Scheduling Heuristics for Steelmaking Continuous Casting Processes

Kangbok Lee

POSTECH, South Korea

Juntaek Hong, Kyungduk Moon, Kwansoo Lee (POSTECH), Michael L. Pinedo (NYU)

Scheduling Seminar

Feb. 16, 2022

Summary

- We consider a practical steelmaking-continuous casting scheduling problem.

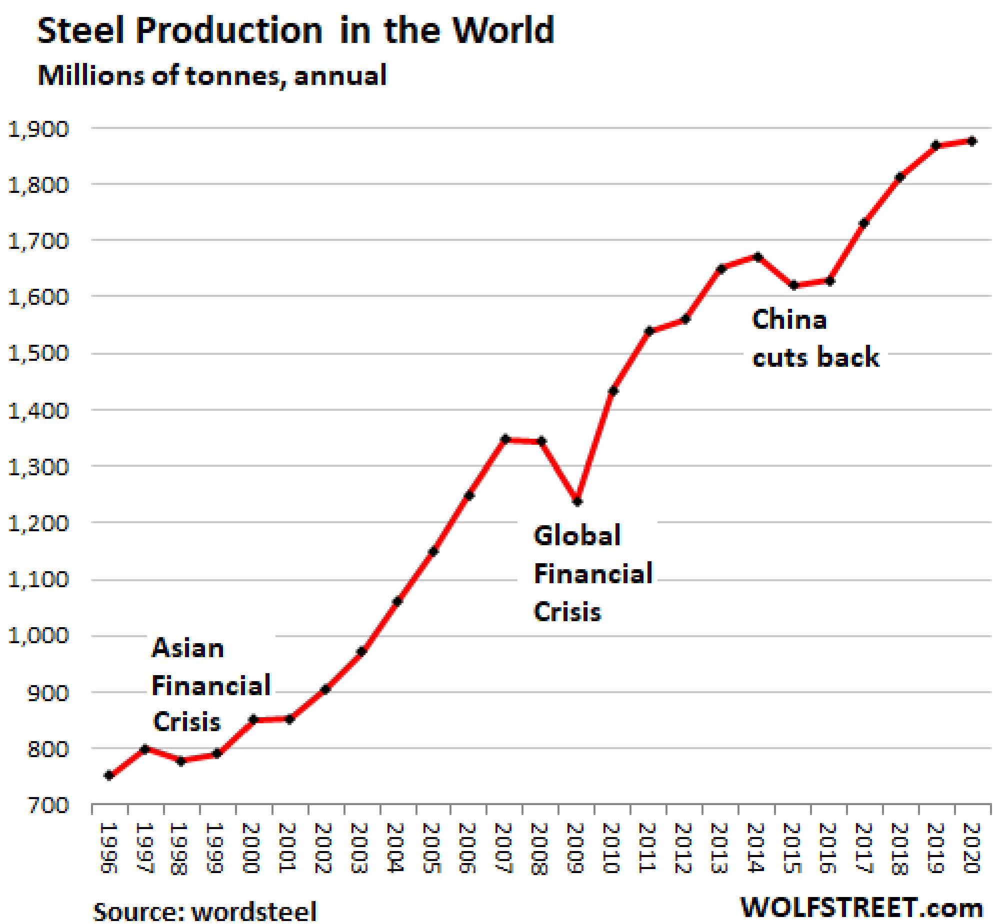
- We propose an **iterated greedy matheuristic(IGM)**, an intuitive method to solve the problem.

- IGM performs well.

Table of Contents

Introduction

World crude steel production (in million metric tons)



Introduction

Pressure on Steelmaking Industry against Facility Expansion

REUTERS

July 13, 2019:

China plans to toughen emission checks on steel mills

BEIJING (Reuters) - China will continue to enforce production restrictions in heavy industry in winter this year and will tighten its emission assessment on steel mills when granting exemptions from curbs already in place, an environment ministry official said

Introduction

Pressure on Steelmaking Industry against Facility Expansion

Bloomberg Green

Subscribe

March 12, 2021:

China Pollution Crackdown Exposes Rule Breakers in Top Steel Hub

China's top environmental official vowed to reinforce pollution curbs after inspections found some steel mills were violating output restrictions and faking documents.

A team led by Huang Runqiu, the minister of ecology and environment, on Thursday found four mills in the steelmaking hub of Tangshan weren't complying with production cuts put in place to reduce heavy pollution.

Introduction

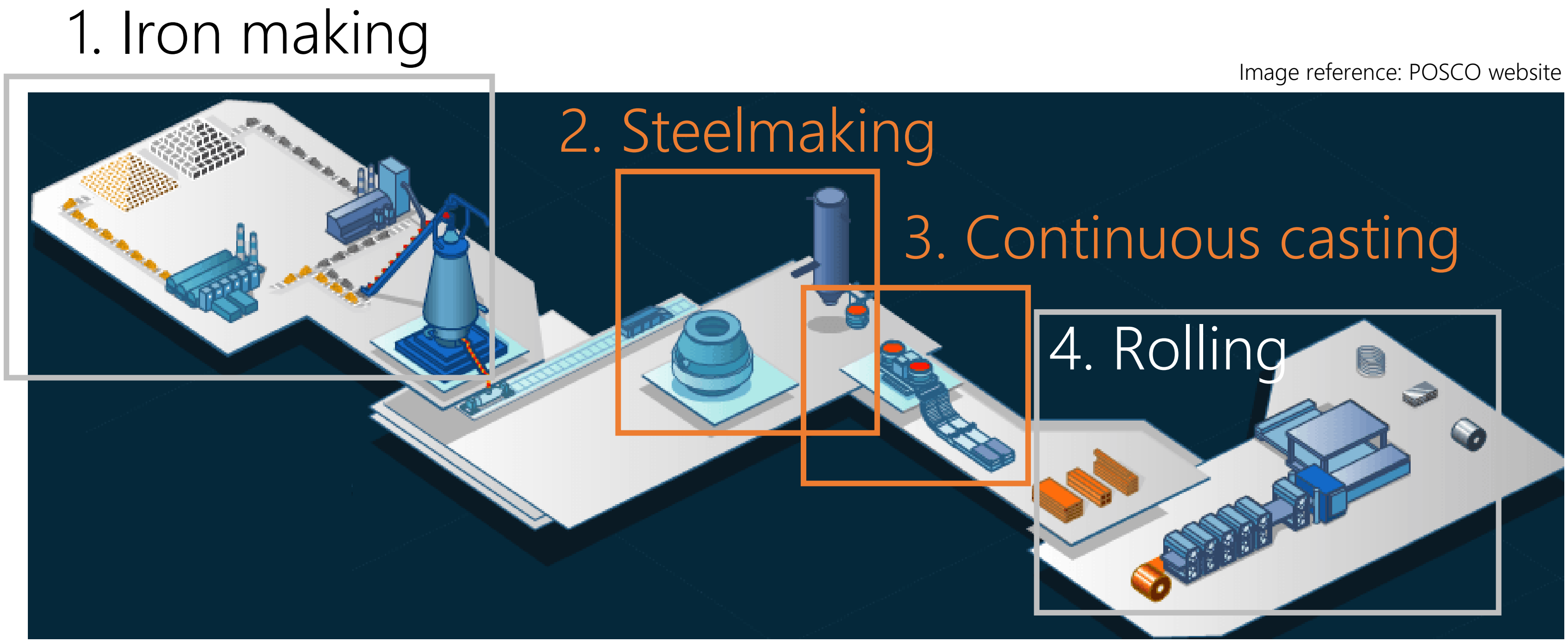
Importance of steel scheduling

- Expansion of conventional facility is limited
- New technology for steel industry is currently inviable.

➢ **Efficient operation of existing facilities is still crucial.**

Introduction

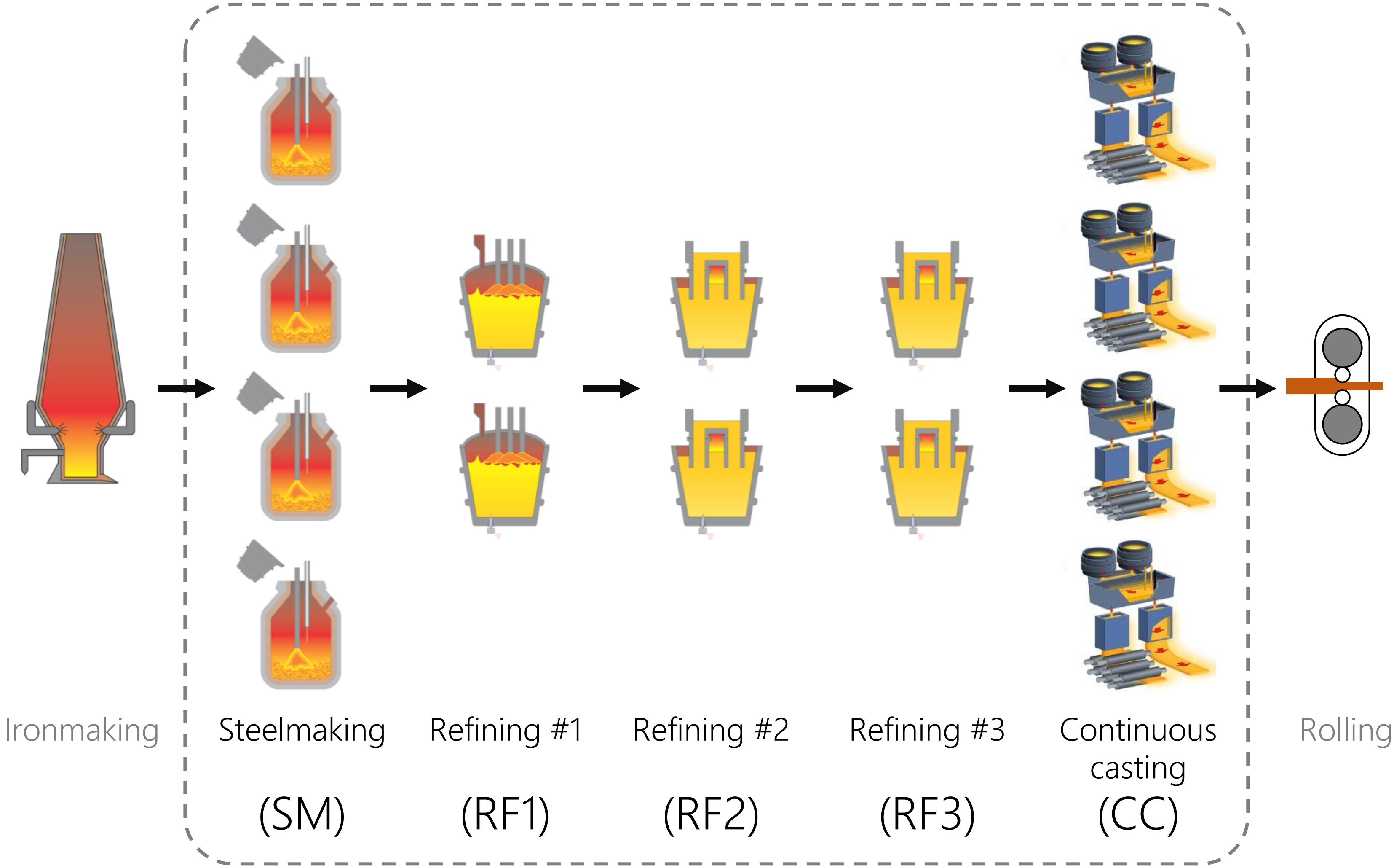
Steel Production



Steelmaking–Continuous Casting (SCC) process is typically the bottleneck

Problem Description

SCC Process



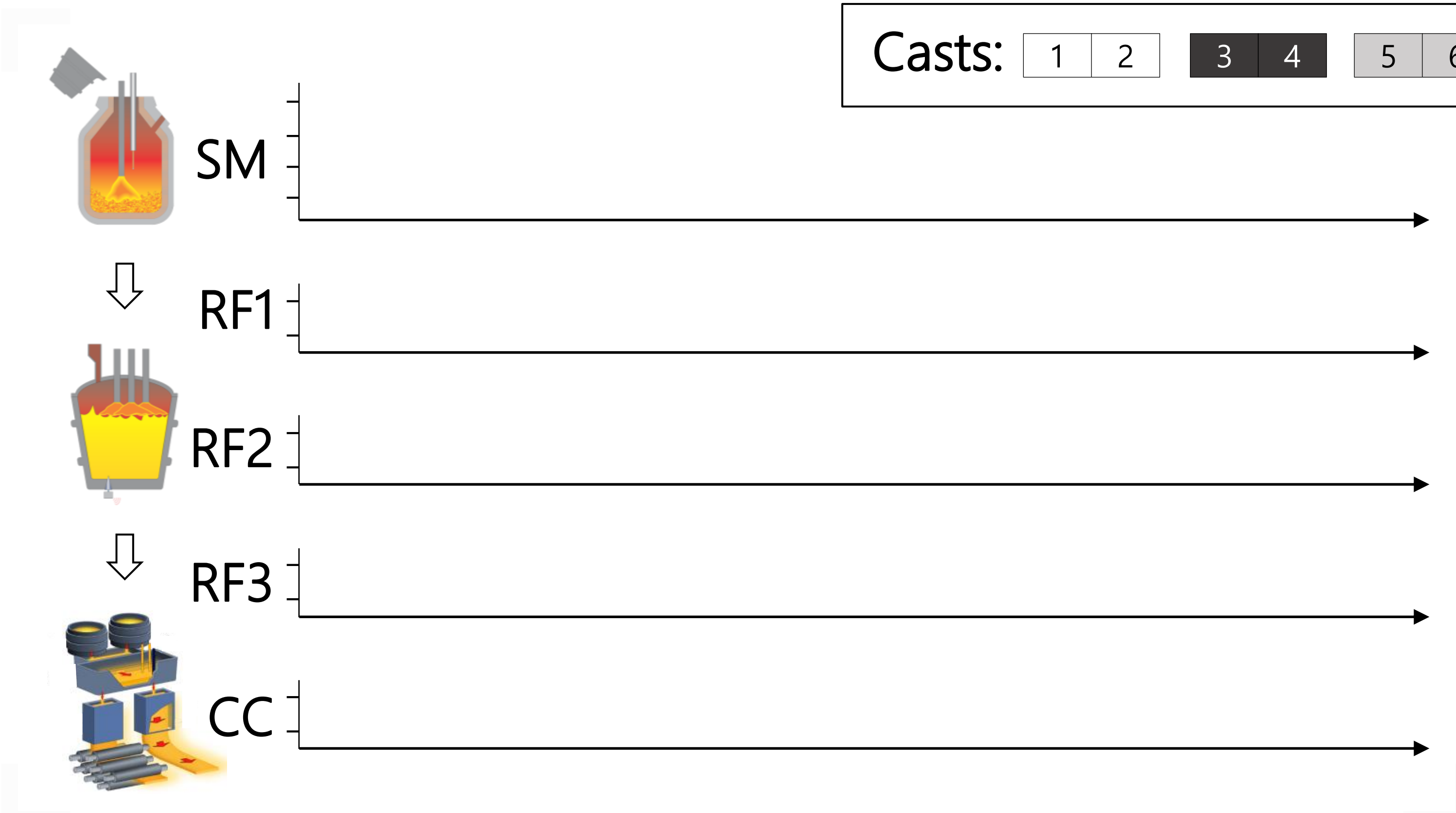
Problem Description

SCC Process schedule example

Required stages

Charge	SM	RF1	RF2	RF3	CC
1	: SM →				CC
2	: SM →	RF1 →			CC
3	: SM →	RF1 →		RF3 →	CC
4	: SM →				CC
5	: SM →				CC
6	: SM →		RF2 →	RF3 →	CC

Casts: [1 | 2] [3 | 4] [5 | 6]



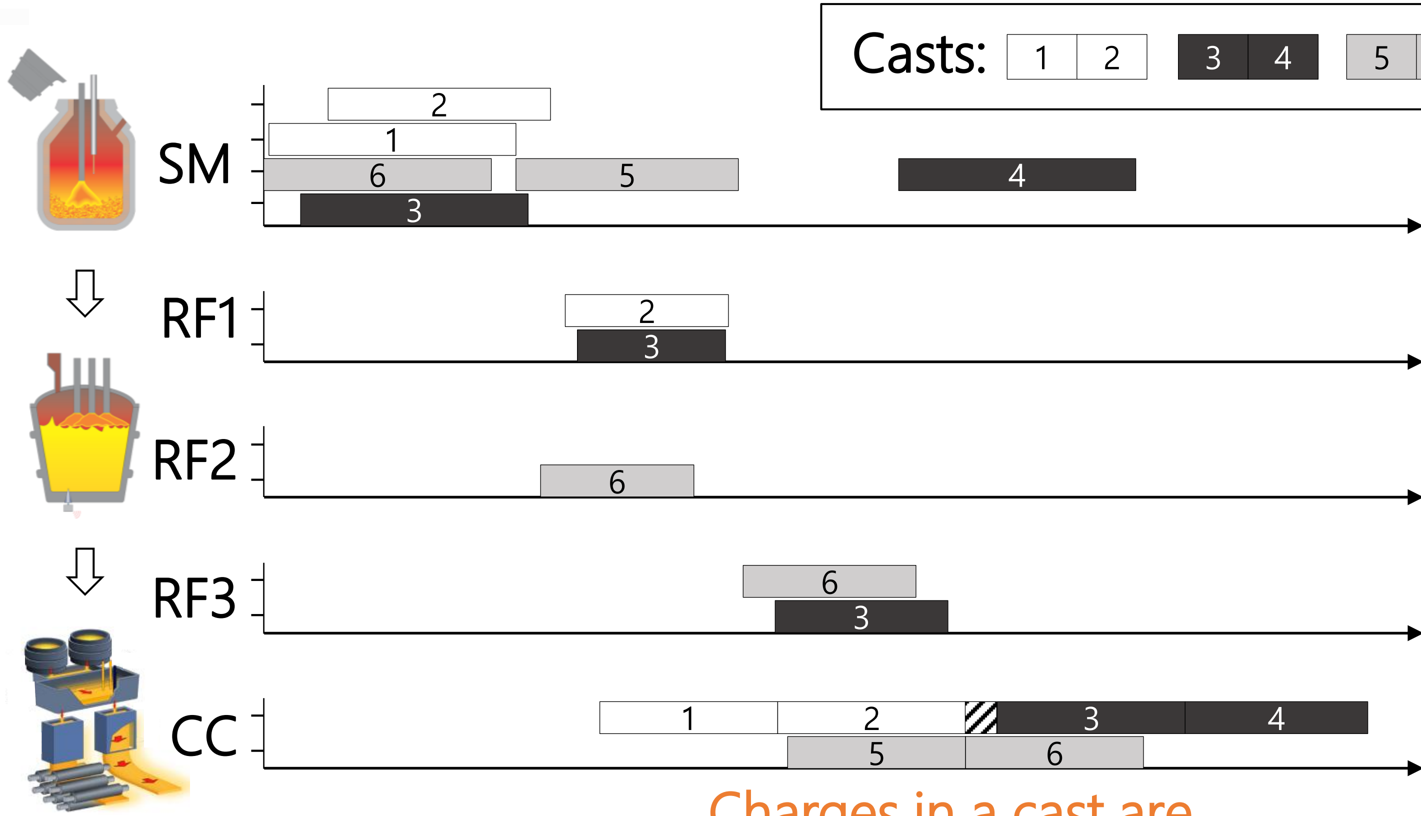
Problem Description

SCC Process schedule example

Required stages

- 1 : SM → CC
- 2 : SM → RF1 → CC
- 3 : SM → RF1 → RF3 → CC
- 4 : SM → CC
- 5 : SM → CC
- 6 : SM → RF2 → RF3 → CC

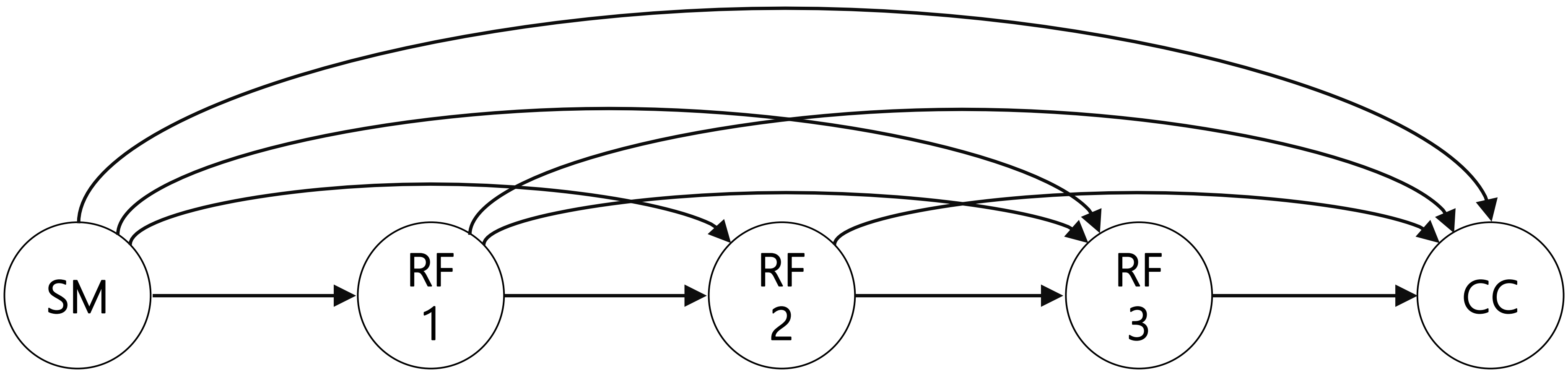
Casts: 1 2 | 3 4 | 5 6



Charges in a cast are continuously casted

Problem Description

SCC Scheduling



Flexible Flowshop with stage skipping

Problem Description

SCC Scheduling Problem

- Parameters
- Variables
- Objective
- Constraints

Problem Description

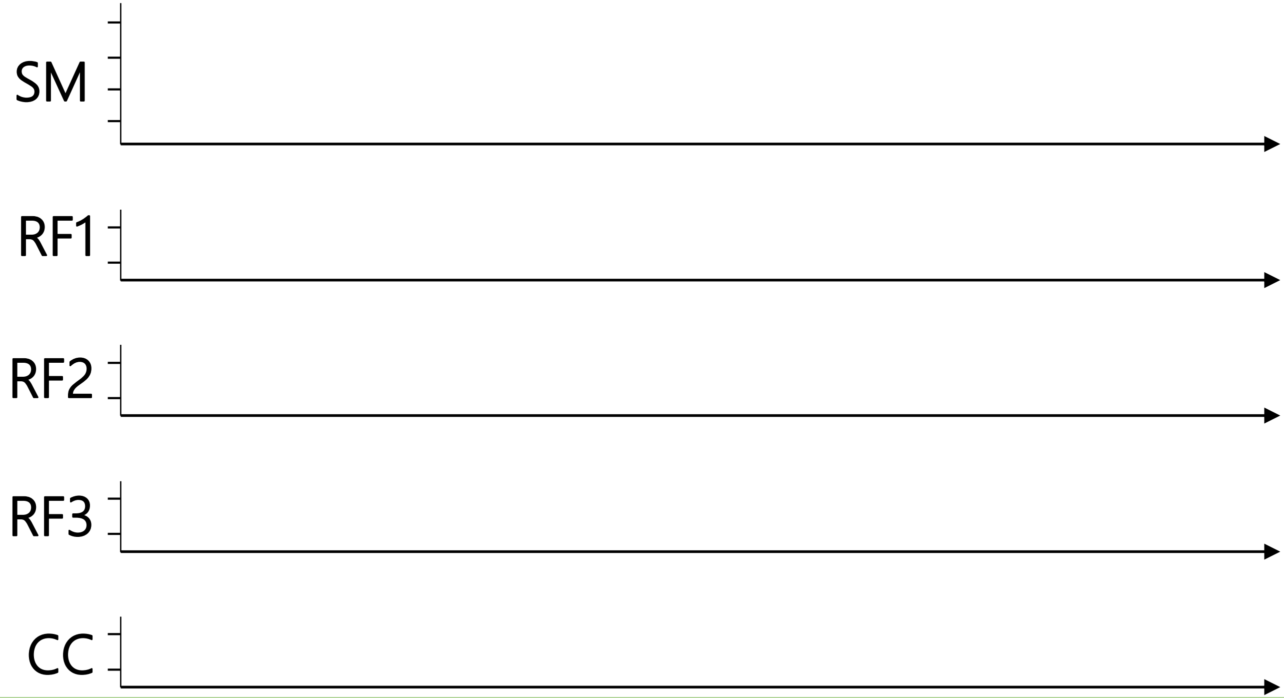
SCC Scheduling Problem

- **Parameters**
  - SCC environment
  - Charge
  - Cast: a sequence of charges

Problem Description

SCC Scheduling Problem: **Parameters**

- SCC environment
  - Stages, machines, transportation time between stages



Problem Description

Required stages

3 : SM → RF1 → RF3 → CC

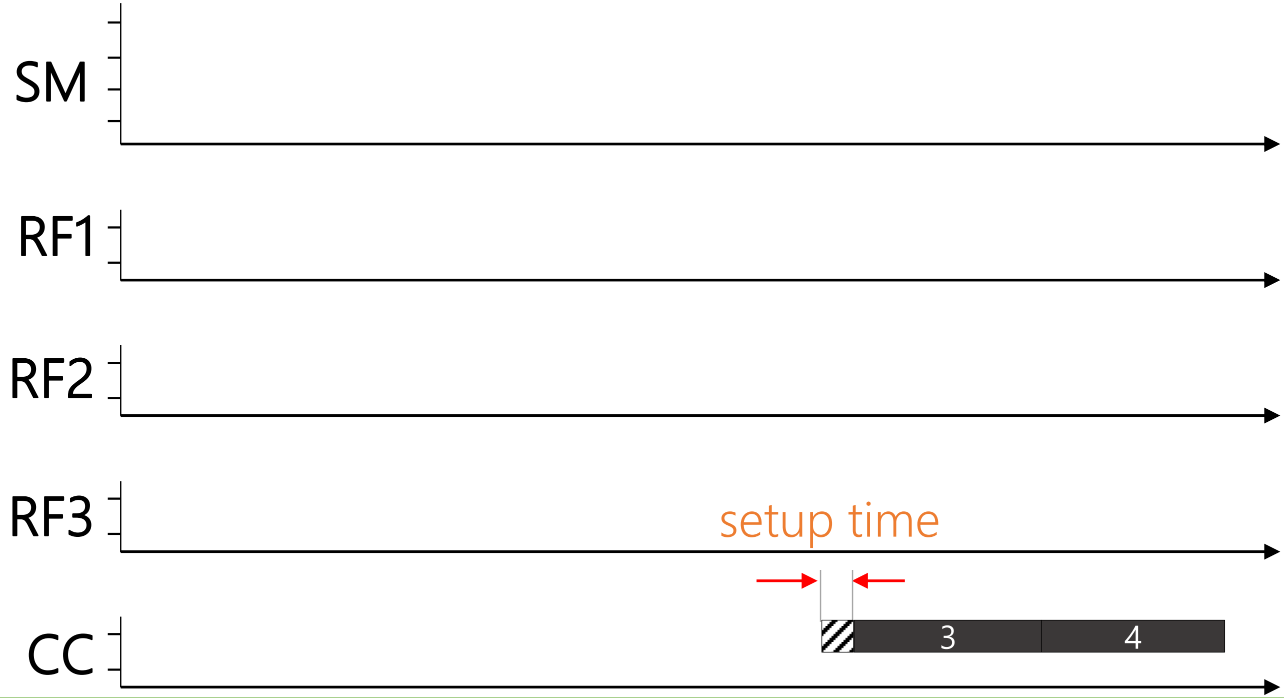
SCC Scheduling Problem: **Parameters**

- Charge
  - Required refining stages (route), Proc. time on each machine
  - Max waiting time, Due date (at the last stage)

Problem Description

SCC Scheduling Problem: **Parameters**

- Cast: a sequence of charges
  - Setup time at the last stage
    before processing the first charge



Problem Description

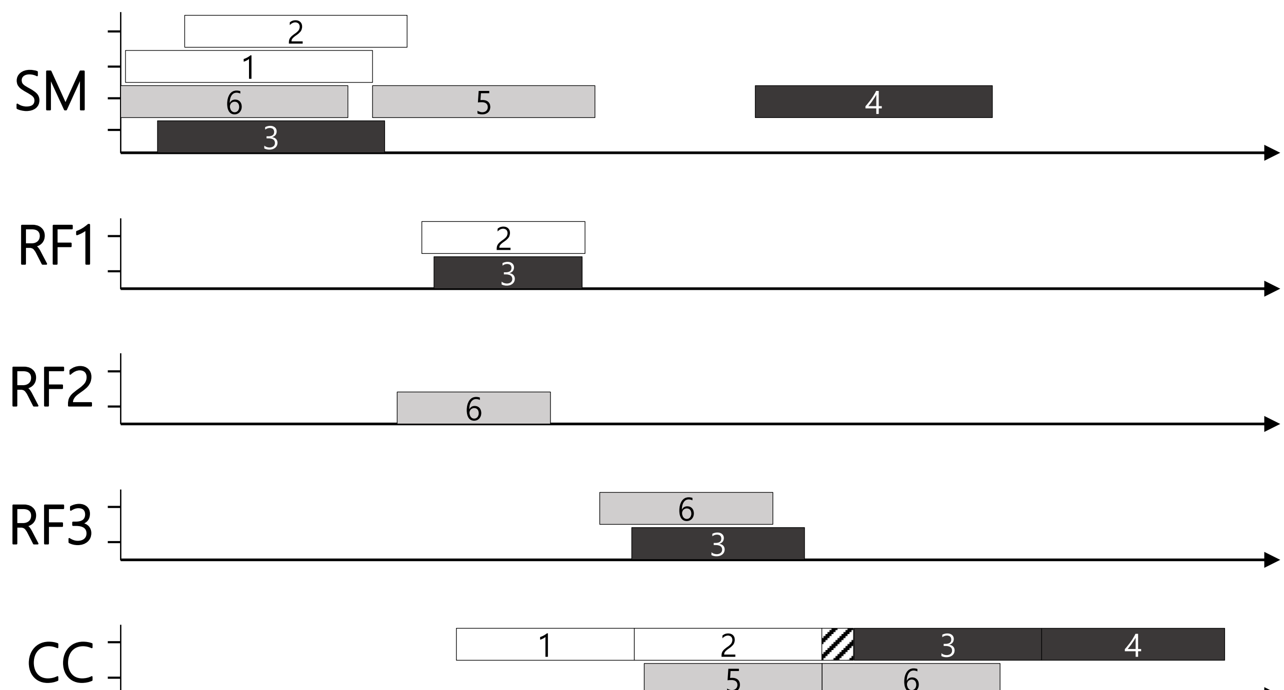
SCC Scheduling Problem: **Variables**

- Machine assignment
- Completion time

Required stages

1 : SM → CC
2 : SM → RF1 → CC
3 : SM → RF1 → RF3 → CC
4 : SM → CC
5 : SM → CC
6 : SM → RF2 → RF3 → CC

Casts: 1 2 | 3 4 | 5 6



Problem Description

SCC Scheduling Problem: **Objective**

- To minimize
  - Cast breaks
  - Total waiting time (between stages)
  - Total earliness
  - Total tardiness

Problem Description

SCC Scheduling Problem: **Constraints**

- Constraints
  - At most one charge at a time in each machine
  - CC stage
    - One CC machine for all charges in a cast
    - No idle time in a cast in the CC stage
  - Maximum waiting time (between stages)

Problem Description

Contribution to the Literature

Author (year)	Problem type*	Ca-CC fix	E&T†	Completion time‡	Waiting time‡	Max waiting time	Diff. Ch routes	MC uniformity††	Controllable time‡‡	# RF stages	Max charges	Algorithm	Time limit (sec)
			Objectives			Constraints				Data		Method	
Tang et al. (2002)	I		Ch		W			P		1	12	LR	222
Pacciarelli and Pranzo (2004)	I			M		O		P		3	114	Heu	324
Bellabdaoui and Teghem (2006)	I	O		M		O		P	C	1	8	MIP	6
Xuan and Tang (2007)	I			W	W			P		1	12	LR	623
Atighehchian, Bijari, and Tarkesh (2009)	I			M	S	O		R		1	108	ACO+NLP	300
Pan et al. (2013)	I	O	Ca		S			P		1	120	ABC	30
Sun and Wang (2013)	I		Ca		S	O	O	R		4	7	Heu	-
Tang, Zhao, and Liu (2014)	R			M	S	O	O	P	A	3	100	DE	60
Mao et al. (2014a)	R	O		S	S		O	P	A	2	120	LR	116
Mao et al. (2014b)	I	O	Ca		S			P		3	40	LR	176
Li et al. (2014)	I	O	Ca		S			P		3	120	FOA	20
Sbihi, Bellabdaoui, and Teghem (2014)	I	O		S		O		R	C	3	49	MIP	∞
Mao et al. (2015)	I	O		S	W			P		2	120	LR	54
Hao et al. (2015)	I	O			W		O	P		1	900	PSO	150
Jiang et al. (2015)	I	O	Ca		S		O	P	C	2	100	DE+VNS	400
Li, Pan, and Mao (2016)	R	O	Ca		S			P	A	1	120	FOA+IG	100
Pan (2016)	I			M	S			P		4	180	ABC	54
Long et al. (2016)	I	O	Ch		S		O	P		2	-	GA+LP	400
Jiang et al. (2016)	I	O		S	S		O	P	C	2	150	Heu	30
Yu, Chai, and Tang (2016)	R	O			S		O	P	A	1	30	Heu	-
Cui and Luo (2017)	I	O	Ca		W			P		2	20	LR	60
Jiang, Liu, and Hao (2017)	I	O	Ca		S		O	P		2	120	GA+LS	600
Long, Zheng, and Gao (2017)	R	O	Ch		W		O	P	A	2	66	GA+VNS	250
Sun et al. (2017)	R	O	Ch		W			P	A	2	40	LR	135
Fazel Zarandi and Dorry (2018)	I	O		M	S	O		P		1	61	PSO+LP	300
Jiang, Zheng, and Liu (2018)	I				S	O		P		1	150	CRO	330
Li et al. (2018)	I			M				P		1	120	ABC	100
Long et al. (2018a)	I				S		O	P	A	5	104	GA	-
Long et al. (2018b)	I	O		M	S		O	P	A	5	140	GA	450
Peng et al. (2018)	R	O	Ca		S			P	A	1	240	ABC	10
Sbihi and Chemangui (2018)	I	O		M		O		R	C	1	49	GA+LP	1800
Cui, Luo, and Wang (2020)	I	O	Ca		W			P		1	45	LR	150
Peng et al. (2020)	R	O	Ca		S			P		1	120	ICA+LS	30
Han et al. (2021)	I			W	W		O	P		3	62	LR	1200
This paper (2021)	I		Ch		S	O	O	R		3	36	IG+MIP	600

Problem Description

Contribution to the Literature

- 34 papers in 2002- 2021

Author (year)
Tang et al. (2002)
Pacciarelli and Pranzo (2004)
Bellabdaoui and Teghem (2006)
Xuan and Tang (2007)
Atighehchian, Bijari, and Tarkesh (2009)
Pan et al. (2013)
Sun and Wang (2013)
Tang, Zhao, and Liu (2014)
Mao et al. (2014a)
Mao et al. (2014b)
Li et al. (2014)
Sbihi, Bellabdaoui, and Teghem (2014)
Mao et al. (2015)
Hao et al. (2015)
Jiang et al. (2015)
Li, Pan, and Mao (2016)
Pan (2016)
Long et al. (2016)
Jiang et al. (2016)
Yu, Chai, and Tang (2016)
Cui and Luo (2017)
Jiang, Liu, and Hao (2017)
Long, Zheng, and Gao (2017)
Sun et al. (2017)
Fazel Zarandi and Dorry (2018)
Jiang, Zheng, and Liu (2018)
Li et al. (2018)
Long et al. (2018a)
Long et al. (2018b)
Peng et al. (2018)
Sbihi and Chemangui (2018)
Cui, Luo, and Wang (2020)
Peng et al. (2020)
Han et al. (2021)
This paper (2021)

Problem Description

Contribution to the Literature

- **5 Categories** for analysis

Assumption | Problem | Experiment

		Objectives			Constraints				Data		Method	
Problem type[*]	Ca-CC fix	E&T[†]	Completion time[‡]	Waiting time[‡]	Max waiting time	Diff. Ch routes	MC uniformity[††]	Controllable time[‡‡]	# RF stages	Max charges	Algorithm	Time limit (sec)

Problem Description

Contribution to the Literature: **Assumption**

- Problem Type
  - Initial schedule
  - Reschedule
- Ca-CC fix:
  - The assignment of cast – the machine in CC stage is given

Problem Description

Contribution to the Literature: **Assumption**

- Problem Type
  - Initial schedule
  - Reschedule
- Ca-CC fix:
  - The assignment of cast – the machine in CC stage is given

In this paper

- Problem Type
  - Initial schedule
  - ~~Reschedule~~
- Ca-CC fix:
  - ~~The assignment of cast – the machine in CC stage is given~~

Problem Description

Contribution to the Literature: **Problem & Experiment**

- Objectives
  - E&T (Charge, Cast)
  - Completion time ($C_{\max}$, ΣC_j)
  - Waiting time (Max, Sum)
- Data
  - # RF stages (1-5)
  - Max charges (7-900)
- Constraints
  - Max waiting time
  - Diff. Ch routes
  - MC uniformity
  - Controllable time
- Method
  - Algorithm
  - Time limit (sec)

Problem Description

In this paper

- **Objectives**
  - E&T (Charge)
  - ~~Completion time~~
  - Waiting time (Sum)
- **Data**
  - # RF stages (3)
  - Max charges (36)
- **Constraints**
  - Max waiting time
  - Diff. Ch routes
  - MC uniformity (unrelated)
  - ~~Controllable time~~
- **Method**
  - Algorithm: IG+MIP
  - Time limit (600 sec)

Problem Description

Contribution to the Literature

- Combination of practical elements that makes the problem hard
  - Charges w/ different routes
    (5/34 w/ # of RF stages $\geq$3)
  - Maximum waiting time constraints
  - Minimizing Total waiting time
    (5/34 w/ waiting time as both objective and constraints)
  - Minimizing Total earliness & Total tardiness
    (4/34 w/ Charge level E/T)

MIP Formulation

Notation: **Parameters**

$\mathcal{S}$ The sequence of all stages, $\mathcal{S} = \{1, 2, ..., l, ..., L\}$

where L is the last stage for CC

J The set of all casts, $J = \{1, 2, ..., j, ..., m\}$

where m is the number of casts

Ω The set of all charges, $\Omega = \{1, 2, ..., k, ..., n\}$

where n is the number of charges

Ω_j The sequence of charges in cast j, $\Omega_j := \{\Omega_j[1], \Omega_j[2], ..., \Omega_j[n_j]\}$

where n_j is the number of charges in cast j $(\forall j \in J)$

$\hat{\Omega}_j$ The set of pairs of two consecutive charges in cast j,

$\hat{\Omega}_j := \{(\Omega_j[\kappa], \Omega_j[\kappa + 1]) : \kappa \in \{1, 2, \ldots, n_j - 1\}\}$ $(\forall j \in J)$

MIP Formulation

Notation: **Parameters**

$\mathcal{S}_k$	The sequence of stages in charge k's route, $\mathcal{S}_k := \{\mathcal{S}_k[1], \mathcal{S}_k[2], ..., \mathcal{S}_k[c_k]\}$ where c_k is the number of stages in charge k's route $(\forall k \in \Omega)$ and $\mathcal{S}_k[1] = 1, \mathcal{S}_k[c_k] = L$
$\hat{\mathcal{S}}_k$	The set of pairs of two consecutive stages in the route of charge k, $\hat{\mathcal{S}}_k := \{(\mathcal{S}_k[\rho], \mathcal{S}_k[\rho+1]) : \rho \in \{1, 2, ..., c_k - 1\}\}$ $(\forall k \in \Omega)$
M_l	The set of machines at stage l $(\forall l \in \mathcal{S})$
p_{ik}	The processing time of charge k on machine i $(\forall k \in \Omega, i \in \bigcup_{l \in \mathcal{S}_k} M_l)$
$\tau_{ii'}$	The transportation time from machine i to i' $(\forall i, i' \in \bigcup_{l \in \mathcal{S}} M_l)$

MIP Formulation

Notation: **Parameters**

r_{kl} The earliest release time of charge k at stage l given as

$r_{k1} := 0$ and $r_{kl'} := r_{kl} + \min_{i \in M_l, i' \in M_{l'}} \{p_{ik} + \tau_{ii'}\}$

$(\forall k \in \Omega, (l, l') \in \hat{\mathcal{S}}_k)$

s_{ij} The setup time of cast j on machine i at the last stage

$(\forall j \in J, i \in M_L)$

d_k The due date of charge k at the last stage $(\forall k \in \Omega)$

$W_{\max}$ The maximum waiting time

π_1-π_4 Coefficients of penalty for

(cast break / waiting time / earliness / tardiness)

Q A sufficiently large number

MIP Formulation

Notation: **Variables**

$X_{kk'l}$ 1 if charge k precedes charge k' on the same machine at stage l, and 0 otherwise $\forall k, k' \in \Omega,\ k \neq k',\ l \in \mathcal{S}_k \cap \mathcal{S}_{k'}$ precedence variable

Y_{ikl} 1 if charge k at stage l is assigned to machine i, and 0 otherwise $\forall k \in \Omega,\ l \in \mathcal{S}_k,\ i \in M_l$ assignment variable

C_{kl} The completion time of charge k at stage l $\forall k \in \Omega,\ l \in \mathcal{S}_k$

U_k The idle time between charge k and its following charge at the last stage $\forall k \in \Omega \setminus \cup_{j \in J}\{\Omega_j[n_j]\}$ $U_k > 0$ → cast break

W_{kl} The waiting time of charge k between stage l and the next stage l' in its route $\forall k \in \Omega, (l, l') \in \hat{\mathcal{S}}_k$ Waiting time

E_k / T_k The earliness / tardiness of charge k $\forall k \in \Omega$ Earliness / Tardiness

MIP Formulation

Minimize

$$\pi_1 \cdot \sum_{j \in J} \sum_{\kappa=1}^{n_j - 1} U_{\Omega_j[\kappa]} + \pi_2 \cdot \sum_{k \in \Omega} \sum_{\rho=1}^{c_k - 1} W_{k, \mathcal{S}_k[\rho]} + \pi_3 \cdot \sum_{k \in \Omega} E_k + \pi_4 \cdot \sum_{k \in \Omega} T_k \quad (1)$$

Cast break | Waiting Time | Earliness | Tardiness

Subject to

$$\textstyle\sum_{i \in M_l} Y_{ikl} = 1 \qquad \forall k \in \Omega, l \in \mathcal{S}_k \quad (2)$$

$$X_{kk'l} + X_{k'kl} \geq Y_{ikl} + Y_{ik'l} - 1 \qquad \forall k, k' \in \Omega, k < k', l \in \mathcal{S}_k \cap \mathcal{S}_{k'}, i \in M_l \quad (3)$$

$$X_{kk'l} + X_{k'kl} \leq 1 - (Y_{ikl} - Y_{ik'l}) \qquad \forall k, k' \in \Omega, k \neq k', l \in \mathcal{S}_k \cap \mathcal{S}_{k'}, i \in M_l \quad (4)$$

$$Y_{ikL} = Y_{ik'L} \qquad \forall j \in J, (k, k') \in \hat{\Omega}_j, i \in M_L \quad (5)$$

$$X_{kk'L} = 1 \qquad \forall j \in J, (k, k') \in \hat{\Omega}_j \quad (6)$$

MIP Formulation

Subject to

$$C_{kl} \geq r_{kl} + p_{ik} \cdot Y_{ikl} \qquad \forall k \in \Omega, l \in \mathcal{S}_k, i \in M_l \tag{7}$$

$$C_{k'l} - C_{kl} \geq p_{ik'} - Q(2 - Y_{ikl} - Y_{ik'l} + X_{k'kl}) \tag{8}$$
$$\forall k, k' \in \Omega,\ k \neq k', l \in \mathcal{S}_k \cap \mathcal{S}_{k'}, i \in M_l$$

$$C_{k'L} - C_{kL} \geq (p_{ik'} + s_{ij'}) - Q(2 - Y_{ikL} - Y_{ik'L} + X_{k'kL}) \tag{9}$$
$$\forall j, j' \in J,\ j \neq j', i \in M_L, (k, k') = (\Omega_j[n_j], \Omega_{j'}[1])$$

$$C_{kl'} - (C_{kl} + W_{kl}) \geq (\tau_{ii'} + p_{i'k}) - Q(2 - Y_{ikl} - Y_{i'kl'}) \tag{10}$$
$$\forall k \in \Omega, (l, l') \in \hat{\mathcal{S}}_k, i \in M_l, i' \in M_{l'}$$

$$C_{kl'} - (C_{kl} + W_{kl}) \leq (\tau_{ii'} + p_{i'k}) + Q(2 - Y_{ikl} - Y_{i'kl'}) \tag{11}$$
$$\forall k \in \Omega, (l, l') \in \hat{\mathcal{S}}_k, i \in M_l, i' \in M_{l'}$$

MIP Formulation

Subject to

$$U_k - (C_{k'L} - C_{kL} - p_{ik'}) \geq -Q(1 - Y_{ik'L}) \quad (12)$$

$$T_k - E_k = C_{kL} - d_k \qquad \forall k \in \Omega \quad (13)$$

$$W_{kl} \leq W_{\max} \qquad \forall k \in \Omega, l \in \mathcal{S}_k \setminus \{L\} \quad (14)$$

$$X_{kk'l} \in \{0,1\} \qquad \forall k, k' \in \Omega, k \neq k', l \in \mathcal{S}_k \cap \mathcal{S}_{k'} \quad (15)$$

$$Y_{ikl} \in \{0,1\} \qquad \forall k \in \Omega, l \in \mathcal{S}_k, i \in M_l \quad (16)$$

$$C_{kl} \geq 0 \qquad \forall k \in \Omega, l \in \mathcal{S}_k \quad (17)$$

$$W_{kl} \geq 0 \qquad \forall k \in \Omega, l \in \mathcal{S}_k \setminus \{L\} \quad (18)$$

$$U_k \geq 0 \qquad \forall k \in \Omega \setminus \cup_{j \in J} \{\Omega_j[n_j]\} \quad (19)$$

$$E_k, T_k \geq 0 \qquad \forall k \in \Omega \quad (20)$$

Iterated Greedy Matheuristic

Overview

- **Lower Bounds**
  - Consider a subproblem with a single cast (i.e., MIP($\{j\}$) $j \in J$).
  - Let σ_j be the optimal solution of MIP($\{j\}$) .
  - Valid LBs: (assuming no cask breaks in MIP($\{j\}$))

$$\pi_2 \cdot \sum_{k \in \Omega_j} \sum_{\rho=1}^{c_k - 1} W_{k, \mathcal{S}_k[\rho]} + \pi_3 \cdot \sum_{k \in \Omega_j} E_k + \pi_4 \cdot \sum_{k \in \Omega_j} T_k \geq Z(\sigma_j)$$

  - Let $S_j^*(\sigma_j)$ be a desired starting time for cast j at CC stage.
  - Sort the casts in a non-decreasing order of $S_j^*(\sigma_j)$ for the algorithm.

Iterated Greedy Matheuristic

Overview

- **Initial heuristic (IH)**
  - On the empty schedule,
  - we put one cast at a time
  - while preserving the former schedule
    - machine assignment of charge
    - precedence relationship between charges
  - ➢ **to achieve a good initial schedule**

Iterated Greedy Matheuristic

Overview

- **Destruction & Construction (DC)**
  - We select some charges to be rearranged
    - DC cast (DA): charges in a cast
    - DC charge (DH): charges in similar period
  - We rearrange selected charges
    by solving an MIP model
    - which is smaller than an MIP model
      describing the whole problem
  - ➢ **to find a better schedule**

Iterated Greedy Matheuristic

Overview

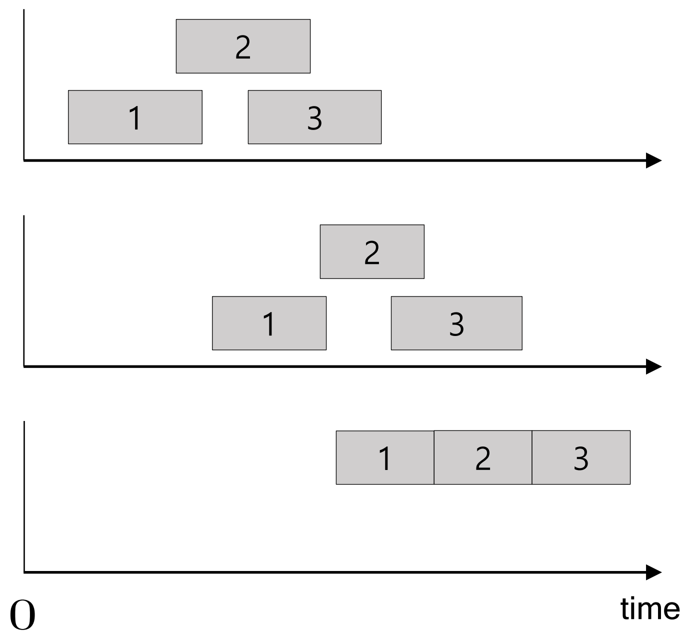
- IGM: Iterated Greedy Matheuristic

IH → n * [DA → DH] → MI (MIP improvement)

Iterated Greedy Matheuristic

Initial Heuristic

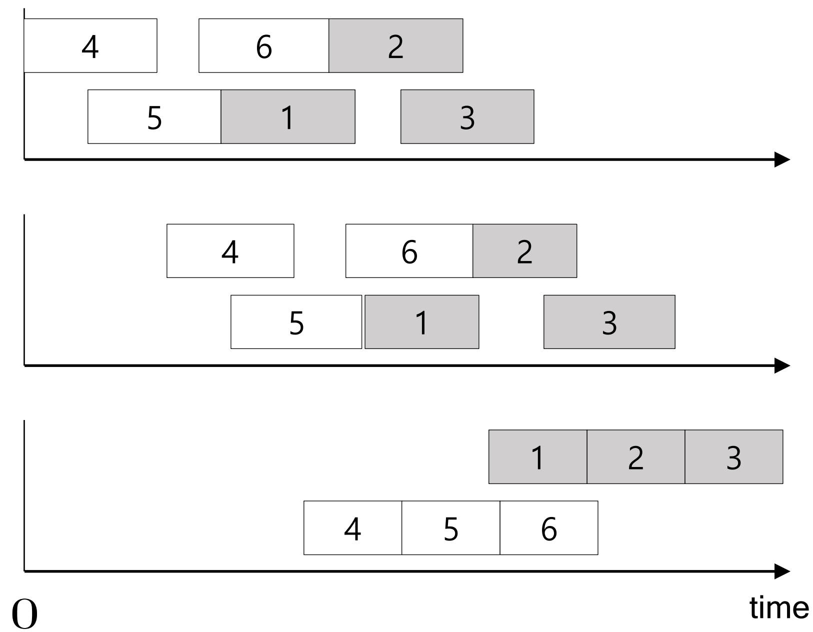
Cast sequence: | 1 | 2 | 3 | | 4 | 5 | 6 | | 7 | 8 | 9 |



Iterated Greedy Matheuristic

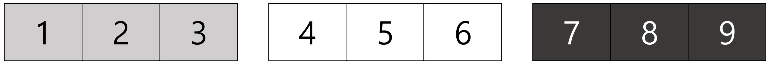
Initial Heuristic

Cast sequence: 1 2 3 | 4 5 6 | 7 8 9

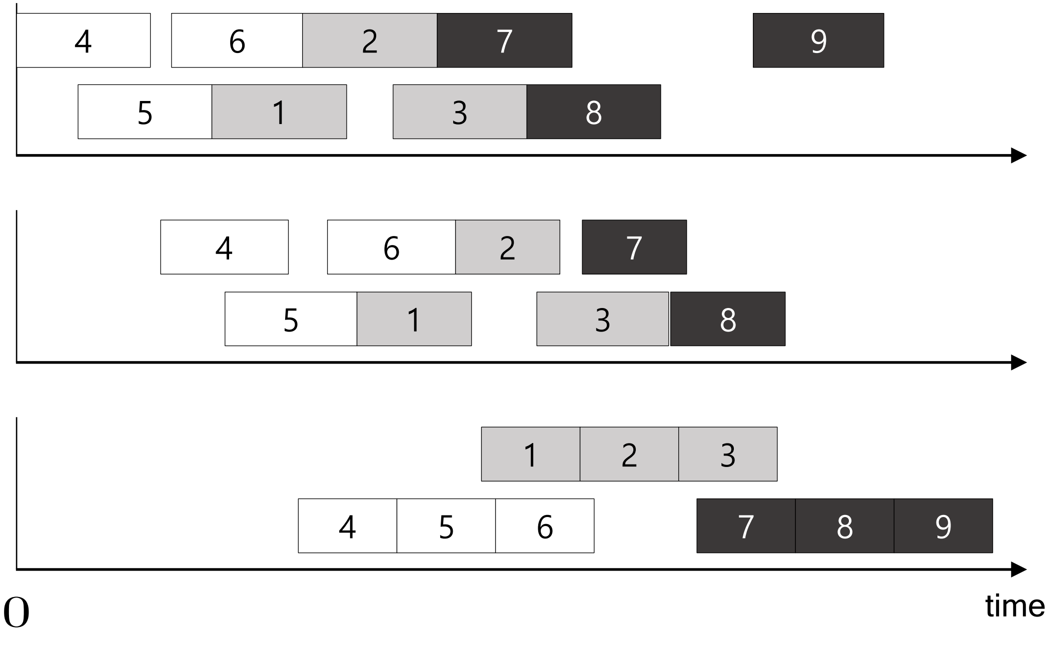


Iterated Greedy Matheuristic

Initial Heuristic

Cast sequence: 

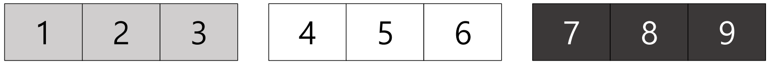
- while preserving the former schedule
  - machine assignment of charge
  - precedence relationship between charges

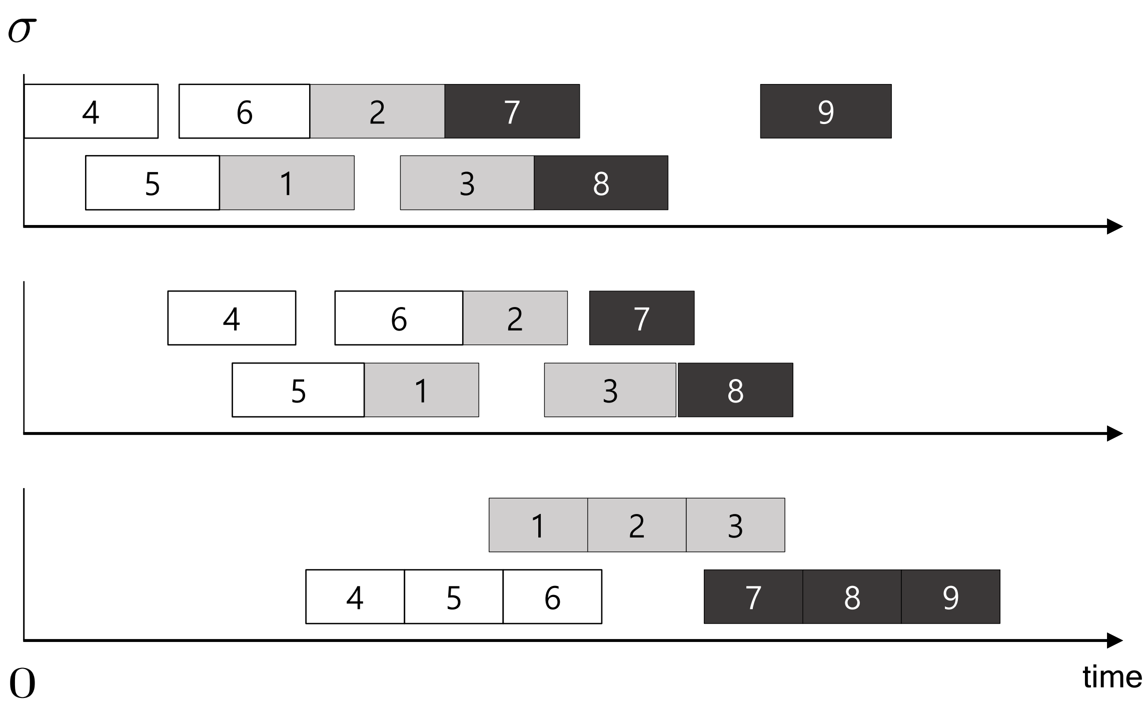


Iterated Greedy Matheuristic

DC Cast

Cast sequence:



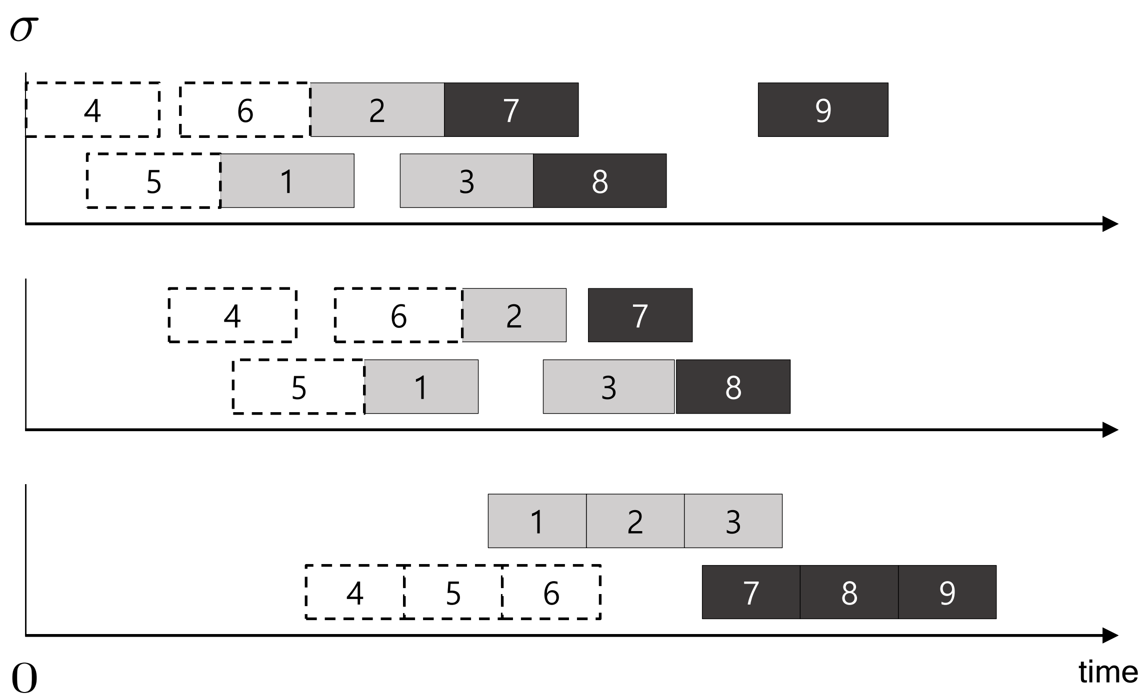


Iterated Greedy Matheuristic

DC Cast

Cast sequence: 1 2 3 | 4 5 6 | 7 8 9

: Charges to be rearranged

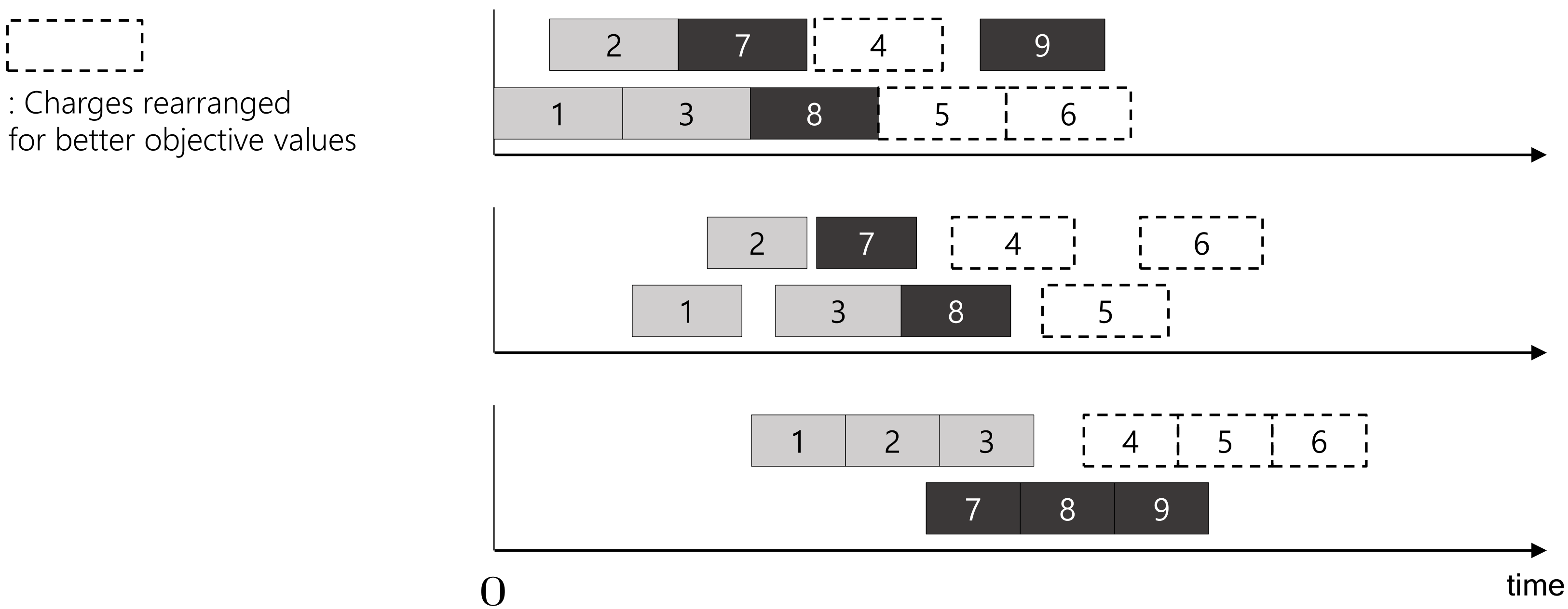


Iterated Greedy Matheuristic

DC Cast

Cast sequence: 1 2 3 | 4 5 6 | 7 8 9

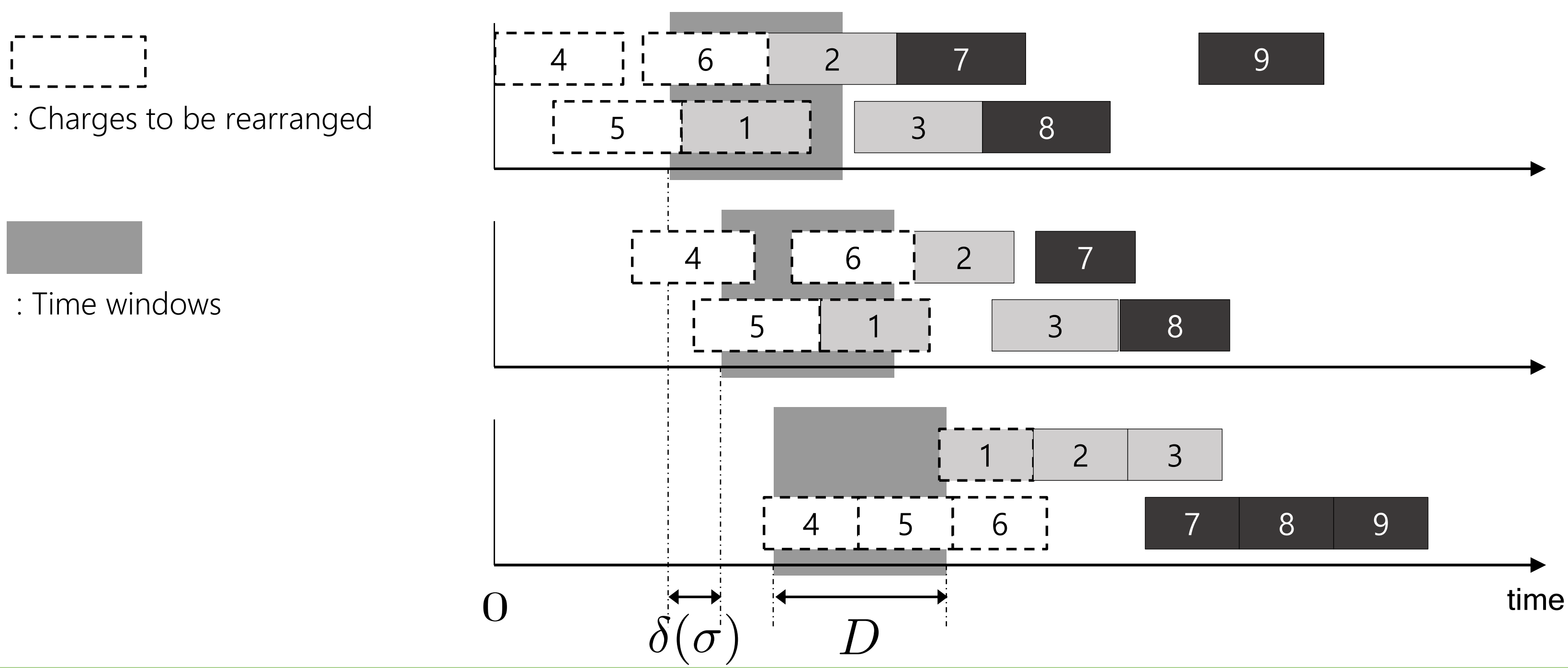
- while preserving the other charges' schedule
  - machine assignment of charge
  - precedence relationship between charges



Iterated Greedy Matheuristic

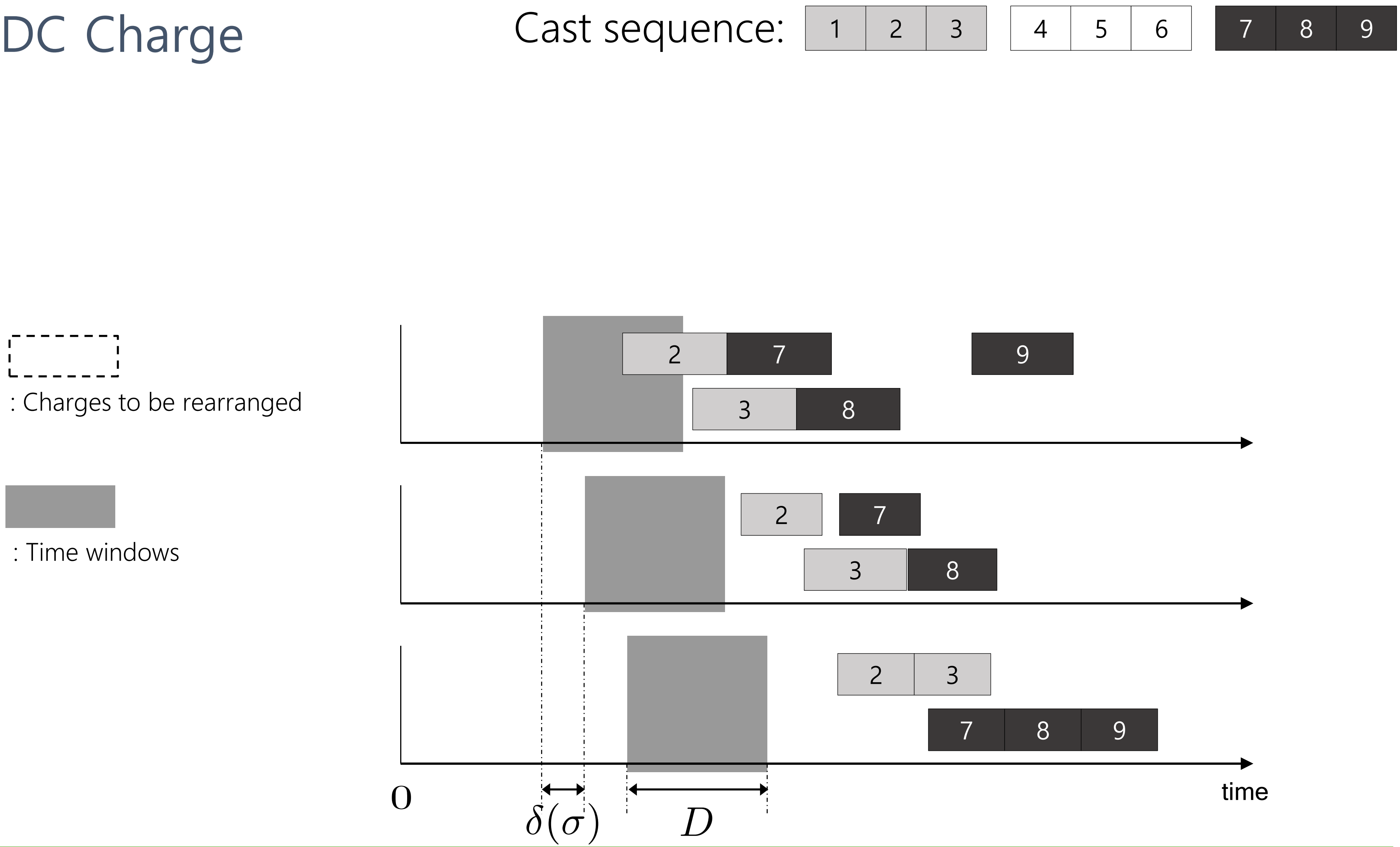
DC Charge

Cast sequence: 1 2 3 | 4 5 6 | 7 8 9



Iterated Greedy Matheuristic

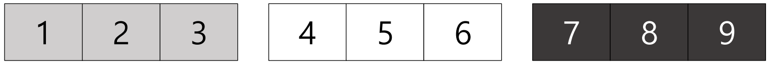
DC Charge



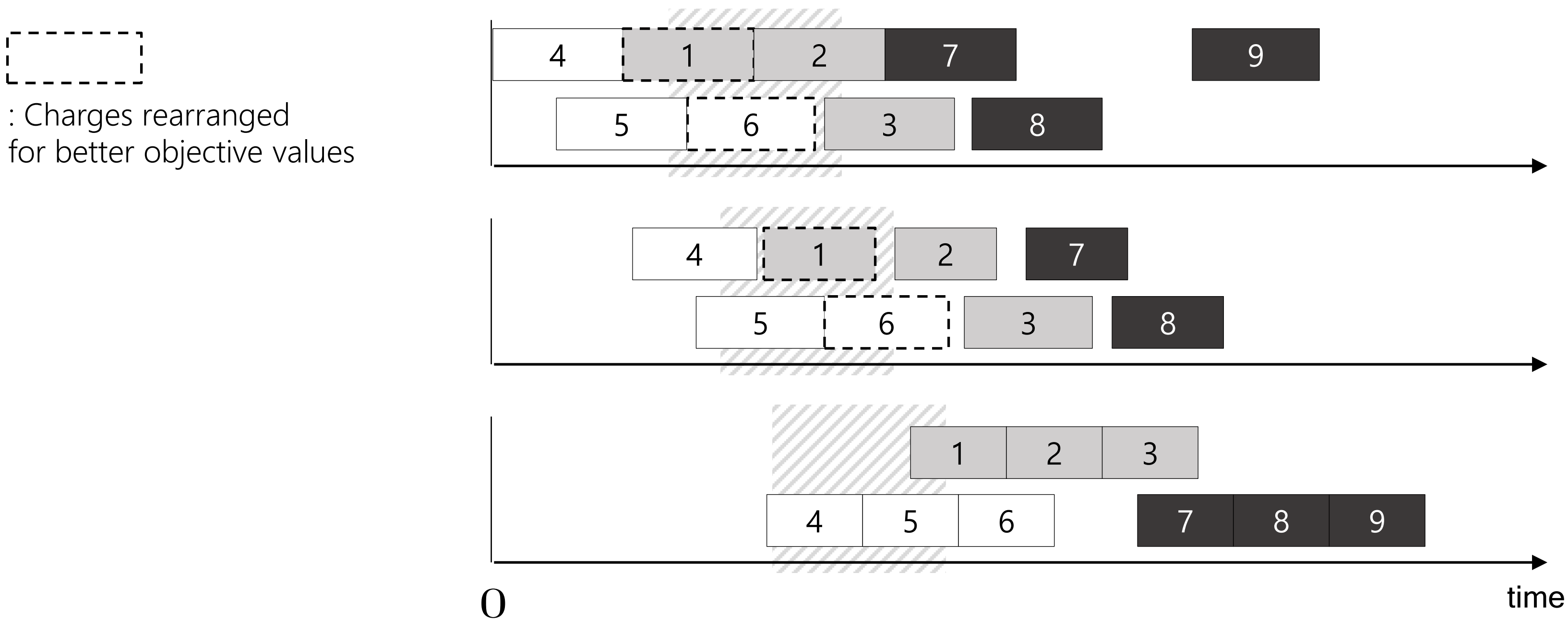
Iterated Greedy Matheuristic

DC Charge

Cast sequence:



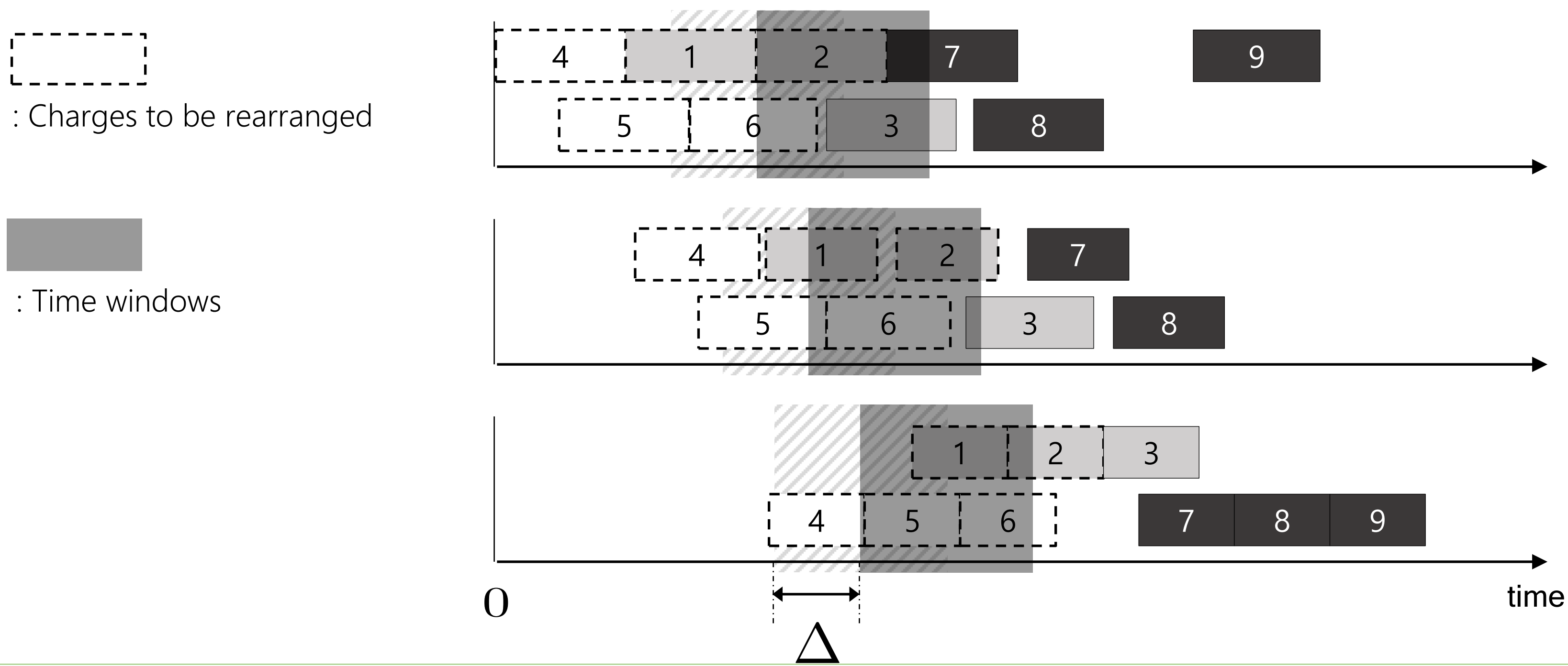
- while preserving the other charges' schedule
  - machine assignment of charge
  - precedence relationship between charges



Iterated Greedy Matheuristic

DC Charge

Cast sequence: | 1 | 2 | 3 | | 4 | 5 | 6 | | 7 | 8 | 9 |



Iterated Greedy Matheuristic

Notation for the Heuristic

MIP(J')	The MIP with restricted set of casts $J' \subseteq J$ (e.g., MIP(J) denotes the master MIP.)
σ	A partial or feasible schedule of MIP(J)
$Z(\sigma)$	The obj. value of σ to a MIP (sub)problem
$\hat{(\cdot)}^{\sigma}$	The value of a variable determined by solution σ
$S_j^*(\sigma)$	The starting time of cast j in CC stage of solution σ

Iterated Greedy Matheuristic

Notation for the Heuristic

$\mathcal{P}^X$	The set of all precedence variables
$\mathcal{A}^Y$	The set of all machine assignment variables
X(or Y)	A variable in $\mathcal{P}^X$ (or $\mathcal{A}^Y$)
$\mathcal{C}^{\text{fix}}$	A set of constraints that fix the values of particular X and Y variables
$\mathcal{C}^{\text{LB}}$	A set of lower bound constraints for the objective terms in the master MIP
V^X(or V^Y)	A set of X(or Y) variables that are not fixed during an iteration

Iterated Greedy Matheuristic

Notation for the Heuristic

$\overline{T}$	A time limit for a MIP subproblem
R	The number of repeated runs of a heuristic
$\langle \sigma, \mathcal{C}, \overline{T} \rangle$	Control parameters in solving a MIP subproblem;

σ : a partial or a feasible incumbent solution ($\varnothing$ if not available),

$\mathcal{C}$: a set of additional constraints, and

$\overline{T}$: a time limit

Iterated Greedy Matheuristic

Algorithm

Algorithm 1: Lower bound computation (LC).

Input : A set of casts J

Output: $\mathcal{C}^{\text{LB}}$, a rearranged sequence of casts J'

begin

$\mathcal{C}^{\text{LB}} \leftarrow \varnothing$;

for j **in** J **do**

$\sigma_j \leftarrow$ Solve MIP($\{j\}$);

$\mathcal{C}^{\text{LB}} \leftarrow \mathcal{C}^{\text{LB}} \cup \{\text{Eq.(21)}\}$

$$\pi_2 \cdot \sum_{k \in \Omega_j} \sum_{\rho=1}^{c_k - 1} W_{k, \mathcal{S}_k[\rho]} + \pi_3 \cdot \sum_{k \in \Omega_j} E_k + \pi_4 \cdot \sum_{k \in \Omega_j} T_k \geq Z(\sigma_j)$$

$J' \leftarrow$ Sort J according to the non-decreasing order of $S_j^*(\sigma_j)$ for $j \in J$;

return $\mathcal{C}^{\text{LB}}, J'$

Iterated Greedy Matheuristic

Algorithm

Algorithm 2: Initial heuristic (IH).

Input : A sorted list of casts J, a time limit $\overline{T}^{\mathrm{IH}}$

Output: A feasible solution σ of the master MIP

begin

$\mathcal{C}^{\mathrm{fix}} \leftarrow \varnothing, \Omega^{\mathrm{fix}} \leftarrow \varnothing;$

for j **in** J **do**

$\sigma \leftarrow$ Solve MIP$(\{1, \ldots, j\})$ with $\langle \varnothing, \mathcal{C}^{\mathrm{fix}}, \overline{T}^{\mathrm{IH}} \rangle$;

for X **in** $\{X_{kk'l}, X_{k'kl} : k \in \Omega_j, k' \in \Omega^{\mathrm{fix}} \cup \Omega_j \setminus \{k\}, l \in \mathcal{S}_k \cap \mathcal{S}_{k'}\}$ **do**

$\mathcal{C}^{\mathrm{fix}} \leftarrow \mathcal{C}^{\mathrm{fix}} \cup \{X = \hat{X}^{\sigma}\};$

for Y **in** $\{Y_{ikl} : k \in \Omega_j, l \in \mathcal{S}_k, i \in M_l\}$ **do**

$\mathcal{C}^{\mathrm{fix}} \leftarrow \mathcal{C}^{\mathrm{fix}} \cup \{Y = \hat{Y}^{\sigma}\};$

$\Omega^{\mathrm{fix}} \leftarrow \Omega^{\mathrm{fix}} \cup \Omega_j;$

return σ

Iterated Greedy Matheuristic

Algorithm

Algorithm 3: DC-cast (DA).

Input : A feasible solution σ, $\mathcal{C}^{\text{LB}}$, $\overline{T}^{\text{DA}}$

Output: An improved solution σ^*

begin

Sort J in the non-decreasing order of $S_j^*(\sigma)$;

$\sigma^* \leftarrow \sigma$;

for j **in** J **do**

$V^X \leftarrow \{X_{kk'l}, X_{k'kl} : k \in \Omega_j, k' \in \Omega \setminus \{k\}, l \in \mathcal{S}_k \cap \mathcal{S}_{k'}\}$;

$V^Y \leftarrow \{Y_{ikl} : k \in \Omega_j, l \in \mathcal{S}_k, i \in M_l\}$;

$\mathcal{C}^{\text{fix}} \leftarrow \{X = \hat{X}^{\sigma^*} : X \in \mathcal{P}^X \setminus V^X\} \cup \{Y = \hat{Y}^{\sigma^*} : Y \in \mathcal{A}^Y \setminus V^Y\}$;

$\sigma^* \leftarrow$ Solve MIP(J) with $\langle \sigma^*, \mathcal{C}^{\text{fix}} \cup \mathcal{C}^{\text{LB}}, \overline{T}^{\text{DA}} \rangle$;

return σ^*

Iterated Greedy Matheuristic

Algorithm

Algorithm 4: DC-charge (DH).

Input : A feasible solution σ, D, Δ, $\mathcal{C}^{\text{LB}}$, $\overline{T}^{\text{DH}}$

Output: An improved solution σ^*

begin

$\delta \leftarrow \delta(\sigma)$ by (22);

for l **in** $\mathcal{S}$ **do**

$[t_l^s, t_l^e] \leftarrow \left[\bar{S}_1(\sigma) + (l-1)\delta, \bar{S}_1(\sigma) + (l-1)\delta + D\right]$;

$\sigma^* \leftarrow \sigma$;

while $\exists l \in \mathcal{S}$ such that $t_l^s \leq \bar{C}_l(\sigma^*)$ **do**

$\Omega^D \leftarrow \{k : k \in \Omega, \exists l \in \mathcal{S}_k \text{ such that } \hat{C}_{kl}^{\sigma^*} \in [t_l^s, t_l^e]\}$;

$V^X \leftarrow \{X_{kk'l}, X_{k'kl} : k \in \Omega^D, k' \in \Omega \setminus \{k\}, l \in \mathcal{S}_k \cap \mathcal{S}_{k'}\}$;

$V^Y \leftarrow \{Y_{ikl} : k \in \Omega^D, l \in \mathcal{S}_k, i \in M_l\}$;

$\mathcal{C}^{\text{fix}} \leftarrow \{X = \hat{X}^{\sigma^*} : X \in \mathcal{P}^X \setminus V^X\} \cup \{Y = \hat{Y}^{\sigma^*} : Y \in \mathcal{A}^Y \setminus V^Y\}$;

$\sigma^* \leftarrow$ Solve MIP(J) with $\langle \sigma^*, \mathcal{C}^{\text{fix}} \cup \mathcal{C}^{\text{LB}}, \overline{T}^{\text{DH}} \rangle$;

for l **in** $\mathcal{S}$ **do**

$[t_l^s, t_l^e] \leftarrow [t_l^s + \Delta, t_l^e + \Delta]$;

return σ^*

Iterated Greedy Matheuristic

Algorithm

Algorithm 5: Iterated greedy matheuristic (IGM).

Input : J, $\overline{T}^{\mathrm{IH}}$, R^{DC}, R^{DA}, $\overline{T}^{\mathrm{DA}}$, R^{DH}, $\overline{T}^{\mathrm{DH}}$, D, Δ, $\overline{T}^{\mathrm{IGM}}$

Output: A feasible solution σ

begin

$\mathcal{C}^{\mathrm{LB}}, J' \leftarrow \mathrm{LC}(J)$; LB Computation

$\sigma \leftarrow \mathrm{IH}(J', \overline{T}^{\mathrm{IH}})$; Initial Heuristic

repeat R^{DC} **times**

DC Cast

repeat R^{DA} **times** $\sigma \leftarrow \mathrm{DA}(\sigma, \mathcal{C}^{\mathrm{LB}}, \overline{T}^{\mathrm{DA}})$ **until** *not improved*;

repeat R^{DH} **times** $\sigma \leftarrow \mathrm{DH}(\sigma, D, \Delta, \mathcal{C}^{\mathrm{LB}}, \overline{T}^{\mathrm{DH}})$ **until** *not improved*;

DC Charge

until *not improved*

$\sigma \leftarrow \mathrm{MI}(\sigma, \mathcal{C}^{\mathrm{LB}}, \overline{T}^{\mathrm{IGM}} - \text{Elapsed time})$;

return σ

Experimental Results

Test Data Summary

- Random processing times
  - SM: 45~55 min
  - RF: 30~40 min
  - CC: 35~45 min
- Random routing
  - Each charge has a 2/3 probability of skipping each RF stage
- Transportation time:
  - 10 min between all machines
- Maximum waiting time:
  - 30 min

Experimental Results

Test Data Summary

- Three problem sizes
  - small: 2~3 casts, 6~12 charges
  - medium: 3~4 casts, 15~24 charges
  - practical: 4~7 casts, 30~36 charges

- Total 90 problem instances
  - 30 small-sized problems
  - 30 medium-sized problems
  - 30 practical-sized problems

Experimental Results

Algorithm Parameters

- For IH,
  - $\overline{T}^{\mathrm{IH}}$ = 60 sec.

- For DC,
  - $R^{\mathrm{DC}} = 4$, $R^{\mathrm{DA}} = 2$, $R^{\mathrm{DH}} = 1$
  - $\overline{T}^{\mathrm{DA}}$ = 60 sec, $\overline{T}^{\mathrm{DH}}$ = 60 sec, D = 90 min, Δ = 45 min.

- For IGM,
  - $\overline{T}^{\mathrm{IGM}}$ = 600 sec.

Experimental Results

Compared algorithms

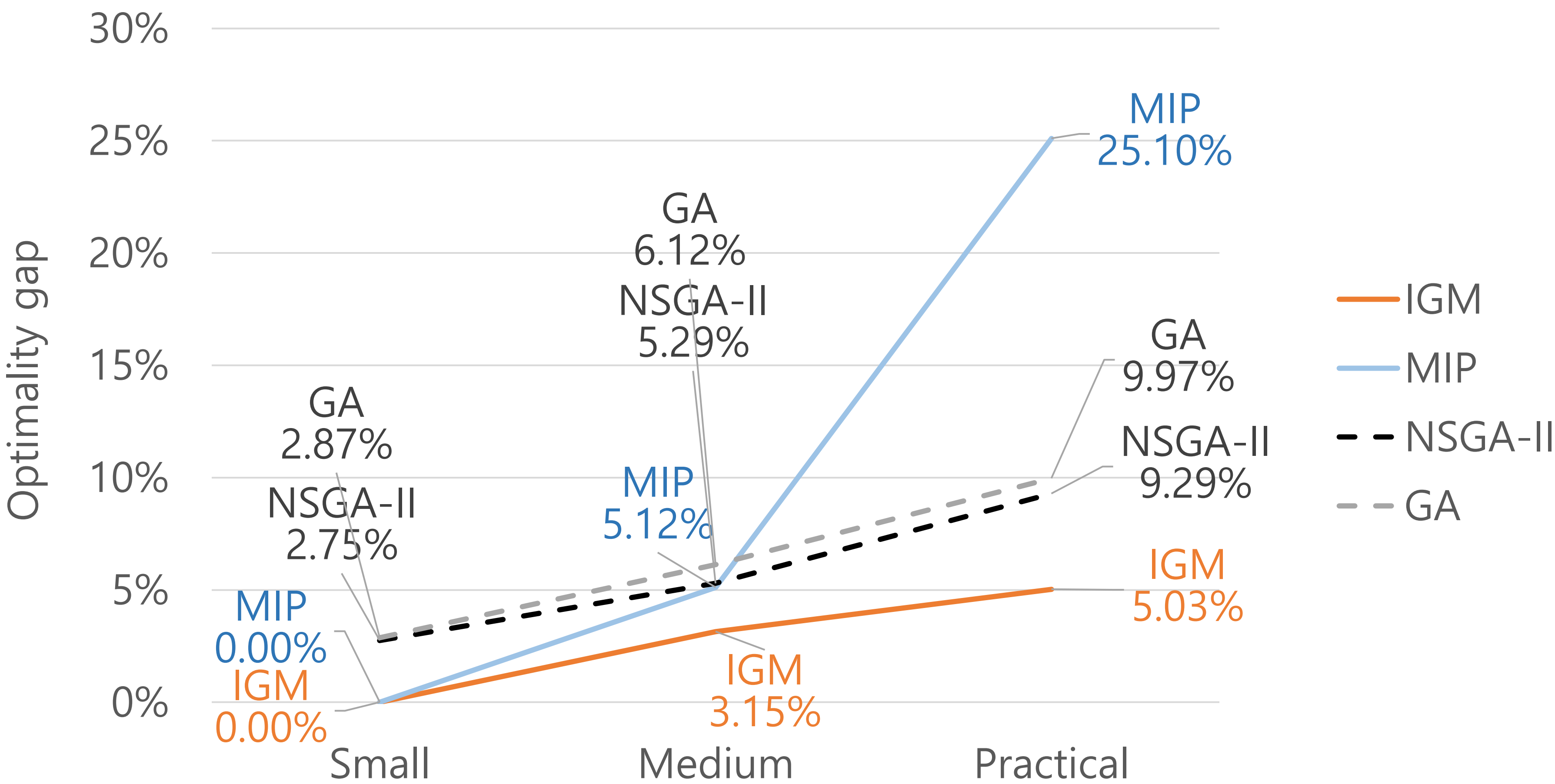
- Iterated greedy matheuristic (IGM) → 10 minutes

- Solving the whole MIP model (MIP)
- NSGA-II
- Simple genetic algorithm (GA)

} → 20 minutes

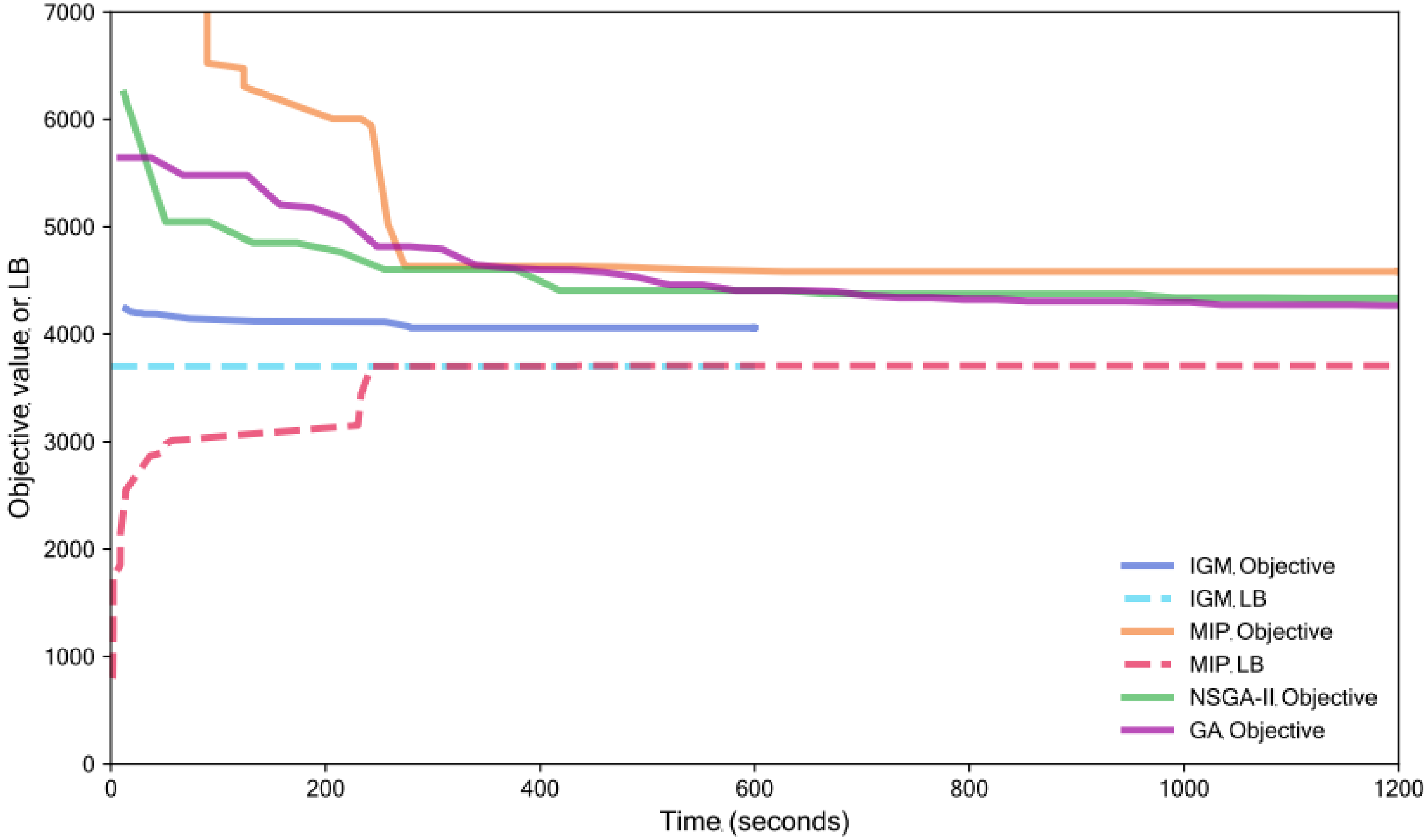
Experimental Results

The average optimality gaps



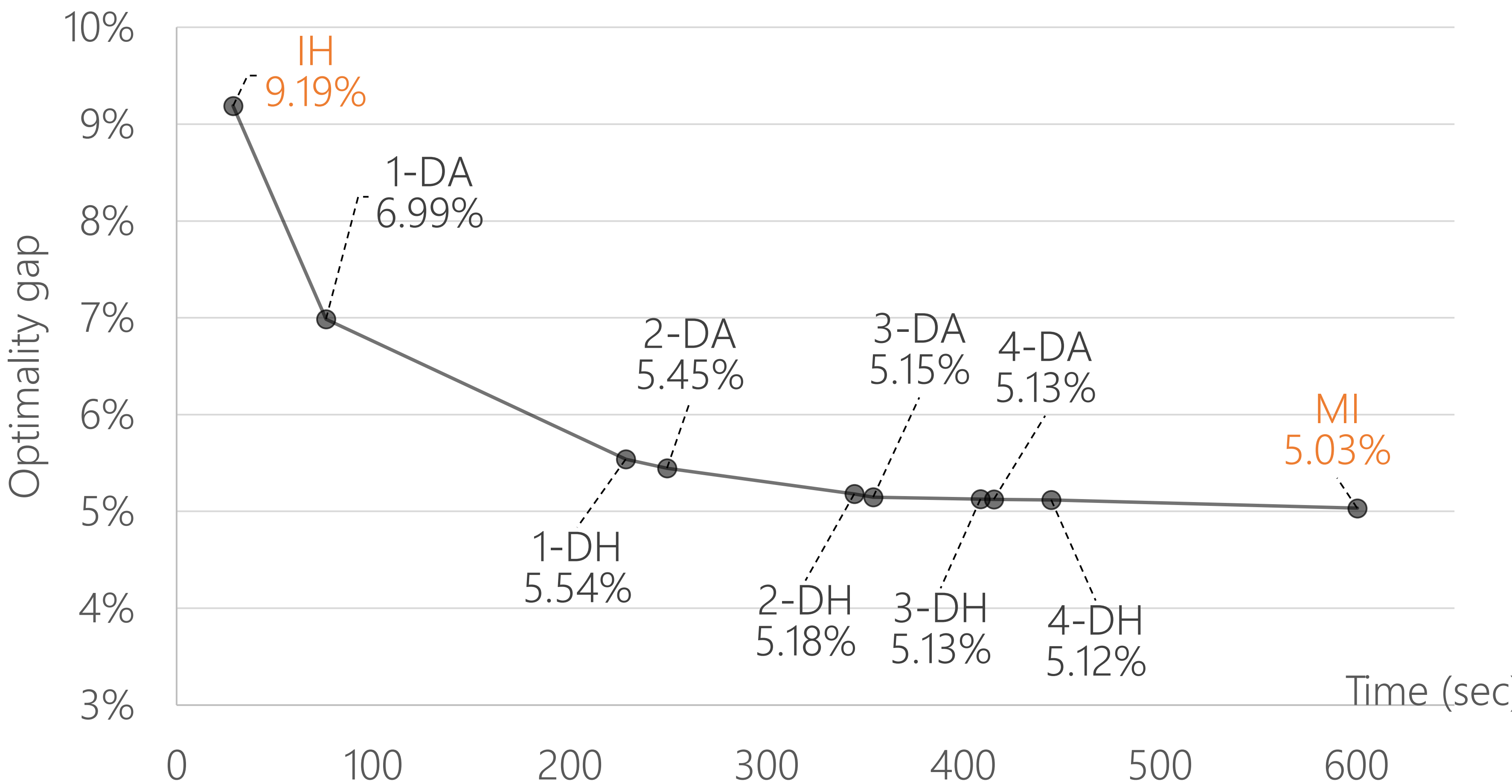
Experimental Results

Example: obj. value and LB over time on a practical size



Experimental Results

Avg. performance of IGM on practical size problems



Conclusion

- We consider a practical steelmaking-continuous casting scheduling problem.
- We establish a general **Mixed Integer Program (MIP)**.
- We propose an **iterated greedy matheuristic (IGM)**, utilizing **MIP** and **it subproblems**.
- IGM performs very well on all different sizes.

Conclusion

- IGM may be applied to various problems since it uses a MIP and its subproblems.

- Practical hybrid flowshop scheduling problems considering:
  - sequence-dependent setup times
  - precedence constraints
  - machine eligibility constraints

- Scheduling problems in more general machine environments (e.g., flexible job shop)

Thank you

Kangbok Lee

POSTECH, South Korea