Machine Learning for Scheduling and Resource Allocation

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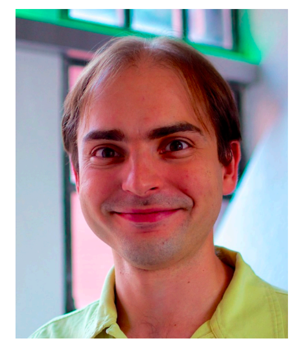

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Online Scheduling via Learned Weights. SODA 2020.
Learnable and Instance-Robust Predictions for Matchings, Flows and Load Balancing. ESA 2021
Using Predicted Weights for Ad Delivery. ACDA 2021
Faster Matching via Learned Duals. NeurIPS 2021

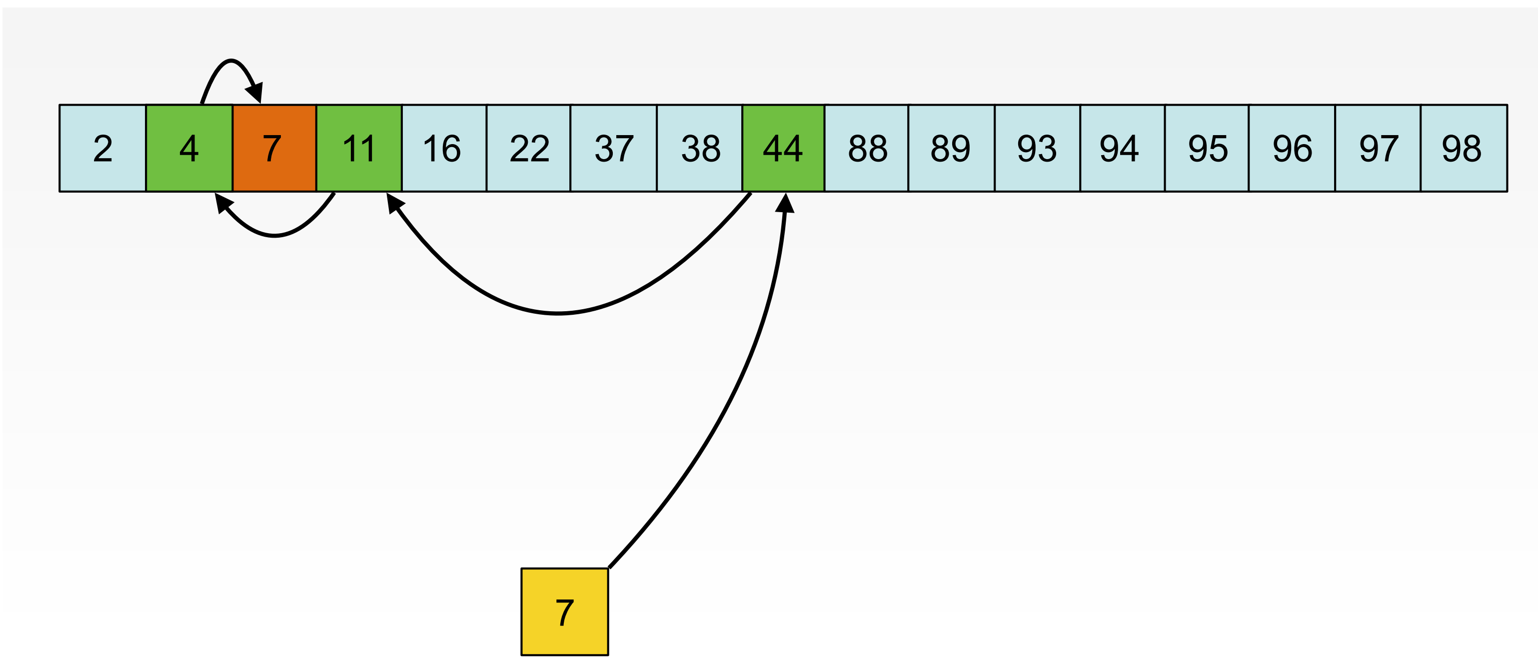
Machine Learning is Transforming Society

- Has not fundamentally changed combinatorial algorithms for resource allocation problems
- However, could it?

Optimization Augmented with Machine Learning

Motivating Example
[Kraska et al. SIGMOD 2018]

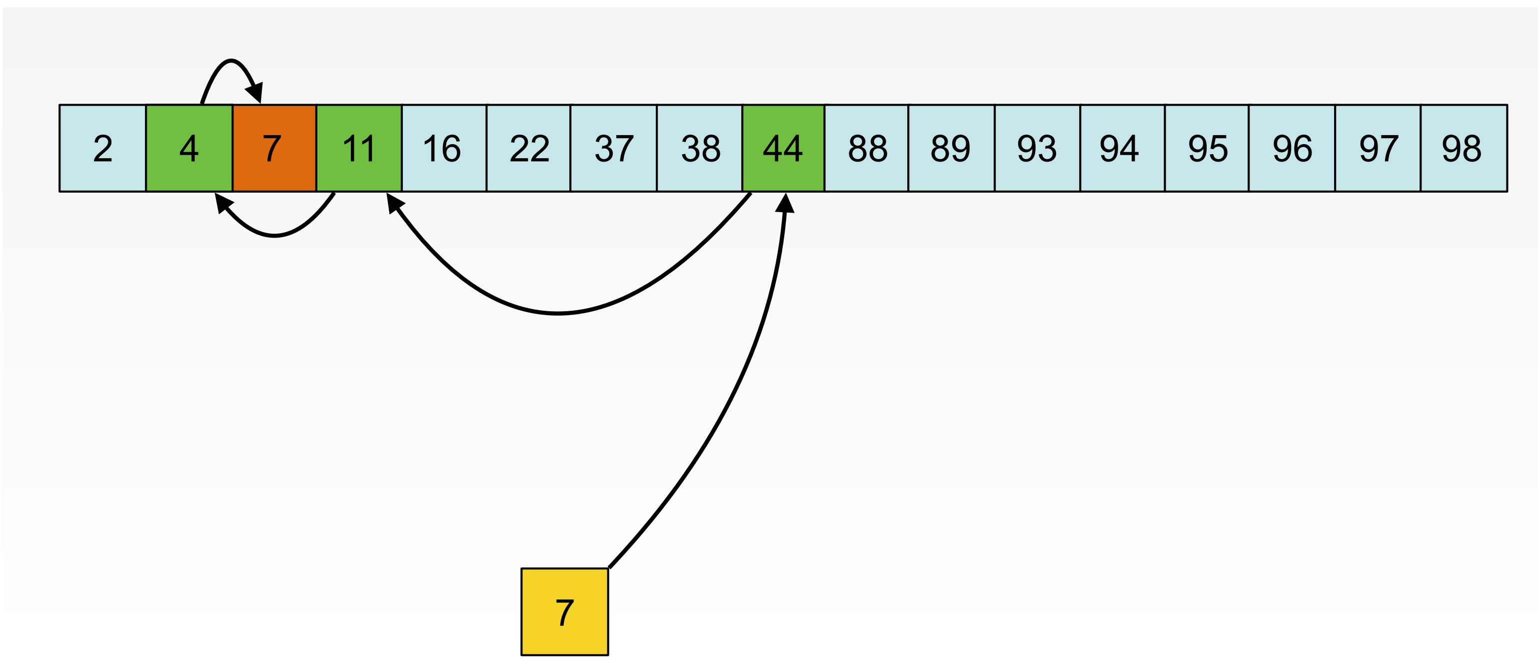
- Array of n integers A
- Over time queries arrive asking if q is in A



Motivating Example

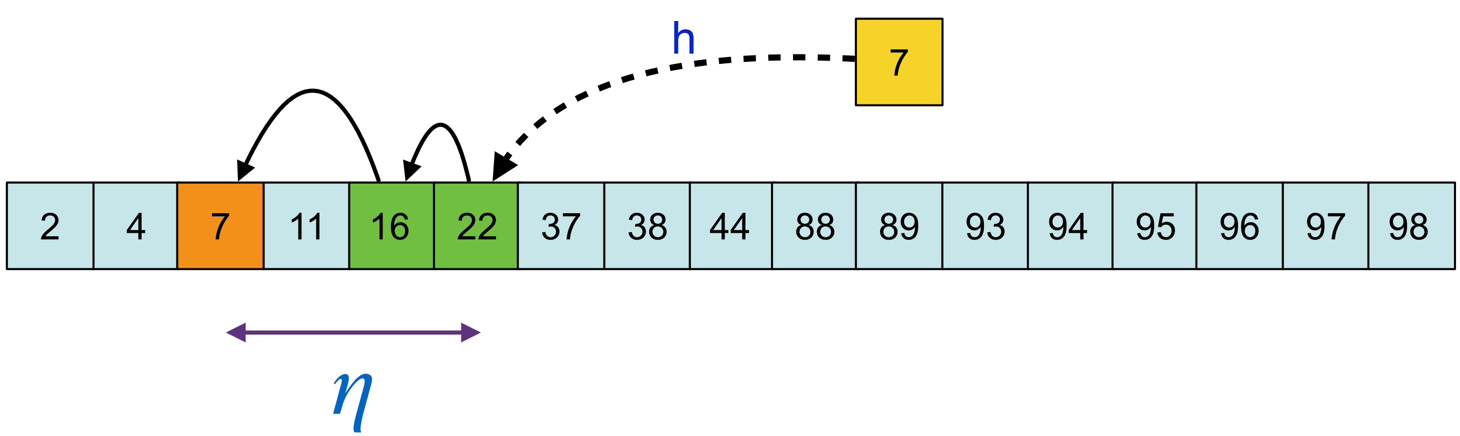
- Array of n integers A
- Over time queries arrive asking if q is in A

O(log n) lookup time



Motivating Example

- Train a predictor h(q) to predict where q is in the array
    - Estimates where the integer is based on prior queries
- Could be wrong, but hopefully not too far off
    - Use doubling binary search from prediction



Motivating Example

- Analysis
  - Let η be the value of $|h(q) - OPT(q)|$, the error in the prediction
  - Run time is $O(\log \eta)$
- Need to be careful about overhead of the prediction
  - Can make this work in practice

Learning Augmented Algorithm

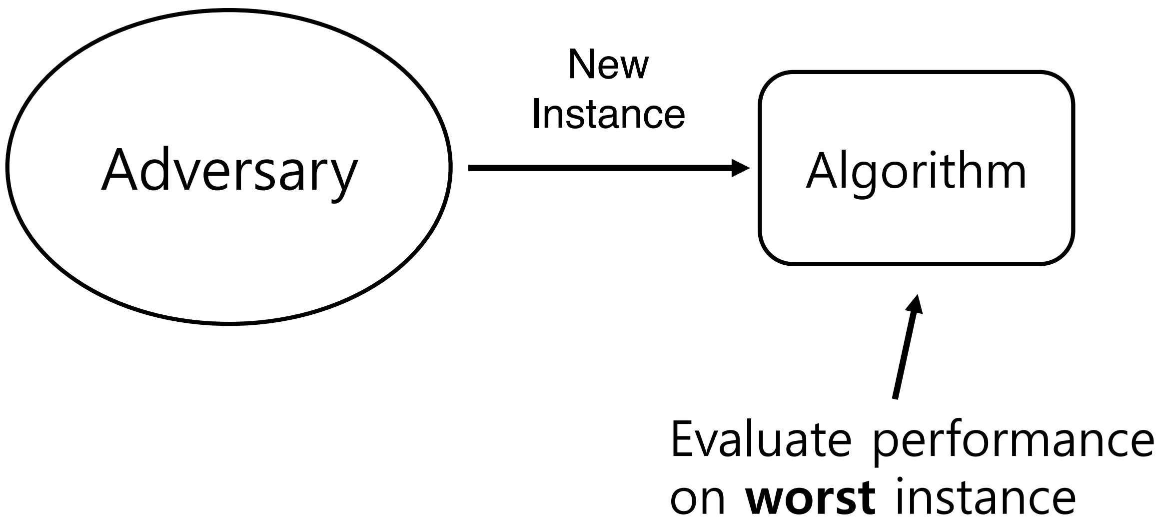
- Run time binary search O(log n)
- Run time prediction O(log η)

- Perfect predictions give constant lookup
- Worst case is the same as the best classical algorithm
    - Gracefully degrades to the worst case
- Omitted empirical results show predictions using little space can give much faster lookups

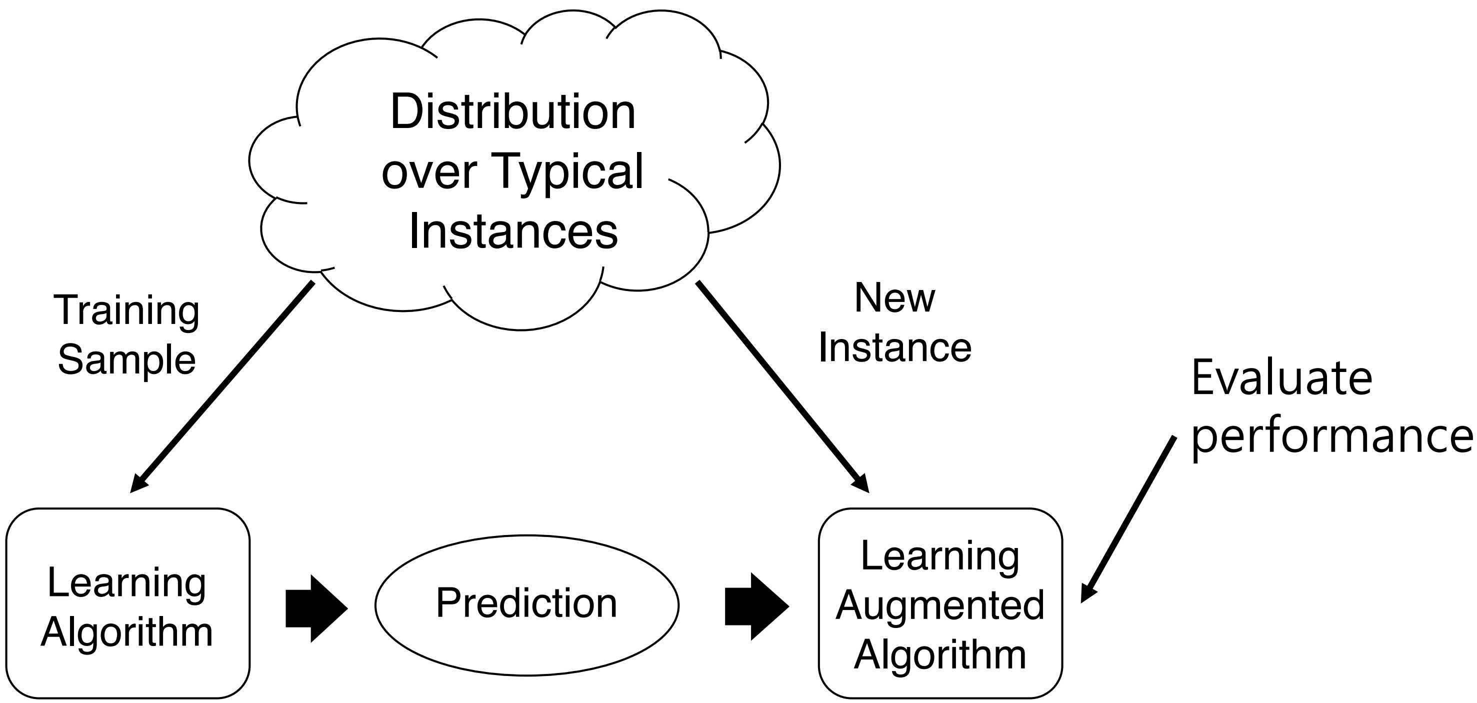
Learning Augmented Algorithms

- Punchline:
  - Machine learning can be combined with classical algorithms to obtain better results
  - Gives us new widely applicable models for beyond worst-case analysis

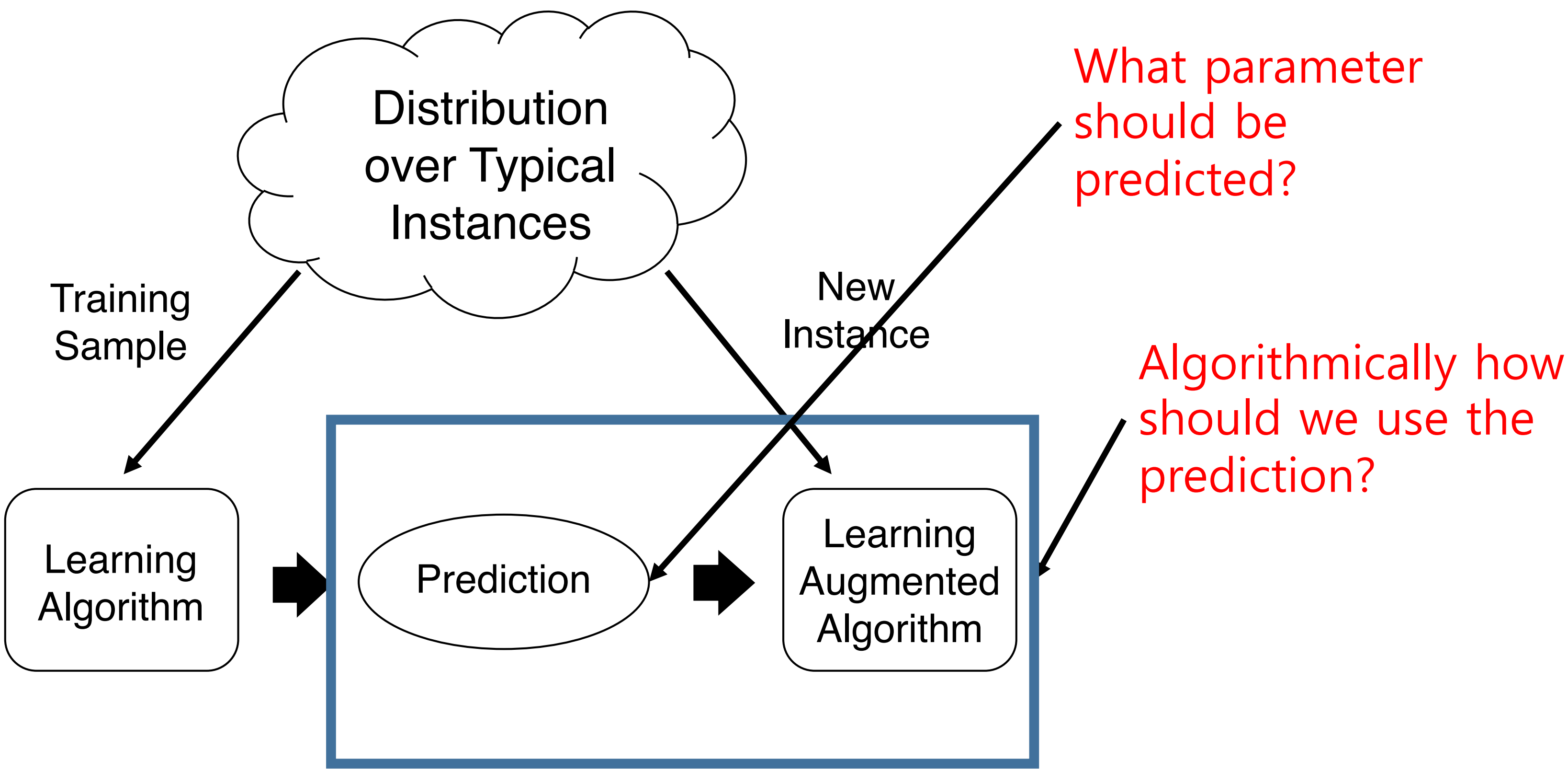
Worst-Case Analysis



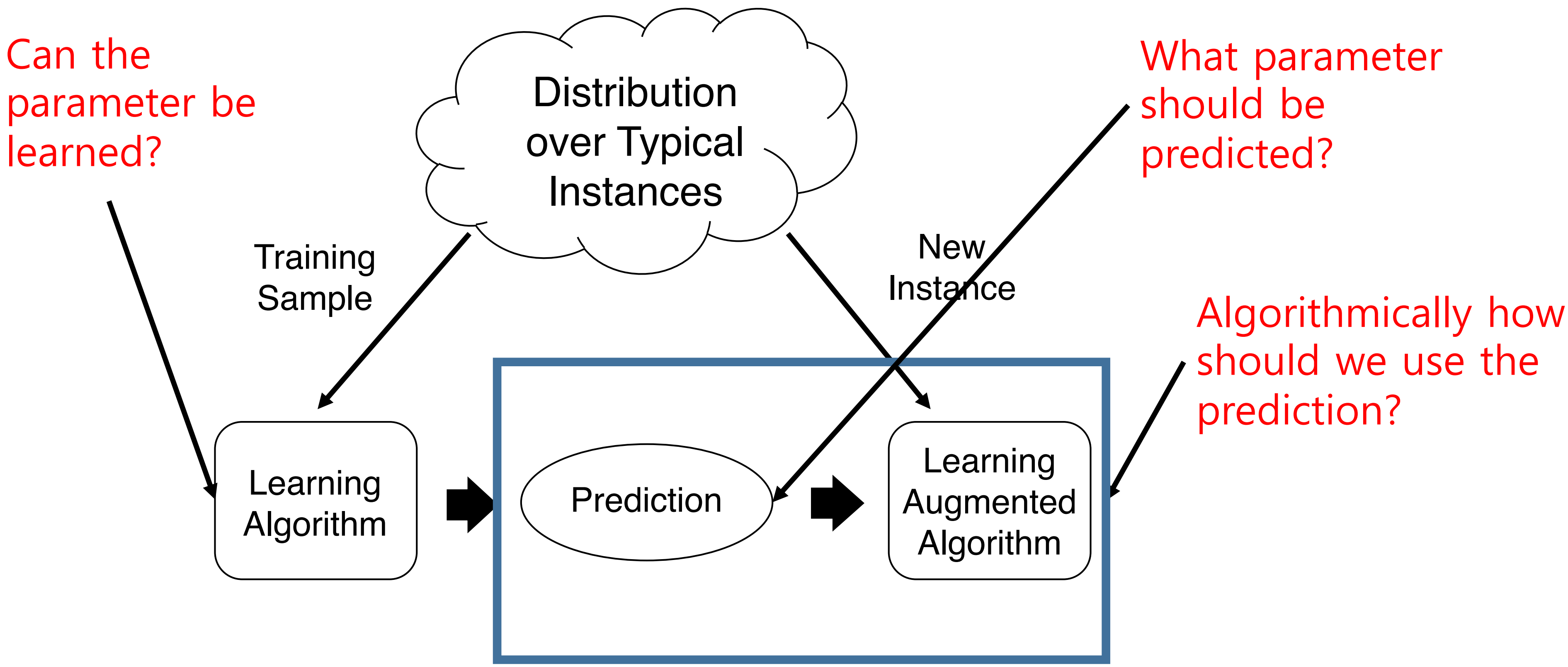
Learning Augmented Algorithms



Learning Augmented Algorithms



Learning Augmented Algorithms



Current Status

ERL: Desirable Analysis Framework

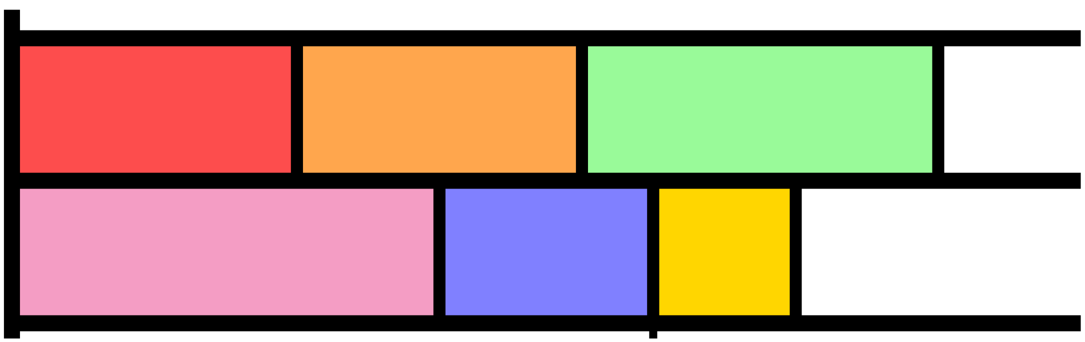
- **Existence:** Predictions should allow the algorithm to go beyond worst-case bounds
  - Location in the array
  - What to predict is often the main question
- **Robustness**: Algorithms are robust to minor changes in the problem input
  - Algorithm is robust to incorrect location in the array
- **Learnability**: Predictions should be learnable if data is coming from a distribution
  - Example: PAC-Learning

Beyond Worst-Case Analysis Frameworks

- Online algorithm design
    - Competitive ratio parameterized by error in the predictions
- Running time
    - Worst case run time parameterized by error in the predictions

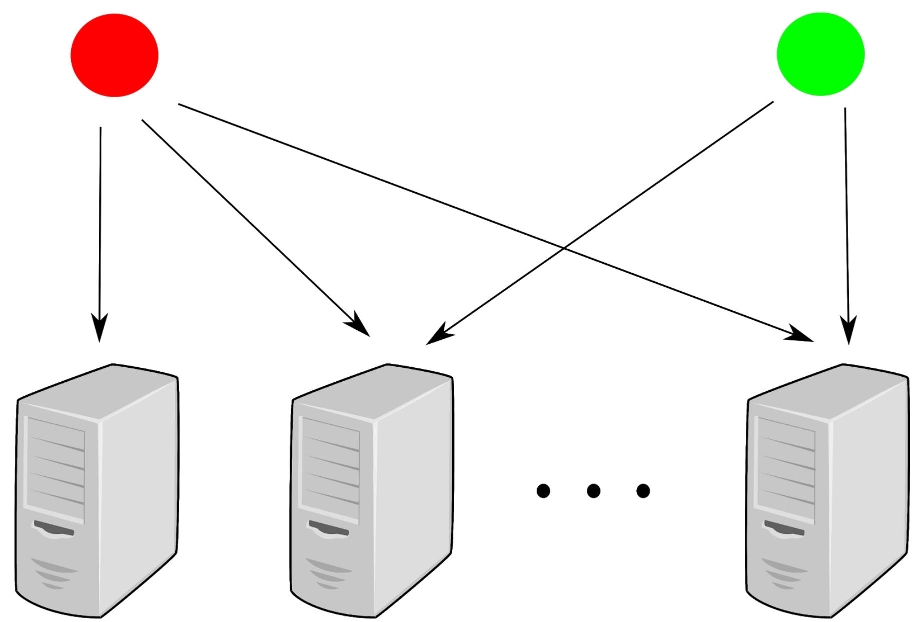
Online Restricted Assignment Makespan Minimization

- Client Server Scheduling
  - Processed in m machines in the **restricted assignment** setting (some results hold for **unrelated machines**)
  - Jobs arrive over time in the **online-list** model
    - **All arrive at time 0**
    - **Jobs revealed one at a time**
  - Assign jobs to the machines to minimize **makespan**



Restricted Assignment Makespan Minimization

- m machines
- n jobs
  - Online list: a job must be immediately assigned before the next job arrives
  - N(j): feasible machines for job j
  - p(j): size of job j (complexity essentially the same if ***unit sized***)
- Minimize the maximum makespan
  - Optimal makespan is T



Online Competitive Analysis Model

- c-competitive $$\frac{ALG(I)}{OPT(I)} \leq c$$
- Worst case relative performance on each input I

- Problem well understood:
  - A $\Omega(\log m)$ lower bound on any online algorithm
  - Greedy is a $O(\log m)$ competitive algorithm [Azar, Naor, and Rom 1995]

Beyond Worst Case via Predictions

- Reasonable assumption:
  - Access to last week's job sequence
  - Predict the future based on the past.
  - What should be predicted?
  - How can it be used?

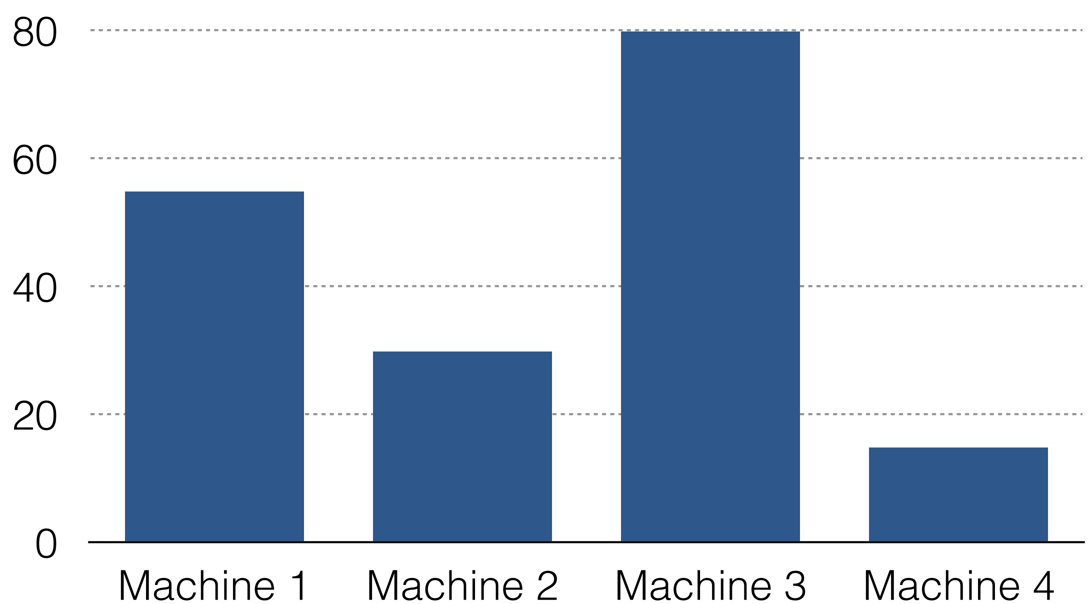
Existence

- First show natural predictions that fail
- Next give a good parameter to predict

What (not) to Predict?

- **Number of jobs assigned to machines** in the optimal solution?
  - Perhaps we can identify the contentious machines?

optimal solution

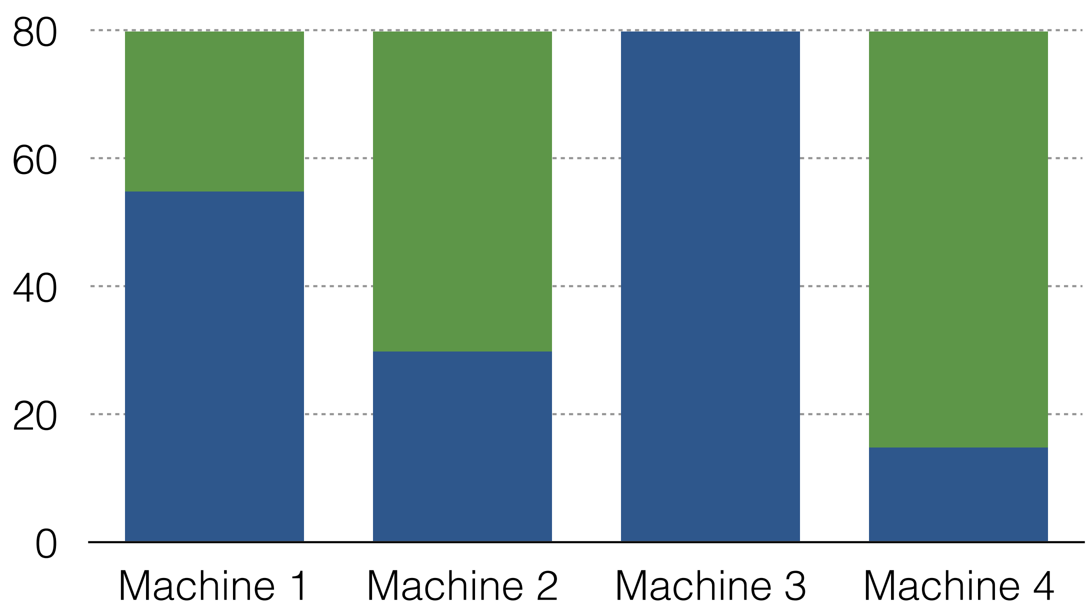


makespan 80

What (not) to Predict?

- **Load** of the machines in the optimal solution?
  - Perhaps we can identify the contentious machines? **No**

optimal solution



new instance padded with dummy jobs

loads the **same**

What (not) to Predict?

- **Number** of jobs that can be assigned to a machine?
  - Perhaps machines that can be assigned more jobs are more contentious?

What (not) to Predict?

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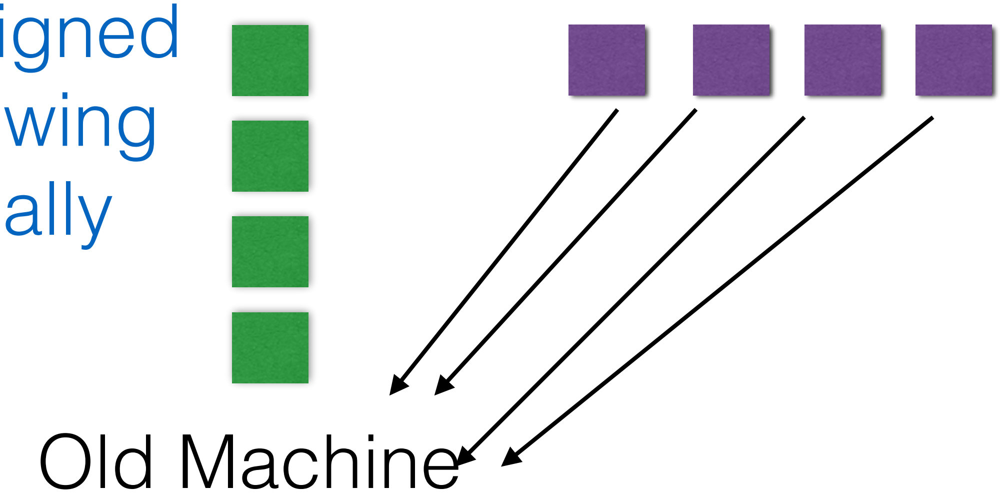
New jobs can be assigned to old machines, skewing ‘degrees’ adversarially



What (not) to Predict?

- Distribution on job types

- Is this the best predictive model?
  - 2^m job types possible
  - Perhaps not the right model if information is sparse

What (not) to Predict?

- Predict **dual variables**
- Known to be useful for **matching** in the **random order model** [Devanur and Hayes, Vee et al.]
  - Read a portion of the input
  - Compute the duals
  - Prove a primal assignment can be (approximately) constructed from the duals online
  - Use duals to make assignments on remaining input

What (not) to Predict?

- Predict **dual variables** for makespan scheduling
  - Can derive primal based on dual
  - Sensitive to small error (e.g. changing a variable by a factor of $1+1/\text{poly}(n)$ has the potential to drastically change the schedule)

What to Predict?

- Idea: capture **contentiousness** of a machine
  - Seems like the most important quantity besides types of jobs

Prediction: Machine Weights

- Predict a weight for each machine
  - **Single** number (compact)
  - Lower weight means more restrictive machine
  - Higher weight less restrictive
- Framework:
  - Predict machine **weights**
  - Using to construct **fractional** assignments online
  - **Round** to an **integral** solution online

Fractional Assignments via Weights

- Each machine i has a weight w_i

- Job j is assigned to machine i **fractionally** as follows:

$$x_{i,j} = \frac{w_i}{\sum_{i' \in N(j)} w_{i'}}$$

Existence

- **Theorem (existence of weights)**: Let T be optimal max load. For any $\varepsilon > 0$, there **exists** machine weights such that the resulting fractional max load is at most $(1+\varepsilon)T$.

- **Theorem (rounding assignments)**: There exists an online algorithm that takes as input fractional assignments and outputs integer assignments for which the maximum load is bounded by $O((\log\log(m))^3 T')$, where T' is maximum fractional load of the input. The algorithm is randomized and succeeds with probability at least $1 - 1/m^c$

- **Theorem (tightness of rounding)**: Any randomized online rounding algorithm has worst case load at least $\Omega(T' \log \log m)$

- **Large makespan case:** [fractional makespan larger than $\log(m)$]

    - Randomized rounding gives gives a $(1+\varepsilon)T'$ where T' is maximum fractional load of the input with probability at least $1 - 1/m^c$.

Parameter Robustness

- Predict a parameter
- η is the l_k-norm error in the prediction for some k
- Prove algorithm is $f(\eta)$ competitive
- Pros
  - Often can show desirable trade-off guarantees
- Cons
  - Difficult to compare across parameters

Results on Robustness

- **Theorem:** Given predictions of the machine weights with **maximum relative error** $\eta > 1$, there exists an online algorithm yielding fractional assignments for which the fractional max load is bounded by $O(T \min\{\log(\eta), \log(m)\})$.

- **Corollary**: There exists an $O(\min\{(\log\log(m))^3 \log(\eta), \log m\})$ competitive algorithm for restricted assignment in the online algorithms with learning setting

Other Robustness

- Additional robustness model
  - Instance robustness

Learnability Model

- Unknown distribution model $\mathscr{D}$
  - Instance drawn from unknown distribution
  - Best prediction $y^* := \texttt{argmax}_y \mathbb{E}_{\mathcal{I} \sim \mathcal{D}}[ALG(\mathcal{I}, y)]$
- How many samples s to compute $\hat{y}$ giving the following performance with high probability

$$\mathbb{E}_{\mathcal{I} \sim \mathcal{D}}[ALG(\mathcal{I}, \hat{y})] \geq (1 - \epsilon)\mathbb{E}_{\mathcal{I} \sim \mathcal{D}}[ALG(\mathcal{I}, y^*)]$$

Learnability Model

- Similar to
  - PAC learning
  - Data-driven algorithm design
- Alternative: competitive analysis
  - Show a small number of samples needed for the following performance with good probability

$$\mathbb{E}_{\mathcal{I}\sim\mathcal{D}}[ALG(\mathcal{I},\hat{y})] \geq (1-\epsilon)\mathbb{E}_{\mathcal{I}\sim\mathcal{D}}[OPT(\mathcal{I})]$$

Learnability

- **Theorem:** Let $\mathscr{D}$ be a product distribution such that $\mathbf{E}_{S \sim \mathscr{D}}[OPT(S)] \geq \Omega(\log m)$. There exists an algorithm that constructs **nearly optimal** weights using a polynomial number of samples in m.

Summary for Restricted Assignment

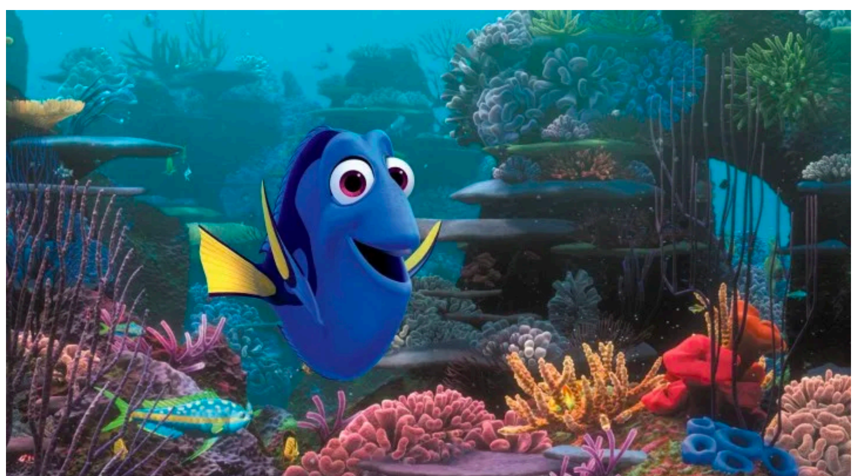
- Existence
  - Weights
- Robustness
  - Parameter and Instance Robustness
- Learnability
  - Low sample complexity

Predictions for Online Algorithms

- Lots of success for online algorithm design
  - Matching
  - Caching
  - Ski-rental
  - Scheduling
  - Online learning
  - Heavy hitters
- What about the original question of speeding up algorithms offline?

Warm-Start

- Many problems are solved repeatedly on ‘similar’ instances
  - e.g. scheduling yesterday versus today
- We solve from scratch



Framework

- Problem instances $X_1, X_2, \ldots$ are drawn from an unknown distribution $\mathcal{D}$

- Learn a starting summary S

- Design an algorithm that runs faster when given S

ERL Framework Pitfalls

- Existence: What to predict?
- Robustness
  - Feasibility: The warm start may not be feasible
  - Optimization: The warm start may not be useful
- Learnability: The starting solution may not be learnable

Weighted Bipartite Matching

- Input a bipartite graph $G = (L \cup R, E)$ with edge costs $c_{i,j}$
- Output the minimum cost perfect matching

Existence
What to Predict?

- Idea 1: Edges in optimal solution
  - Brittle
- Idea 2: LP duality

Existence

Primal

$$\min \sum_{e \in E} c_e x_E$$

$$\text{subject to:} \sum_{e \in N(i)} x_e = 1 \qquad \forall i \in V$$

$$x_e \geq 0 \qquad \forall e \in E$$

Dual

$$\max \sum_{i \in V} y_i$$

$$\text{subject to: } y_i + y_j \leq c_{ij} \qquad \forall (i,j) \in E$$

- Dual:
  - Assigns prices to vertices
- Complementary slackness
  - Edges in the matching have tight dual constraints

Existence

Primal

$$\min \sum_{e \in E} c_e x_E$$

$$\text{subject to: } \sum_{e \in N(i)} x_e = 1 \qquad \forall i \in V$$

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$$\max \sum_{i \in V} y_i$$

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- Hungarian algorithm (popular in practice)
  - Start with dual values at 0
  - Compute max cardinality matching on tight edges
  - If not done, find a set violating Hall's theorem. Update duals

Existence

Primal

$$\min \sum_{e \in E} c_e x_E$$

$$\text{subject to: } \sum_{e \in N(i)} x_e = 1 \qquad \forall i \in V$$

$$x_e \geq 0 \qquad \forall e \in E$$

Dual

$$\max \sum_{i \in V} y_i$$

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- Hungarian algorithm (popular in practice)
  - Predict dual values
  - Compute max cardinality matching on tight edges
  - If not done, find a set violating Hall's theorem. Update duals

Robustness
Main Idea

Idea:

- Predict the dual values, i.e. predict $\hat{y}_i$
- "Warm start" Hungarian algorithm from predicted duals.

Feasibility issue:

- Hungarian algorithm slowly increases duals. Always has a feasible solution
- But, predicted dual may be infeasible
- Have an edge s.t.: $\hat{y}_i + \hat{y}_j > c_{ij}$

Approach:

- Minimally reduce predicted duals to attain feasibility
- Must do it quickly (since speed is of the essence)

Robustness
Making Duals Feasible

- Write LP for the feasibility problem:

$$\min \sum_{i \in V} \delta_i$$
$$\text{subject to: } \delta_i + \delta_j \geq (\hat{y}_i + \hat{y}_j - c_{ij})^+ \qquad \forall (i,j) \in E$$
$$\delta_i \geq 0 \qquad \forall i \in V$$

Algorithm (greedy):

- Pick any vertex i. Set its δ_i value to the minimum that satisfies all of the constraints
- Remove i from the graph and repeat.
- Theorem: Resulting solution is a 2-approximation for the LP, runs in linear time!

Overview

Existence:

- Predict the dual values, i.e. predict $\hat{y}_i$
- "Warm start" Hungarian algorithm from predicted duals.

Feasibility:

- Quickly round predicted duals $\hat{y}_i$ to feasible ones, y_i' .

Optimization:

- Run Hungarian algorithm starting from rounded duals, y_i'.

Learnability:

- Can show duals have small sample complexity.

Robustness

Overall approach:

- Obtain (learn) duals: $\hat{y}_1, \ldots, \hat{y}_n$
- Given a new matching instance, $G = (V, E)$ find feasible duals $y'_1, \ldots, y'_n$
- Run Hungarian method starting with $y'_1, \ldots, y'_n$

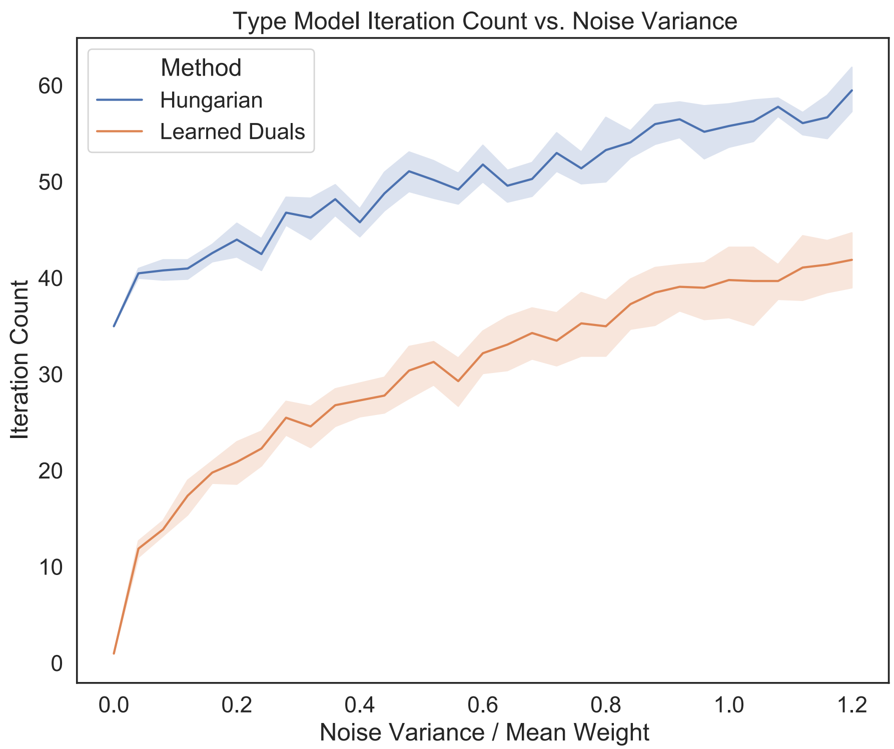
Theorem: The overall running time is: $O(\|\hat{y} - y^*\|_1) \cdot m\sqrt{n}$

- Strictly better when the error is small
- Can prove that it's no worse than vanilla Hungarian algorithm

Does it Work?

Experiment 1(a):

- Start with a bipartite graph with a planted min cost perfect matching
- Generate new instances by adding random noise of increasing magnitude to the edge weights

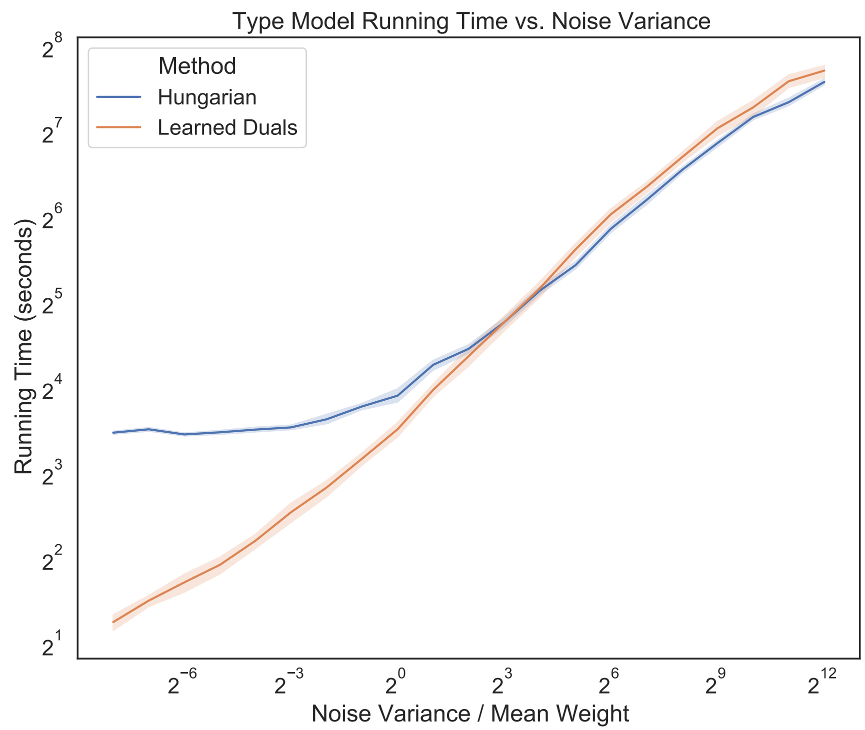


- When noise is low, learning approach dominates.

Does it Work?

Experiment 1(b):

- Start with a bipartite graph with a planted min cost perfect matching
- Generate new instances by adding random noise of increasing magnitude to the edge weights

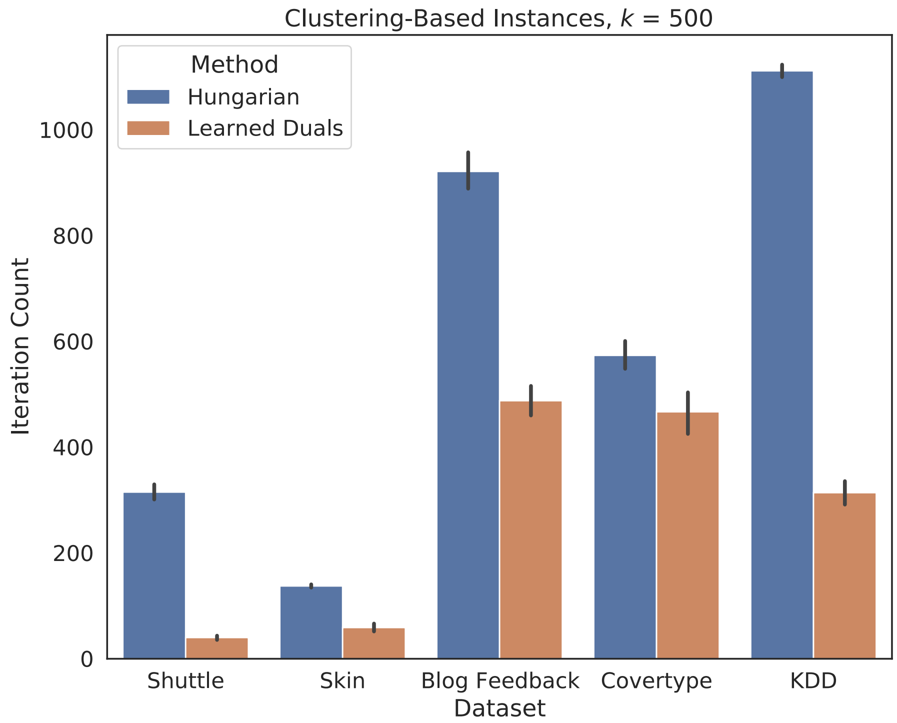


- When noise gets high, nothing to be learned, so converge to Hungarian method.

Does it Work?

Experiment 2:

- Perfect matching problems derived from geometric datasets

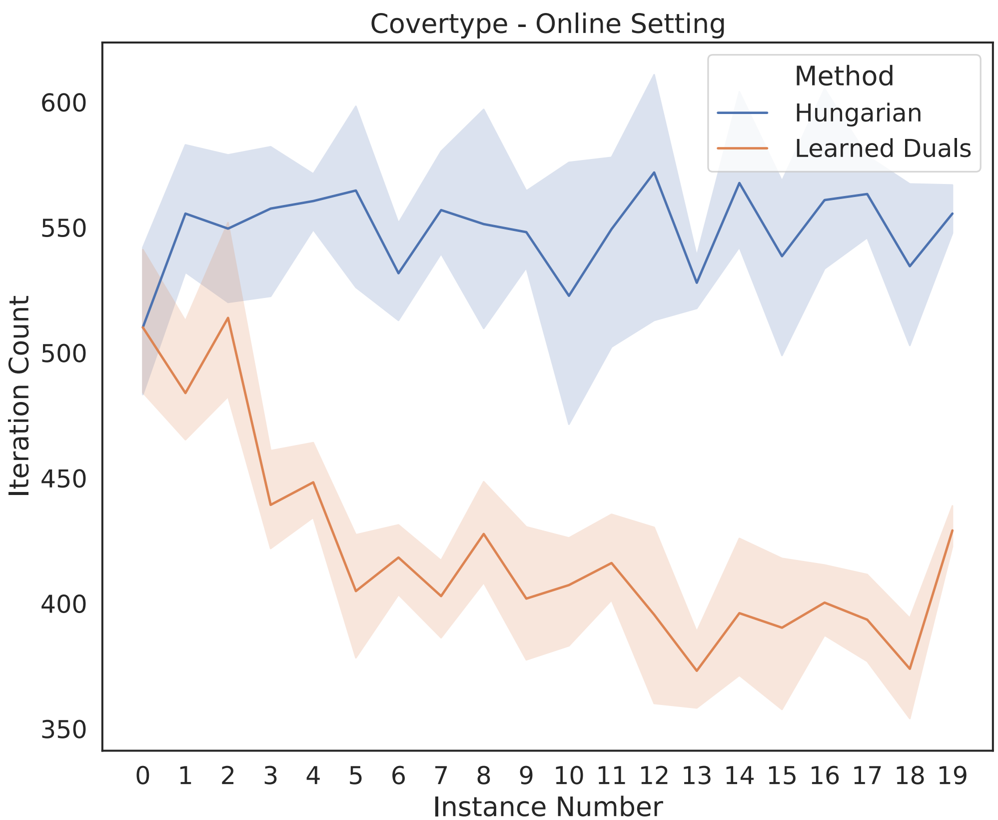


- Learned gains can be substantial (10x in some cases)

Does it Work?

Experiment 3:

- How many samples do you need to learn?



- Many fewer than the theory predicts

Future Work

- How useful is this new paradigm empirically and theoretically
  - **Rich area: Online algorithms to cope with uncertainty, running time off-line, other applications?**

Thank you!

Questions?