	1.1030	0.8040
		33	1.2300	0.8972
		50	1.4346	0.8983
200	580	25	1.1398	0.8963
		50	1.3710	0.9740

Conclusion and perspectives

Contributions

- Innovative techniques to evaluate neighborhood solutions in constant time for the time-dependent vehicle routing problem with time windows on a road network.
- High-quality solutions in very reasonable computation times on benchmark instances recently reported in the literature.
- The approach can be integrated in a dynamic environment for managing routes in real-time.

Perspectives

- What about stochastic models to account for uncertainty in travel speed predictions?
- How could this approach be extended to account for revisions of travel speeds in real-time?
- How can Machine Learning and advanced OR methods can be used/combined in this context?