

# Enhancing project resilience: a risk-averse approach to payment delays

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Joint work with Öncü Hazır

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# Late payments: the silent killer

## European Payment Report 2023

The European Payment Report (EPR) highlights the impact late payments has on the development and growth among European businesses. The insights are based on a survey of 10,556 companies across 29 European countries, conducted between November 2022 and March 2023. You can download the full European report from [intrum.com/epr2023](https://intrum.com/epr2023)



⬇ Download the UK report

### Inflation and interest rates are creating challenges

Across Europe, growth is slowing down while supply-chain disruption and soaring energy costs drive inflation at a rate not seen for decades. Almost 6 in 10 companies are worried that the risk of late

### Strengthening cash flow and managing credit risk top the agenda

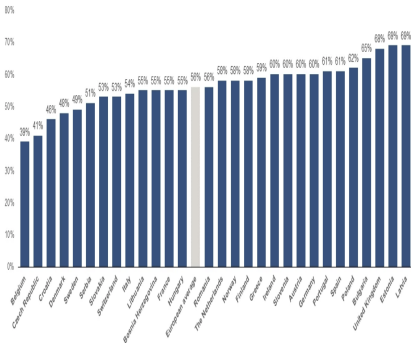
Liquidity, cash flow and credit risk management are the main strategic priorities for UK businesses as they seek to secure their financial positions.

### Late payments increasingly seen as a significant barrier to growth

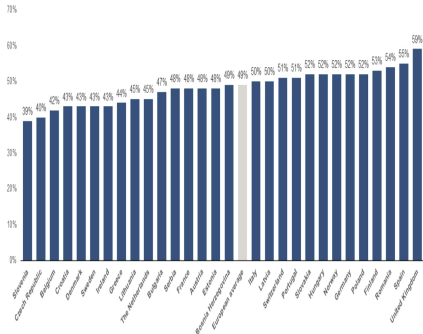
Late payments are hindering the growth of UK companies, hampering the economic and social development of the economy. More than 8 in 10 UK businesses have been asked to accept longer payment terms than

# Late payments: the silent killer

Due to inflation, we are finding it increasingly difficult to pay our suppliers on time (agree)



In my entire career, cash and financial debt management has never been more of a boardroom priority than it is today (agree)



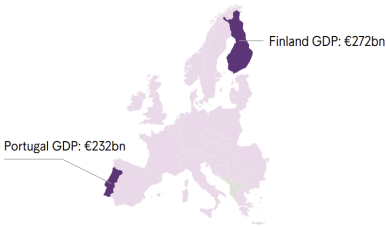
Source: Intrum's European Payment Report 2023

# Late payments: the silent killer

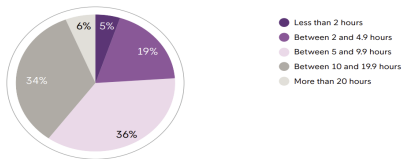
Chasing payments is costing Europe a quarter of a trillion euros a year

Total annual cost to the European economy:

# €275bn



To your best estimate, how many hours on average does your business spend each week chasing clients/customers for payment (such as sending reminders and making phone calls, etc)?



**10.4** = **74**  
hours in a week on European average = days on average in a year chasing late payments

Source: Intrum's European Payment Report 2023

# Late payments: the silent killer

## How do late payments impact EU businesses?

500 invoices are sent every second in the EU. However, only 200 of them will be paid on time. Here are some facts about late payments.



### More than 60% of EU businesses

are still not paid on time and SMEs are most affected.



### 1 in 4 bankruptcies

are due to invoices not being paid on time.



### €158 million in financing costs

for EU companies could be saved per day by reducing payment delays.



### 900,000 jobs

could be created in the public sector thanks to timely payments.



### 20% of businesses

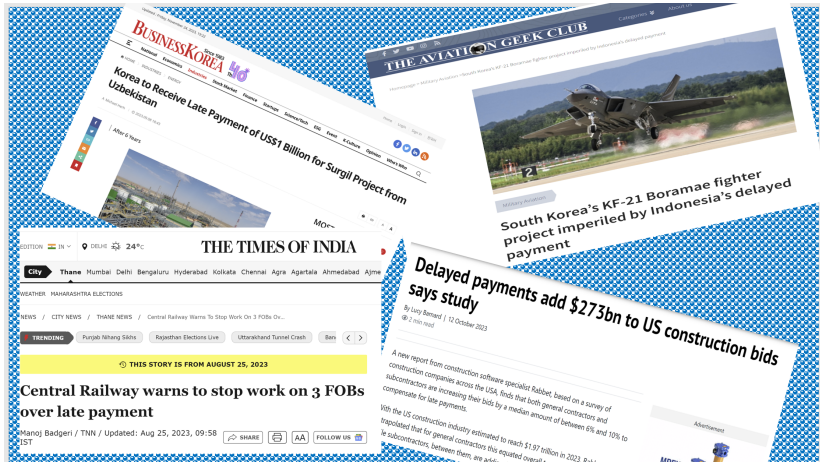
consider payment delays as a barrier to their green transition.

Source: European Commission

Source: <https://www.euronews.com/business/2023/09/05/>

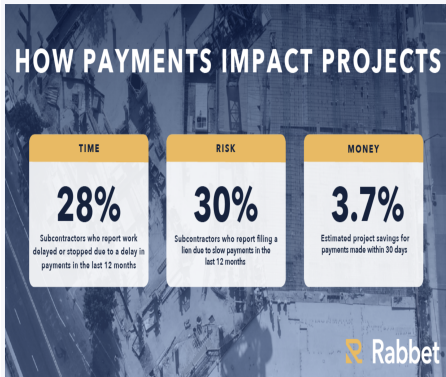


# Projects



# Construction sector

Construction companies depend on positive cash flows for their daily functioning



# Research motivation

How do firms compensate the lack of liquidity generated by (uncertain) payment delays?

## Features of the problem

- NPV with delayed payments
- Financing costs
- Uncertainty
- Risk aversion

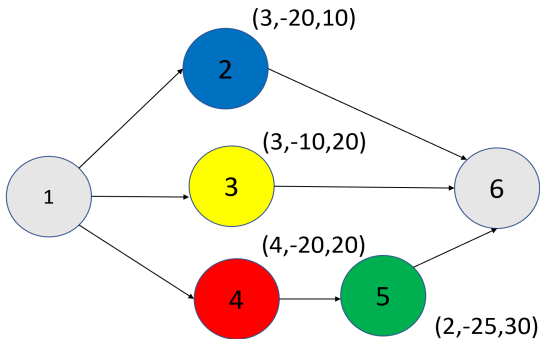


# Literature review

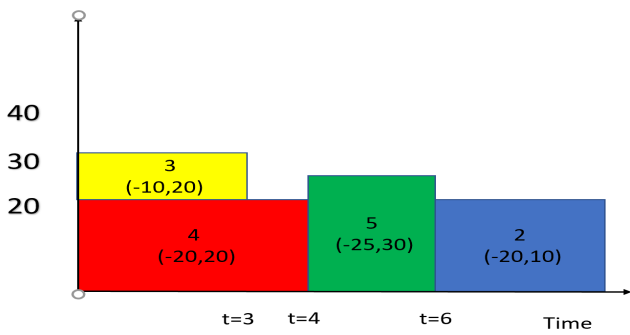
Year	Authors	Payments	Uncertainty	Risk	DR	Financing cost
2003	Vanhoucke et al.	Progress	X	X	X	X
2005	Mika et al.	Different modes	X	X	X	X
2005	Ke and Liu	X	Activity duration	Chance constraints	X	X
2010	Wiesemann et al.	X	Activity duration & Cash flows	X	X	X
2016	Leyman and Vanhoucke	At compl. times	X	X	X	X
2017	Leyman and Vanhoucke	At compl. times	X	X	X	X
2019	Liang et al.	X	Activity duration	X	X	X
2020	Rezaei et al.	X	Activity duration	CVaR	X	X
2021	Peymankar et al.	X	Cash Flows	X	X	X
	<b>This paper</b>	<b>At compl. times</b>	<b>Payments delays</b>	<b>CVaR</b>	<b>Yes</b>	<b>Yes</b>

Two-stage distributionally robust model with and without risk aversion for the NPV maximization with financing costs

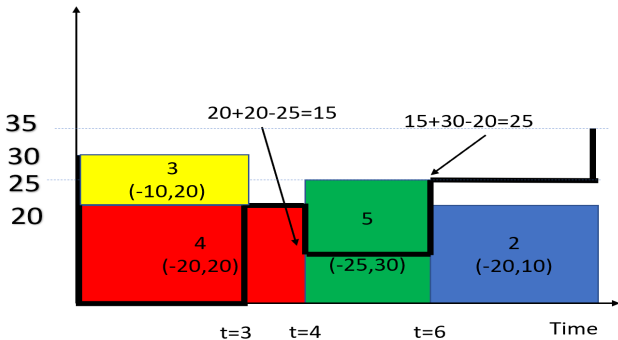
# Example



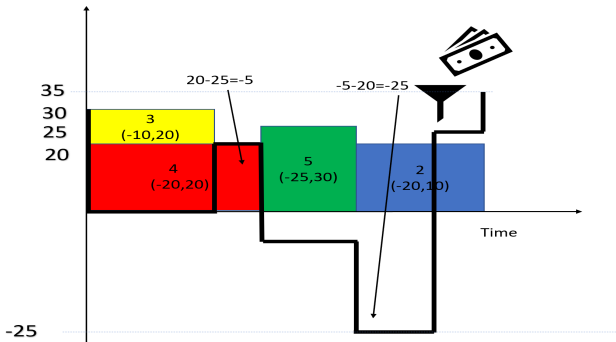
# Example



# Example

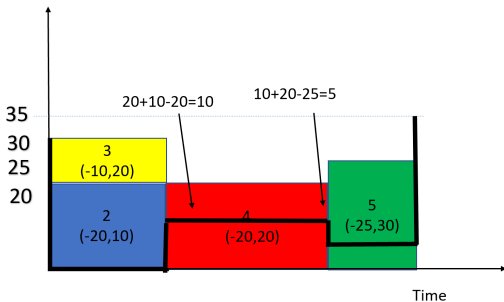


# Example



Delay in payment for activities 4 and 5

# Example



# Feature of the problem

## Activities

- Precedence relations

## Time related concerns

- Deadline

## Cost related concerns

- Discounted cash flow analysis (NPV)

NPV is calculated by taking a sum of all the income and expenses over the time period chosen, and discounting it back to the present



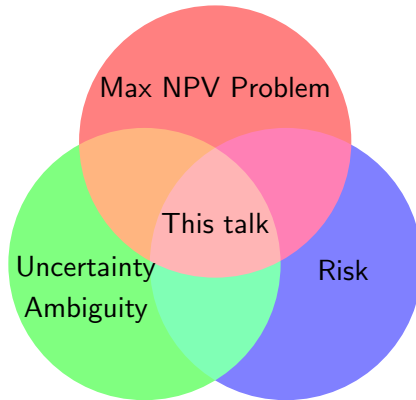
# Deterministic problem

## Input

- $V$ : set of all nodes
- $E$ : set of all arcs (i.e. immediate precedence relationships)
- $T$ : time horizon
- $\bar{d}$ : deadline for the completion of the project
- $\beta$ : discount rate
- $p_i$ : duration of activity  
 $i \in V$
- $c_i^{in}/c_i^{out}$ : cash inflow/outflow of activity  
 $i \in V$  ( $c_i^{in} > 0$ ), ( $c_i^{out} < 0$ )

$$\begin{aligned} \max \quad & \sum_{i \in V} \sum_{t \in T} \frac{c_i^{out}}{(1 + \beta)^t} x_{it} + \frac{c_i^{in}}{(1 + \beta)^t} q_{it} \\ \sum_{t \in T} x_{it} &= 1 \quad \forall i \in V \\ \sum_{t \in T} t x_{jt} &\geq \sum_{t \in T} t x_{it} + p_i \quad \forall (i, j) \in E \\ \sum_{t \in T} t q_{it} &= \sum_{t \in T} t x_{it} + p_i \quad \forall i \in V \\ \sum_{t \in T} q_{it} &= 1 \quad \forall i \in V \\ \sum_{t \in T} t x_{(n+1)t} + p_{n+1} &\leq \bar{d} \\ q_{it}, x_{it} &\in \{0, 1\} \quad \forall i \in V, t \in T \end{aligned}$$





# Setting the problem as a two-stage SIP

(Stage 1 decision) — Uncertainty — Stage 2 decision

Find a schedule

Goal: Maximize first stage objective + Expected value of the second stage decisions

Hedge against uncertainty = Enhance resilience

# Setting the problem as a two-stage SIP

(Stage 1 decision ) – **Uncertainty** — Stage 2 decision

Goal: Maximize first stage objective + Expected value of the second stage decisions

Hedge against uncertainty = Enhance resilience

# Setting the problem as a two-stage SIP

(Stage 1 decision ) –Uncertainty—Stage 2 decision

If, when, and how much to use short-term financing

Goal: Maximize first stage objective+ Expected value of the second stage decisions

Hedge against uncertainty= Enhance resilience

# General SIP formulation

$$\begin{aligned} \max f(x) &= c^T x + E[Q(x, \omega)] \\ \text{s.t.} \quad Ax &\geq b, x \in \mathcal{R}^{n_1-p_1} \times \mathcal{Z}^{p_1} \end{aligned}$$

$$\begin{aligned} Q(x, \omega) &= \max q(\omega)^T z \\ \text{s.t.} \quad Wz &\geq h(\omega) - T(\omega)x \\ z &\in \mathcal{R}^{n_2-p_2} \times \mathcal{Z}^{p_2} \end{aligned}$$

where

- $\omega$  random event
- $Q(x, \omega)$  is the second-stage objective function
- $T(\omega)$  is the recourse matrix
- $x$  are first-stage decisions
- $z$  are second-stage decisions

# Decision Variables

## First-stage

$$x_{it} = \begin{cases} 1 & \text{if activity } i \in V \text{ starts at time } t \in T; \\ 0 & \text{otherwise} \end{cases}$$

## Second-Stage

$$q_{it}^{\omega} = \begin{cases} 1 & \text{if payment for activity } i \text{ is received at time } t \\ & \text{under scenario } \omega \in \Omega; \\ 0 & \text{otherwise} \end{cases}$$

$l_t^{\omega} \geq 0$  short-term loan contracted at period  $t$  in scenario  $\omega$

# First-stage model

$$ENPV = \max \sum_{i \in V} \sum_{t \in T} \frac{c_i^{out}}{(1 + \beta)^t} x_{it} + E_{\xi}[Q(\mathbf{x}, \xi(\omega))]$$

$$\sum_{t \in T} x_{it} = 1 \quad \forall i \in V$$

$$\sum_{t \in T} tx_{jt} \geq \sum_{t \in T} tx_{it} + p_i \quad \forall (i, j) \in E$$

$$\sum_{t \in T} tx_{(n+1)t} + p_{n+1} \leq \bar{d}$$

$$x_{it} \in \{0, 1\} \quad \forall i \in V, t \in T$$

## Second stage constraints ( $\forall \omega \in \Omega$ )

$$\sum_{t \in T} tq_{it}^{\omega} = \sum_{t \in T} tx_{it} + p_i + \xi_i^{\omega} \quad \forall i \in V$$

$$\sum_{t \in T} q_{it}^{\omega} = 1 \quad \forall i \in V$$

$$C_0 + \sum_{i \in V} \sum_{t' \leq t} c_i^{\text{out}} x_{it'} + \sum_{i \in V} \sum_{t' \leq t} c_i^{\text{in}} q_{it'}^{\omega} + \sum_{t' \leq t} l_t^{\omega} \geq 0 \quad \forall t \in T$$

$$q_{it}^{\omega} \in \{0, 1\} \quad \forall i \in V, t \in T$$

$$l_t^{\omega} \geq 0 \quad \forall t \in T$$



# Recourse function

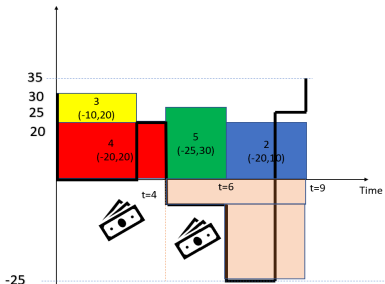
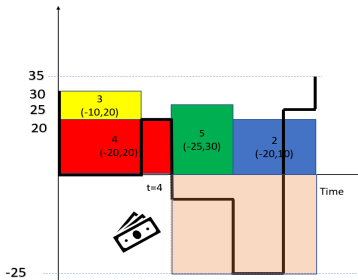
$$Q(\mathbf{x}, \xi(\omega)) =$$

$$= \max \sum_{i \in V} \sum_{t \in T} \frac{c_i^{in}}{(1 + \beta)^t} q_{it}^\omega + \sum_{t \in T} \frac{l_t^\omega}{(1 + \beta)^t} - \sum_{t \in T} \frac{l_t^\omega (1 + r)^{T-t}}{(1 + \beta)^T}$$

# Borrowing policy

$$\frac{25}{(1 + \beta)^4} - \frac{25(1 + r)^{9-4}}{(1 + \beta)^9}$$

$$\frac{5}{(1 + \beta)^4} + \frac{20}{(1 + \beta)^6} - \frac{5(1 + r)^{9-4}}{(1 + \beta)^9} - \frac{20(1 + r)^{9-6}}{(1 + \beta)^9}$$



If  $\beta > r$  then A is better, otherwise B

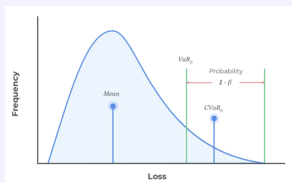
# Well known risk and safety measures

$$VaR(\tilde{z}) = \min_{\eta}(\eta | \mathbf{P}(\tilde{z} \leq \eta) = F_{\tilde{z}}(\eta) \geq \beta)$$

## Conditional Value at Risk

Quantifies the expected value of the random variable  $\tilde{z}$  in the worst  $1 - \beta\%$  of cases described as follows:

$$CVaR = E[\tilde{z} | \tilde{z} \geq VaR].$$



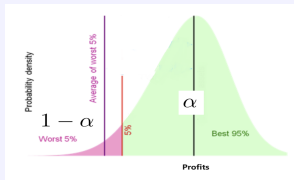
# Well known risk and safety measures

$$SVaR(\tilde{z}) = \max_{\eta} (\eta | \mathbf{P}(\tilde{z} \geq \eta) \geq \alpha)$$

## Conditional Value at Risk

Quantifies the expected value of the random variable  $\tilde{z}$  in the worst  $1 - \alpha\%$  of cases described as follows:

$$SCVaR = E[\tilde{z} | \tilde{z} \leq SVaR].$$



# Reformulation for discrete set of scenarios

## Conditional Value at Risk

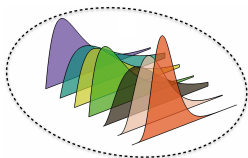
$$SCVaR[Q(\mathbf{x}, \xi(\omega))] = \max_{\eta \in \mathcal{R}^+} \eta - \frac{E_{\xi}[(\eta - Q(\mathbf{x}, \xi(\omega)))^+]}{1 - \alpha}$$

## Reformulation

$$\begin{aligned} \max_{\eta \in \mathcal{R}^+} \eta - \frac{\sum_{\omega} \mathcal{P}^{\omega} \gamma^{\omega}}{1 - \alpha} \\ \gamma^{\omega} = \eta - Q(\mathbf{x}, \xi(\omega)) & \quad \forall \omega \\ \gamma^{\omega} \geq 0 & \quad \forall \omega \end{aligned}$$

## Ambiguity in the probability distribution

Let  $\Xi$  be a family of probability distributions with given characteristics.



## Box Ambiguity Set

$$\Xi = \{\mathbf{p} = \mathbf{p}_0 + \boldsymbol{\pi} \mid \mathbf{e}^T \boldsymbol{\pi} = 0, \|\boldsymbol{\pi}\|_1 \leq \Psi\}$$

where  $\boldsymbol{\pi}$  is a perturbation vector and  $\Psi \in [0, 1]$  is the upper bound of the fluctuation.

$$\inf_{\xi \in \Xi} \max_{\eta \in \mathcal{R}^+} \eta - \frac{E_{\xi}[(\eta - Q(\mathbf{x}, \xi(\omega)))^+]}{1 - \alpha}$$

$$\inf_{\xi \in \Xi} \max_{\eta \in \mathcal{R}^+} \eta - \frac{E_{\xi}[(\eta - Q(\mathbf{x}, \xi(\omega)))^+]}{1 - \alpha}$$

$$\max_{\eta \in \mathcal{R}^+} \eta - \frac{1}{1 - \alpha} \min_{\xi \in \Xi} E_{\xi}[(\eta - Q(\mathbf{x}, \xi(\omega)))^+]$$



# Toward a deterministic reformulation

$$\max_{\eta \in \mathcal{R}^+} \eta - \frac{1}{1 - \alpha} \min_{\xi \in \Xi} E_{\xi}[(\eta - Q(\mathbf{x}, \xi(\omega)))^+]$$

$$\min \gamma^T \mathbf{p} = \sum_{\omega} p^{\omega} \gamma^{\omega}$$

$$\gamma^{\omega} = \eta - Q(\mathbf{x}, \xi(\omega)) \quad \forall \omega$$

$$\gamma^{\omega} \geq 0 \quad \forall \omega$$

$$\min_{\pi} \gamma^T \mathbf{p}$$

$$\gamma^{\omega} = \eta - Q(\mathbf{x}, \xi(\omega)) \quad \forall \omega$$

$$\gamma^{\omega} \geq 0 \quad \forall \omega$$

$$\mathbf{p} = \mathbf{p}_0 + \pi$$

$$\mathbf{e}^T \pi = 0$$

$$\|\pi\|_1 \leq \Psi$$

# Toward a deterministic reformulation

$$\min_{\pi} \gamma^T \mathbf{p}$$

$$\gamma^{\omega} = \eta - Q(\mathbf{x}, \xi(\omega)) \quad \forall \omega$$

$$\gamma^{\omega} \geq 0 \quad \forall \omega$$

$$\mathbf{p} = \mathbf{p}_0 + \pi$$

$$\mathbf{e}^T \pi = 0$$

$$\|\pi\|_1 \leq \Psi$$

$$\gamma^T \mathbf{p}_0 + \min_{\pi} \gamma^T \pi$$

$$\gamma^{\omega} = \eta - Q(\mathbf{x}, \xi(\omega)) \quad \forall \omega$$

$$\gamma^{\omega} \geq 0 \quad \forall \omega$$

$$\mathbf{e}^T \pi = 0$$

$$-\pi \leq \Psi$$

$$\pi \leq \Psi$$

$$\Psi = \Psi \mathbf{e}$$

# Toward a deterministic reformulation

$$\gamma^T \mathbf{p}_0 + \min_{\pi} \gamma^T \pi$$

$$\mathbf{e}^T \pi = 0$$

$$-\pi \leq \Psi$$

$$\pi \leq \Psi$$

# Toward a deterministic reformulation

$$\gamma^T \mathbf{p}_0 + \min_{\pi} \gamma^T \pi$$

$$\mathbf{e}^T \pi = 0 \quad \mu$$

$$-\pi \leq \Psi \quad \beta$$

$$\pi \leq \Psi \quad \delta$$

# Toward a deterministic reformulation

$$\gamma^T \mathbf{p}_0 + \min_{\pi} \gamma^T \pi$$

$$\mathbf{e}^T \pi = 0 \quad \mu$$

$$-\pi \leq \Psi \quad \beta$$

$$\pi \leq \Psi \quad \delta$$

$$\gamma^T \mathbf{p}_0 + \max_{\beta, \delta, \mu} \Psi^T \beta + \Psi^T \delta$$

$$\mathbf{e}\mu - \beta + \delta = \gamma$$

$$\beta \geq 0$$

$$\delta \geq 0$$

$$\mu \text{ free}$$

# The risk-averse distributional robust model

$$\max \sum_{i \in V} \sum_{t \in T} \frac{c_i^{out}}{(1 + \beta)^t} x_{it} + \eta - \frac{\gamma^T \mathbf{p}_0 + \boldsymbol{\Psi}^T \beta + \boldsymbol{\Psi}^T \delta}{1 - \alpha}$$

First and second stage constraints

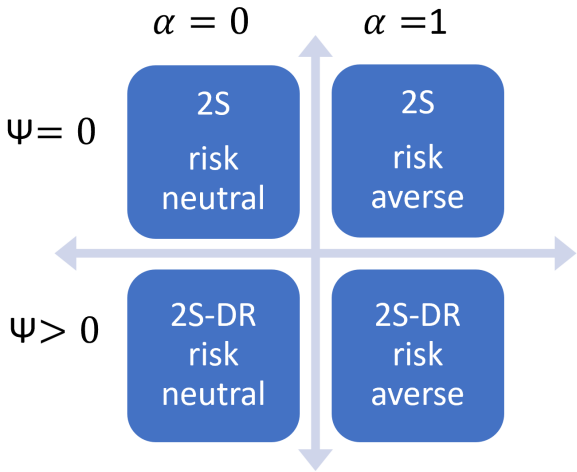
$$\mathbf{e}\mu - \beta + \delta = \gamma$$

$$\beta \geq 0$$

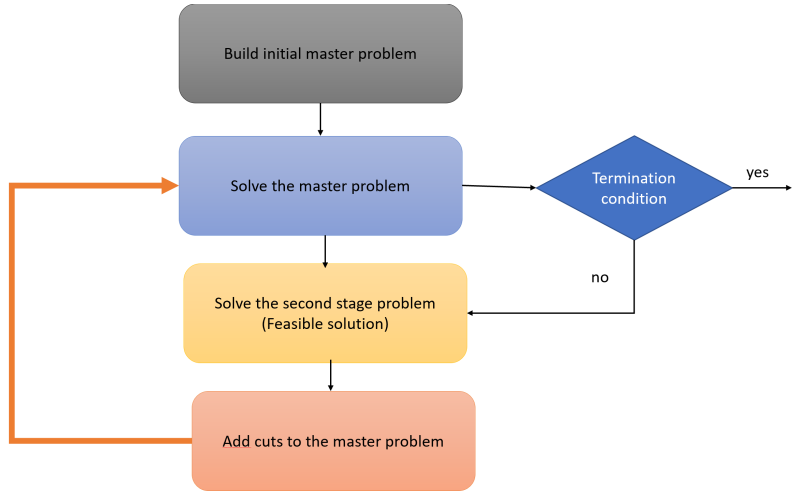
$$\delta \geq 0$$

$$\gamma \geq 0$$

$$\mu \text{ free}$$



# Heuristic





# Master problem

$$MP^0 = \max \sum_{i \in V} \sum_{t \in T} \frac{c_i^{out}}{(1 + \beta)^t} x_{it} + \zeta$$

$$\zeta \leq \sum_{i \in V} \sum_{t \in T} \frac{c_i^{in}}{(1 + \beta)^t} \hat{q}_{it} + \sum_{t \in T} \frac{\hat{t}_t}{(1 + \beta)^t} - \sum_{t \in T} \frac{\hat{t}_t(1 + r)^{T-t}}{(1 + \beta)^T}$$

$$\sum_{t \in T} x_{it} = 1 \quad \forall i \in V$$

$$\sum_{t \in T} tx_{jt} \geq \sum_{t \in T} tx_{it} + p_i \quad \forall (i, j) \in E$$

$$\sum_{t \in T} tx_{(n+1)t} + p_{n+1} \leq \bar{d}$$

$$\sum_{t \in T} t\hat{q}_{it} = \sum_{t \in T} tx_{it} + p_i + \hat{\xi}_i \quad \forall i \in V$$

$$\sum_{t \in T} \hat{q}_{it} = 1 \quad \forall i \in V$$

$$C_0 + \sum_{i \in V} \sum_{t' \leq t} c_i^{out} x_{it'} + \sum_{i \in V} \sum_{t' \leq t} c_i^{in} \hat{q}_{it'} + \sum_{t' \leq t} \hat{t}_{t'} \geq 0 \quad \forall t \in T$$

$$x_{it} \in \{0, 1\} \quad \forall i \in V, t \in T$$

$$\hat{q}_{it} \in \{0, 1\} \quad \forall i \in V, t \in T$$

$$\hat{t}_t \geq 0 \quad \forall t \in T$$

# Subproblem

$$q_{it}^{k\omega} = \begin{cases} 1, & \text{if } \sum_{t' \in T} (t' x_{it'}^k + p_i + \xi_i^{k\omega}) = t \\ 0, & \text{otherwise.} \end{cases}$$

# Subproblem

$$Q_{LB}^{\kappa} = \max_{\eta} \eta - \frac{\sum_{\omega \in \Omega} [\gamma^{\omega} p_0^{\omega} + \Psi^{\omega} \beta^{\omega} + \Psi^{\omega} \delta^{\omega}]}{1 - \alpha}$$

$$\sum_{t' \leq t} I_t^{\omega} \geq \sum_{i \in V} \sum_{t' \leq t} c_i^{\text{out}} x_{it'}^{\kappa} + \sum_{i \in V} \sum_{t' \leq t} c_i^{\text{in}} q_{it'}^{\kappa \omega} \quad \forall t \in T, \forall \omega \in \Omega$$

$$\gamma^{\omega} = \eta - \sum_{t \in T} \frac{c_i^{\text{in}}}{(1 + \beta)^t} q_{it}^{\kappa \omega} + \sum_{t \in T} \frac{I_t^{\omega}}{(1 + \beta)^t} - \sum_{t \in T} \frac{I_t^{\omega} (1 + r)^{T-t}}{(1 + \beta)^T} \quad \forall \omega \in \Omega$$

$$\mu - \beta^{\omega} + \delta^{\omega} = \gamma^{\omega} \quad \forall \omega \in \Omega$$

$$\beta^{\omega} \geq 0 \quad \forall \omega \in \Omega$$

$$\delta^{\omega} \geq 0 \quad \forall \omega \in \Omega$$

$$\gamma^{\omega} \geq 0 \quad \forall \omega \in \Omega$$

$$I_t^{\omega} \geq 0 \quad \forall t \in T, \forall \omega \in \Omega$$

$$\mu \quad \text{free}$$

# Cuts

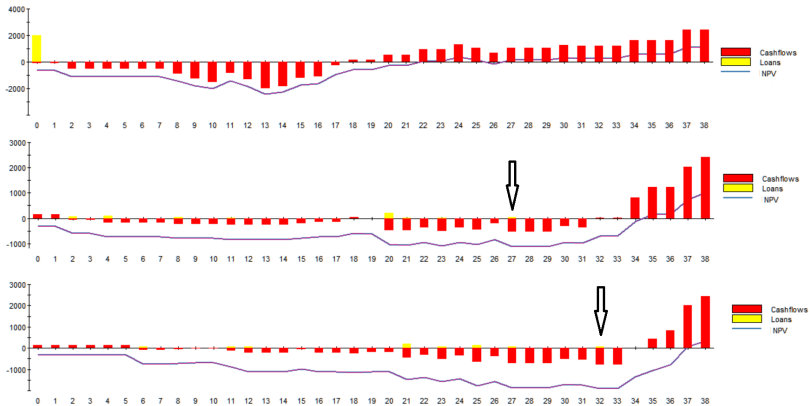
## Integer cut

$$\zeta \leq (Q_{LB}^k)[1 + \sum_{i \in V, t \in T | x_{it}^k = 1} (1 - x_{it}) + \sum_{i \in V, t \in T | x_{it}^k = 0} x_{it}]$$

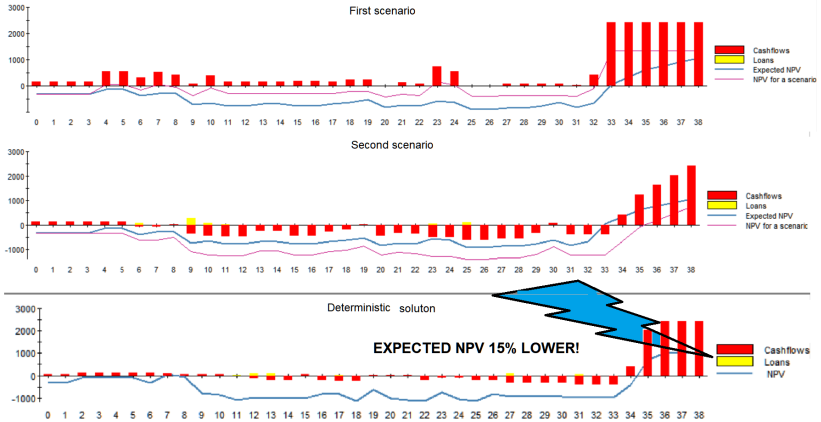
## No-good cut

$$\sum_{(i, t) | x_{it}^k = 1} (1 - x_{it}) \geq 1$$

# Sensitivity to the loan interest rate



# Two-stage stochastic model



# Sensitivity to $\psi$ and $\alpha$ values

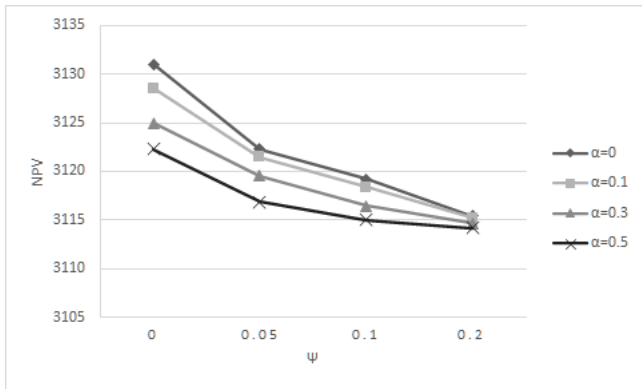


Figure: NPV function for different  $\psi$  and  $\alpha$  values.

# Price of distributional robustness

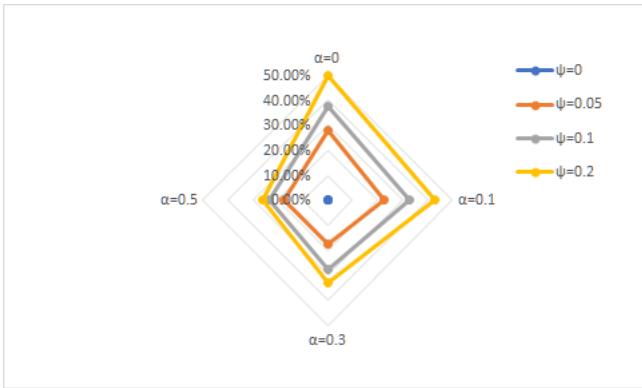


Figure: Price of distributional robustness for different values of  $\psi$  and  $\alpha$ .



# Heuristic performance

- 10 instances from the DC2 data set of Vanhoucke (2010)
- 9 combinations of the distribution of the cash flow
- Three values (0.25, 0.50, 0.75) for the capital constrainedness (CC)
- Three scenarios cardinalities  $|\Omega| = \{20, 40, 60\}$
- Four values for  $\psi = 0, 0.05, 0.1, 0.2$
- Four values for  $\alpha = 0, 0.1, 0.3, 0.5$

# Heuristic performance

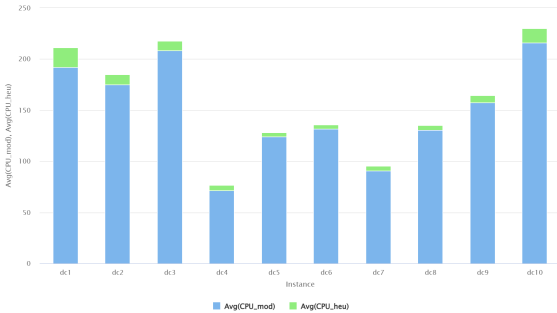


Figure: Heuristic versus exact solution: CPU time.



Figure: Heuristic Gap% for different scenario cardinalities.

## Limitations

- The size of the model

## Future research

- Design of a tailored solution approach
- Different payments modes
- Hybridization with forecasting techniques