Educational Timetabling: Problems, Benchmarks, Algorithms, and Practical Issues

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Outline of the Talk

1. Educational Timetabling: Problems & Benchmarks
2. Algorithms: Local Search
3. Experimental Results
4. Practical Issues
5. Discussion and Conclusions

History of Timetabling

Pioneering articles:

- Gotlieb (1963). The Construction of Class-Teacher Timetables.
- Csima (1965). Investigations on a Timetable Problem.
- de Werra (1971). Construction of School Timetables by Flow Methods.

NP-completeness:

- Even, Itai, & Shamir (1976). On the Complexity of Timetabling and Multicommodity Flow Problems.

Early surveys:

- Schmidt & Strohlein (1980). Timetable Construction — An Annotated Bibliography [200+ papers].
- de Werra (1985). An Introduction to Timetabling.
- Schaerf (1999). A Survey of Automated Timetabling.

Current Activities

- PATAT conference series (`https://patatconference.org`):
  - established in 1995
  - the 13th edition will be in Bruges in August 2022
- International Timetabling Competitions:
  - First competition in 2002
  - Latest one (5th) in 2021
- PATAT EURO Working Group on Automated Timetabling (`https://patat.cs.kuleuven.be`)

Educational Timetabling

"Assign teacher/student meetings to timeslots and rooms"

Main problems:

- High school timetabling
- University course timetabling
- University examination timetabling

Others:

- Event/Conference timetabling
- Student sectioning
- Balance academic curriculum

Non-Educational Timetabling Problems

- Employee Timetabling: nurses, call centers, assembly lines, ...
- Train Timetabling: scheduling, platforming, ...
- Sport Timetabling: round robin tournaments, ...

Motivations for Standards & Benchmarks in Timetabling

Early papers in timetabling:

- Define a brand-new problem
- Apply the authors' favorite technique
- Compare with the straw man:
  - Manual solution
  - Authors' naive implementation of an alternative technique

Required Steps for Better Timetabling Research

- Standard formulations:
  - general vs. specific
  - simplified vs. realistic
- Benchmark instances: repositories, language, format, ...
- Reproducibility: solution checker, execution checker
- Statistical tests for comparisons

Standards & Benchmarks for Timetabling

High School Timetabling:

- "Standard" formulation proposed in 2008
- By researcher from Netherlands, Australia, England, Finland, Brazil, Greece, Austria, and Italy
- Very general (and quite complex) formulation
  "No concession to judicious simplifications"
- Uses an XML data format
- 40+ instances available at present
- Repository: `https://www.utwente.nl/ctit/hstt/`

Standards & Benchmarks for Timetabling

Examination Timetabling:

- Uncapacitated formulation [Carter *et al*, 1996]
  - 13 real-world benchmark instances: (No. of exams: 81 – 2419)
  - Still not solved to optimality
  - Extended by other researchers (by adding data)
- New complex formulation, with 12 instances [McCollum *et al*, 2007]
- Many others [Müller, 2016; Battistutta *et al*, 2020]

Standards & Benchmarks for Timetabling

Course Timetabling:

- Post-Enrolment (PE-CTT) [Paetcher, 2002]
  - many artificial instances available
- Curriculum-Based (CB-CTT) [Di Gaspero & Schaerf, 2002]
  - 50+ real-world instances from many universities
  - high-quality generator [Lopes & Smith-Miles, 2010]
- Other formulations
  - Rich, structured, and real-world [Müller *et al*, 2020]
  - ...

International Timetabling Competitions (1/2)

1. ITC-2002
   - Problem: PE-CTT
   - Fixed deadline: 6 months for writing the solver
   - Instances: 20 (10 Early + 10 Late), artificial
   - CPU time limit ($\approx$ 10min)
   - Final place-list based on average scores on soft constraints
2. ITC-2007
   - Three tracks: Exams, PE-CTT (revised), CB-CTT
   - More realistic formulations and real-world data
   - Early, Late & Hidden instances
   - Finalists' software re-run by organizers $\rightarrow$ No *Mongolian Horde!*
   - Adjudication based on ranks on 10 runs per instance

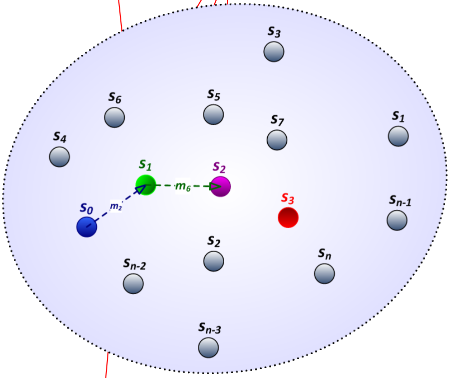
International Timetabling Competitions (2/2)

1. ITC-2011
   - On high-school timetabling
   - Rules similar to ITC-2007
   - A separate competition on best scores
2. ITC-2019
   - Complex structured problem: course timetabling + student sectioning
   - Adjudication on best scores only (no timeout, no code)
3. ITC-2021
   - Sport timetabling problem
   - Same rules of ITC-2019

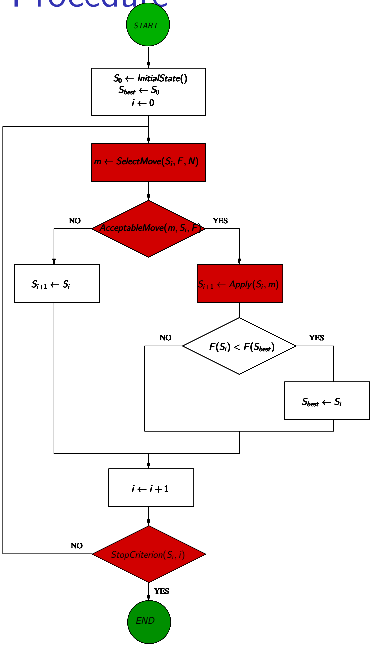
Outline of the Talk

Local Search Procedure

neighborhood analysis and move selection



initial state



Simulated Annealing with Cut-off

```
procedure SimulatedAnnealing(SearchSpace 𝒮, Neighborhood 𝒩,
                             CostFunction F, Parameters T_0, T_f, α, N_s, N_a)
    T ← T_0
    s ← RandomState(𝒮)
    s_best ← s
    while T ≥ T_f
        n_s ← 0; n_a ← 0
        while n_s < N_s ∧ n_a < N_a
            m ← RandomMove(s, 𝒩)
            ΔF ← F(s ⊕ m) − F(s)
            if (ΔF ≤ 0)
                s ← s ⊕ m; n_a ← n_a + 1
                if (F(s) < F(s_best))
                    s_best ← s
            else
                if (RandomReal(0,1) < e^(−ΔF/T))
                    s ← s ⊕ m; n_a ← n_a + 1
            n_s ← n_s + 1
        T ← T · α
    return s_best
```

Curriculum-Based Course Timetabling Problem

Basic Entities:

- Courses & Lectures: Databases (3 times a week), ...
- Periods & Days: Mon_8:30-10:30, ..., Fri_16:30-18:30
- Rooms: A (312 seats), N (25 seats), ...
- Curricula:
  - Civil Eng. (II year): Math2, Structural Eng., ...
  - Mechanical Eng. (III year): Math3, Metallurgy, ...
- No student enrolment matrix

Curriculum-Based Course Timetabling Problem:

Cost Components:

- Conflicts (curriculum based) $\boxed{\text{Hard}}$
- Room Occupancy $\boxed{\text{Hard}}$
- Teacher Availability $\boxed{\text{Hard}}$
- Room Capacity $\boxed{\text{Soft}}$
- Minimum Working Days $\boxed{\text{Soft}}$
- Isolated Lectures $\boxed{\text{Soft}}$
- Room Stability $\boxed{\text{Soft}}$

"Simplifications a-go-go (not necessarily judicious!)"

Solution by Local Search

Search space: ▸ Example

Neighborhoods:

MoveLecture: Move one lecture to a new period and a new room

SwapLectures: Swap period and room of two lectures

Search method:

Simulated Annealing

Simulated Annealing

Sampling: Draw a random move

Acceptance rule:

- if $\Delta \leq 0$ always accept m
- if $\Delta > 0$ accept m with probability $e^{-\Delta/T}$

Cooling scheme: after either N_s iterations or N_a accepted moves:

$T := \alpha \cdot T$

Stop criterion: $T = T_f$

Parameters:

- Initial and final temperatures (T_0 and T_f)
- Cooling rate (α)
- Sampled moves per temperature (N_s)
- Accepted moves per temperature (N_a)
- Ratio: MoveLecture/SwapLectures (ρ)
- Weight of hard constraints (H)

Outline

Parameter tuning

- Fixed number of iterations $\mathcal{I}$: compute N_s from the other parameters
- Parameters: T_0, T_f, α, $\sigma = N_a/N_s$, ρ, w_H
- DoE: Hammersley point set [Hammersley & Handscomb, 1964]
  ▸ Example
- F-Race procedure [Birattari *et al*, 2010]
- Friedman rank-sum and Wilcoxon tests
- Software tool: `json2run` [Urli, 2013]

Results (fixed time) on comp Instances

Instance	Müller	LüHao	Abdullah	SA (us)	Best
01	**5.00**	**5.00**	**5.00**	**5.00**	5
02	61.30	60.60	53.90	**53.00**	24
03	94.80	86.60	84.20	**79.03**	66
04	42.80	47.90	51.90	**38.33**	35
05	343.50	**328.50**	339.50	365.20	290
06	56.80	69.90	64.40	**50.40**	27
07	33.90	28.20	**20.20**	23.80	6
08	46.50	51.40	47.90	**43.60**	37
09	113.10	113.20	113.90	**105.07**	96
10	21.30	38.00	24.10	**20.57**	4
11	**0.00**	**0.00**	**0.00**	**0.00**	0
12	351.60	365.00	355.90	**340.57**	300
13	73.90	76.20	72.40	**71.37**	59
14	61.80	62.90	63.30	**57.93**	51
15	94.80	87.80	88.00	**78.83**	66
16	41.20	53.70	51.70	**34.83**	18
17	86.60	100.50	86.20	**75.70**	56
18	91.70	82.60	85.80	**80.80**	62
19	68.80	75.00	78.10	**67.03**	57
20	**34.30**	58.20	42.90	38.87	4
21	108.00	125.30	121.50	**100.10**	75
Avg	87.22	91.26	88.13	**82.38**	63.71

† Best contributions up to 2012

Application to the Post-Enrolment Formulation

- Main difference:

  Room assignment does not contribute to the objective function
- Preprocessing:

  Identify Any-room events, which fit in any room
- Neighborhoods:
  - Select the period
  - Assign the room deterministically: to the "least attractive" one
- Comment: no matching

Main Results for Post-Enrolment Course Timetabling

Instance	Cambazard		Lewis	Mayer	SA (us)
1	830	547	1492	613	**399.2**
2	924	403	1826	556	**142.2**
3	224	254	457	680	**209.9**
4	352	361	589	580	**349.6**
5	**3**	26	193	92	7.7
6	14	16	689	212	**8.6**
7	11	8	421	**4**	4.9
8	**0**	**0**	206	61	1.5
9	1649	1167	2312	**202**	258.8
10	2003	1297	2262	**4**	186.4
11	311	361	541	774	**269.5**
12	408	**380**	741	538	400.0
13	**89**	135	631	360	120.0
14	**1**	15	660	41	3.6
15	80	47	344	**29**	48.0
16	**19**	58	194	101	50.1
Avg	432.4	317.2	847.4	302.9	**153.7**

† Best contributions up to 2011

Application to Uncapacitated Examination Timetabling

- Larger set of neighborhoods: Kempe chain, multi-swap, kick ...
- Principled neighborhood portfolio approach
- Large number of parameters: multi-stage tuning

Main Results for Uncapacitated Examination Timetabling

	Yang & Petrovic (2005)		Burke & Bykov (2008)		Burke *et al* (2010)		Fong *et al* (2015)		Leite *et al* (2018)		Burke & Bykov (2016)	Mandal *et al* (2020)		SA (us)	
Inst.	min	avg	min	avg	min	avg	min	avg	min	avg	avg	min	avg	min	avg
car91	4.5	4.53	4.58	4.68	4.6	4.9	4.79	4.85	5.31	5.46	5.19	4.58	4.72	4.38	4.44
car92	3.93	3.99	3.81	3.92	3.9	4.1	3.89	4.27	4.27	4.37	4.23	3.82	3.93	3.75	3.80
ear83	33.71	34.87	32.65	32.91	32.8	34.1	33.43	34.48	33.21	33.81	33.69	33.23	34.49	32.61	32.89
hec92	10.83	11.36	10.06	10.22	10	10.6	10.49	10.61	10.11	10.2	10.36	10.32	11.09	10.05	10.16
kfu93	13.82	14.35	12.81	13.02	13	13.4	13.72	13.76	13.34	13.42	13.43	13.34	13.97	12.87	13.06
lse91	10.35	10.78	9.86	10.14	10	10.8	10.29	10.39	10.22	10.45	10.41	10.24	10.62	9.92	10.09
pur93			4.53	4.71					6.17	6.24	4.82			4.22	4.32
rye93	8.53	8.79	7.93	8.06					8.65	8.72	8.45	9.79	10.29	7.99	8.10
sta83	151.52	158.02	157.03	157.05			157.07	157.37	157.03	157.03	157.07	157.14	157.64	157.03	157.05
tre92	7.92	8.1	7.72	7.89	7.9	8.2	7.86	8.04	8.3	8.36	8.16	7.74	8.03	7.72	7.85
uta92	3.14	3.2	3.16	3.26	3.2	3.4	3.1	3.31	3.59	3.64	3.52	3.13	3.22	3.05	3.13
ute92	25.39	26.1	24.79	24.82	24.8	25	25.33	26.04	24.84	24.87	24.9	25.28	26.04	24.76	24.82
yor83	36.53	36.88	34.78	36.16	34.9	36.6	36.12	36.83	35.49	36.38	36.65	35.68	36.79	34.56	34.93
Time	min	max	min	max	min	max	min	max	min	max	avg	max		min	max
(secs)	740	2773	450	901	28	3084	178	588	1020	18000	300	3600		127.8	2055.9

† Best contributions up to Today, solutions available at `https://opthub.uniud.it`

Main Results for ITC-2007 Examination Timetabling

Ins.	McCollum *et al*		Bykov & Petrovic		Hamilton-Bryce		Alzaqebah		SA (us)	
	$\bar{f}$	F%	$\bar{f}$	F%	$\bar{f}$	F%	$\bar{f}$	F%	$\bar{f}$	F%
1	4799	100	4008	100	5469	100	5517	100	**3950**	100
2	425	100	404	100	450	100	538	100	**402**	100
3	9251	100	**8012**	100	10444	100	10325	100	8827	78
4	15821	100	**13312**	100	20241	100	16589	100	—	0
5	3072	100	**2582**	100	3185	100	3632	100	2697	91
6	25935	100	**25448**	100	26150	100	26275	100	25912	95
7	4185	100	3893	100	4568	100	4592	100	**3727**	100
8	7599	100	**6944**	100	80812	100	8328	100	7734	100
9	1071	100	**949**	100	1061	100	—	—	981	100
10	14552	100	**12985**	100	15294	100	—	—	13880	100
11	29358	100	**25194**	100	44820	100	—	—	29788	32
12	5699	100	**5181**	100	5464	100	—	—	5454	88

† Best contributions up to 2016

Comments on Results

- We use SA, a rather simple and "old-fashioned" technique (see [Franzin & Stützle, 2019])
- It outperforms many "modern" ideas (see [Sörensen, 2012])
- Complex neighborhood structures are needed
- A systematic, comprehensive, and statistically-principled tuning is crucial

Outline of the Talk

Practical Timetabling at University of Udine

- Course timetabling: Many more objectives
  - Student's Workload
  - Double Lectures
  - Room Suitability
  - Preassignments
  - Lectures in external rooms
  - Consecutive days for teachers (commuters)
  - Lunch break
  - Max daily lectures for teacher
  - Simultaneity of courses
  - Fairness [Mühlenthaler, PhD, 2014]
  - ...
- Examination timetabling: "Italian" formulation [Battistutta et al, 2020]
  - No student enrolment (curriculum-based)
  - Multiple rooms for one exam
  - Teachers' preferences
  - Written and oral parts (in different days)

Practical Considerations and Recommendations

- Considerations:
  1. Hard to keep apart combinatorial, psychological, political aspects
  2. Post-publication changes are very time-consuming
  3. Users do not:
     - feel the combinatorial complexity
     - understand the objective function
- Recommendations:
  1. Convince teachers to specify requirements in advance!
  2. Limit preassignments as much as possible
  3. The graphical interface is at least as important as the solver
  4. Hide the objective function (do not search consensus upon it)

Outline of the Talk

Future Trends in Timetabling (IMHO)

1. New requirements for COVID-19 emergency
2. Standard timetabling languages (JSON, XML), parsers, generators, ...
3. Benchmark repositories (and solution checkers)
4. Feature-based algorithm selection
5. Interactive rescheduling ("post-publication" tools)
6. Commercial software with state-of-the-art optimizers inside

Thank you!

A search state

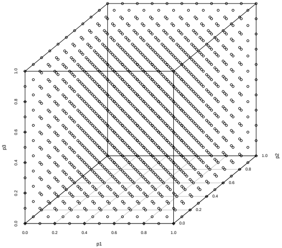
Periods

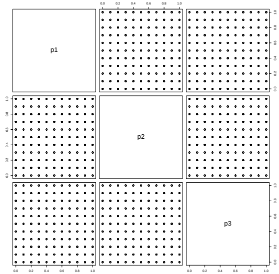
		Mon					Tue			
Courses	c1	—	R1	—	×	—	R3	—	—	...
	c2	R1	—	—	—	R1	—	×	R1	...
	c3	—	R3	R8	—	—	R9	—	R1	...
	c4	—	—	—	—	R3	—	—	—	...
	c5	—	—	×	×	—	—	—	R2	...

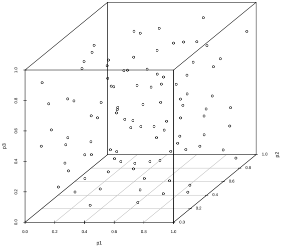
- Conflicts Violations: Possible
- Room Occupancy Violations: Possible
- Teacher Availability Violations: Never

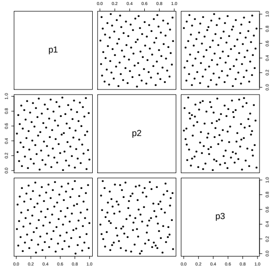
Go back

Full Factorial (1000 points) vs. Hammersley (100 points)









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